On Soft B- Open Sets In Soft Bitopological Space

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Abstract
In this paper, we introduce (1,2)*-soft b-open sets and (1,2)*-soft b-closed sets in soft bitopological space and exhibit the properties of (1,2)*-soft regular-open, (1,2)*-soft preopen, (1,2)*-soft semi open, (1,2)*-soft α-open, (1,2)*-soft β-open in soft bitopological space. Also we study about (1,2)*-soft b-interior, (1,2)*-soft b-closure in soft bitopological space and analyze the relation between other functions.

Keywords: (1,2)*-soft b-open set, (1,2)*-soft b-closed set, (1,2)*-soft regular-open, (1,2)*-soft preopen, (1,2)*-soft semi open, (1,2)*-soft α-open, (1,2)*-soft β-open, (1,2)*-soft b-interior, (1,2)*-soft b-closure.

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Introduction
In the year 1999, Russian researcher Molodtsov [7], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modelling problems in computer science, engineering physics, economics, social sciences and medical sciences. In 2003, Maji, Biswas and Roy [8], studied the theory of soft sets. They discussed the basic soft sets definition with examples. In 2011, Muhammad Shabir and Munazza Naz [12] defined the theory of soft topological over an initial universe with a fixed set of parameters. N. Cagmen and S. Karatas [5] introduced a topology on a soft set called “soft topology” and presented the foundations of the theory of soft topological spaces. In 1963, J.C. Kelly [6], first intiated the concept of bitopological spaces. He defined a bitopological space (X, τ1, τ2) to be a set X with two topologies τ1 and τ2 on X and intiated the systematic study of bitopological spaces. In 2014, Basavaraj M. Ittanagi [2] introduced the concept of Soft bitopological spaces. D. Andrijevic [1] introduced b-open sets which are some of the weak forms of open sets.

2. Preliminaries
In this section, we have presented some of the basic definitions and results of soft theory, soft topological space, bitopological spaces and soft bitopological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X, and A ⊆ X

Definition 2.1: [4] A soft set F_A on the universe X is defined by the set of ordered pairs F_A = { (x, f_A(x)) : x ∈ E }, where f_A : E → P(X) such that f_A(x) = φ if x ∉ A. Here f_A is called approximate function of the soft set F_A. The value of f_A(x) may arbitrary, some of them maybe empty, some may have non empty intersection. The set of all soft sets over X will be denoted by S(X).

Example 2.2: [4] Suppose that there are six houses in the universe X = {h1, h2, h3, h4, h5, h6} under consideration, and that E = { e1, e2, e3, e4, e5 } is a set of decision parameters. The e_i (i = 1, 2, 3, 4, 5) stand for the parameters “expensive”, “beautiful”, “wooden”, “cheap”, and in “green surroundings” respectively. Consider the mapping f_A given by “houses(.)”, where(.) is to be filled by one of the parameters e_i ∈ E. For instance, f_A(e1) means “houses (expensive)” and its functional value is the set { h ∈ X : h is an expensive house }. 
Suppose that \( A = \{ e_1, e_2, e_4 \} \subseteq E \) and \( f_{A}( e_1) = \{ h_2, h_3 \}, f_{A}( e_2) = X, f_{A}( e_4) = \{ h_1, h_3, h_5 \} \).

Then the soft set \( F_A \) as consisting of the following collection of approximations, \( F_A = \{ (e_1, \{ h_2, h_3 \} ),(e_2, X),(e_4, \{ h_1, h_3, h_5 \}) \} \).

**Definition 2.3:** [4] Let \( F_A \in S(X) \). If \( f_{A}(x) = \phi \) for all \( x \in E \), then \( F_A \) is called an empty set, denoted by \( F_A, f_{A}(x) = \phi \) means that there is no element in \( X \) related to the parameter \( x \in E \). Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

**Definition 2.4:** [4] Let \( F_A \in S(X) \). If \( f_{A}(x) = \phi \) for all \( x \in A \), then \( F_A \) is called an \( A \)- universal soft set, denoted by \( X \). If \( A = E \), then the \( A \)- universal soft set is called a universal soft set, denoted by \( X \).

**Definition 2.5:** [4] Let \( F_A, F_B \in \sim S(X) \). Then \( F_A \) is a soft subset of \( F_B \), denoted by \( F_A \subseteq F_B \) if \( f_{A}(x) \subseteq f_{B}(x) \) for all \( x \in E \). Let \( F_A \) and \( F_B \) are soft equal denoted by \( F_A = F_B \) if \( f_{A}(x) = f_{B}(x) \) for all \( x \in E \).

**Definition 2.6:** [8] Let \( F_A, F_B \in \sim S(X) \). Then soft union of \( F_A \) and \( F_B \), denoted by \( F_A \cup F_B \), is defined by \( BAF = C \), where \( C = A \cup B \), and for all \( e \in C \).

\[
H(e) = \begin{cases} 
F(e) & \text{if } e \in A \setminus B \\
G(e) & \text{if } e \in B \setminus A \\
F(e) \cup G(e) & \text{if } e \in A \cap B 
\end{cases}
\]

**Definition 2.7:** [5] Let \( F_A \in S(X) \). The soft power set of \( F_A \) is defined by \( \mathbb{P}(F_A) = \{ F_A \subseteq F_A : i \in I \} \) and its cardinality is defined by \( |\mathbb{P}(F_A)| = 2^{\sum_{x \in f_{A}(x)}} \), where \( |f_{A}(x)| \) is cardinality of \( f_{A}(x) \).

**Example 2.8:** [5] Let \( X = \{ x_1, x_2 \} \), \( E = \{ e_1, e_2 \} \) and \( \tilde{X} = \{ (e_1, \{ x_1, x_2 \} ), (e_2, \{ x_1, x_2 \} ) \} \). Then the soft subsets over \( \tilde{X} \) are

\[
F_{E_1} = \{ (e_1, \{ x_1 \} ) \} \quad F_{E_2} = \{ (e_1, \{ x_2 \} ) \} \\
F_{E_3} = \{ (e_2, \{ x_1 \} ) \} \quad F_{E_4} = \{ (e_2, \{ x_2 \} ) \} \\
F_{E_5} = \{ (e_2, \{ x_1, x_2 \} ) \} \\
F_{E_6} = \{ (e_1, \{ x_1, x_2 \} ) \} \\
F_{E_7} = \{ (e_1, \{ x_1 \} ), (e_2, \{ x_1 \} ) \} \\
F_{E_8} = \{ (e_2, \{ x_1 \} ), (e_2, \{ x_2 \} ) \}.
\]

Are all subsets of \( \tilde{X} \). So \( |\tilde{X}| = 2^{4} = 16 \).

**Definition 2.9:** [12] Let \( \tilde{T} \) be the collection of soft sets over \( \tilde{X} \), then \( \tilde{T} \) is said to be a soft topology on \( X \) if satisfies the following axioms.

(i) \( F_\emptyset, \tilde{X} \) belong to \( \tilde{T} \).

(ii) the union of any member of soft sets in \( \tilde{T} \) belongs to \( \tilde{T} \).

(iii) the intersection of any two soft sets in \( \tilde{T} \) belongs to \( \tilde{T} \). The triplet \((\tilde{X}, \tilde{T}, E)\) is called soft topological space over \( X \). The members of \( \tilde{T} \) are said to be soft open set.

**Example 2.10:** Let us consider the soft subsets of \( \tilde{X} \) that are given in Example 2.8. Then \( \tilde{T}_1 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_2} \} \), \( \tilde{T}_2 = \{ \tilde{X}, F_\emptyset, F_{E_1}, F_{E_2} \} \), \( \tilde{T}_3 = \{ \tilde{X}, F_\emptyset, F_{E_1}, F_{E_2} \} \) are soft topologies on \( \tilde{X} \).

**Definition 2.11:** [16] Let \( X \neq \phi \), \( \tau_1 \) and \( \tau_2 \) are two different topologies on \( X \). Then \((X, \tau_1, \tau_2)\) is called a Bitopological space.
Definition 2.12: [6] Let S be a subset of X. Then, (i) The τ_{1,2}-closure of S, denoted by τ_{1,2}-cl(S), is defined by \( \bigcap \{ F : S \subseteq F, F \) is τ_{1,2}-closed \}. (ii) The τ_{1,2}-interior of S, denoted by τ_{1,2}-int(S), is defined by \( \bigcup \{ A : A \subseteq S, A \) is a τ_{1,2}-open set \}.

Example 2.13: [6] Let X = \{a, b, c\}, \( \tau_1 \) = \{φ, X, \{a\}, \{b\}, \{a,b\}\} and \( \tau_2 \) = \{φ, X, \{c\}, \{b\}, \{a,b\}\}. The sets in \{φ, X, \{a\}, \{b\}, \{a,b\}\} are called τ_{1,2}-open set and sets in \{φ, X, \{c\}, \{b\}, \{a,b\}\} are called τ_{1,2}-closed set.

Definition 2.14: [6] Let S be a nonempty soft set on the universe X, \( X = \{a, b, c\} \). The complement of \( \tau_{1,2} \)-soft open set is \( \tau_{1,2} \)-soft closed set.

Definition 2.15: [13] Let \( \widehat{X} \) be a nonempty soft set on the universe X, \( \widehat{t}_1 \) and \( \widehat{t}_2 \) two different soft topologies on \( \widehat{X} \). Then \( (\widehat{X}, \widehat{t}_1, \widehat{t}_2) \) is called a soft Bitopological space.

Definition 2.16: [13] Let \( (\widehat{X}, \widehat{t}_1, \widehat{t}_2) \) be a soft bitopological space and \( F_A \subseteq \widehat{X} \). Then \( F_A \) is called \( \tau_{1,2} \)-soft open if \( F_A = F_B \cup F_C \), where \( F_B \subseteq \widehat{t}_1 \) and \( F_C \subseteq \widehat{t}_2 \). Then \( \tau_{1,2} \)-soft closed set is called \( \tau_{1,2} \)-soft closed.

Example 2.17: Let us consider the soft subsets \( \widehat{X} \) of \( \tau_1 \) and \( \tau_2 \) are given in Example 2.8 and \( \widehat{t}_1 = \{ \widehat{X}, F_\phi, F_E, F_{E_1}, F_{E_2}\} \), \( \widehat{t}_2 = \{ \widehat{X}, F_\phi, F_{E_1}, F_{E_2}, F_{E_3}\} \). Then \( \tau_{1,2} \)-soft open set are \{ \widehat{X}, F_\phi, F_{E_1}, F_{E_2}, F_{E_3}\} \) and \( \tau_{1,2} \)-soft closed set are \{ \widehat{X}, F_\phi, F_{E_1}, F_{E_2}, F_{E_3}\} \).

Definition 2.18: [12] Let \( F_A \) be a soft subset of \( \widehat{X} \), Then

(i) The \( \tau_{1,2} \)-soft closure of \( F_A \), denoted by \( \tau_{1,2} \)-cl(\( F_A \)), is defined by \( \bigcap \{ F_K : F_A \subseteq F_K, F_K \) is \( \tau_{1,2} \)-soft closed \}.

(ii) The \( \tau_{1,2} \)-soft interior of \( F_A \), denoted by \( \tau_{1,2} \)-int(\( F_A \)), is defined by \( \bigcup \{ F_C : F_C \subseteq F_A, F_C \) is \( \tau_{1,2} \)-soft open \}.

Note that \( \tau_{1,2} \)-int(\( F_A \)) is the biggest \( \tau_{1,2} \)-soft open set that contained by \( F_A \) and \( \tau_{1,2} \)-cl(\( F_A \)) is the smallest \( \tau_{1,2} \)-soft closed set that containing \( F_A \).

Definition 2.19: [11] A subset A of the topological space \((X, \tau)\) is called \( \mathcal{D} \)-open set if \( A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \), and \( \mathcal{D} \)-closed if \( \text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A \).

The following concepts are used in the sequel.

Definition 2.20: A softset \( F_A \) in a soft topological space \( \widehat{X} \) is called

(i) \((1, 2)^*\)-soft regular open set if \( F_A = \tau_{1,2} \)-int(\( \tau_{1,2} \)-cl(\( F_A \))) and \((1, 2)^*\)-soft regular closed set if \( F_A = \tau_{1,2} \)-cl(\( \tau_{1,2} \)-int(\( F_A \)))

(ii) \((1, 2)^*\)-soft α-open set if \( F_A \subseteq \tau_{1,2} \)-int(\( \tau_{1,2} \)-cl(\( F_A \))) and \((1, 2)^*\)-soft α-closed set if \( F_A \subseteq \tau_{1,2} \)-cl(\( \tau_{1,2} \)-int(\( F_A \)))

(iii) \((1, 2)^*\)-soft α-open pre if \( F_A \subseteq \tau_{1,2} \)-int(\( \tau_{1,2} \)-cl(\( F_A \)))

(iv) \((1, 2)^*\)-soft semi open if \( F_A \subseteq \tau_{1,2} \)-cl(\( \tau_{1,2} \)-int(\( F_A \)))

(v) \((1, 2)^*\)-soft β-open if \( F_A \subseteq \tau_{1,2} \)-cl(\( \tau_{1,2} \)-int(\( F_A \)))

The family of all \((1, 2)^*\)-soft regular open (resp. \((1, 2)^*\)-soft α-open, \((1, 2)^*\)-soft preopen, \((1, 2)^*\)-soft semi open, \((1, 2)^*\)-soft β-open) sets in may be denoted by \((1, 2)^*\)-sr open (resp. \((1, 2)^*\)-α open, \((1, 2)^*\)-sp open, \((1, 2)^*\)-ss open, \((1, 2)^*\)-β open) sets.

Lemma 2.21: In \((\widehat{X}, \widehat{t}_1, \widehat{t}_2)\) be a soft bitopological space. We have the following results.

(i) Every \((1, 2)^*\)-soft regular open set is \((1, 2)^*\)-soft open

(ii) Every \((1, 2)^*\)-soft open set is \((1, 2)^*\)-soft α-open.

(iii) Every \((1, 2)^*\)-soft α open set is \((1, 2)^*\)-soft semi open.
(iv) Every $(1, 2)^*$-soft preopen set is $(1, 2)^*$-soft β-open.
(v) Every $(1, 2)^*$-soft semi open set is $(1, 2)^*$-soft β-open.
(vi) Every $(1, 2)^*$-soft α-open set is $(1, 2)^*$-soft preopen

**Proof:** Let $(\widetilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space and $F_A \in \widetilde{X}$. Suppose $F_A$ be a $(1, 2)^*$-soft regular open set. Then $F_A = \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A))$ since $\tilde{\tau}_{1,2} - \text{cl}(F_A)$ is a closed set in soft bitopological space and interior of any set is open. Therefore,

(i) proved.

Let $F_A$ be a $(1, 2)^*$-soft open set. This implies $F_A = \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A))$. Since $F_A \subseteq \tilde{\tau}_{1,2} - \text{cl}(F_A) = \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A))$. Thus,

(ii) proved.

Let $F_A$ be a $(1, 2)^*$-soft α-open set. This implies $F_A \subseteq \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A))$. Thus, (iii) proved.

Let $F_A$ be a $(1, 2)^*$-soft preopen set. This implies $F_A \subseteq \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \subseteq \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A))$. Thus, (iv) proved.

Let $F_A$ be a $(1, 2)^*$-soft semi open set. This implies $F_A \subseteq \tilde{\tau}_{1,2} - \text{cl}(F_A) \subseteq \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A))$. Thus, (v) proved.

Let $F_A$ be a $(1, 2)^*$-soft α-open set. This implies $F_A \subseteq \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \subseteq \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{cl}(F_A))$. Thus, (vi) proved.

**Remark 2.22:** The converse of the above lemma is need not be true as seen in the following examples.

**Example 2.23:** Let $(\widetilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space, where

$\tilde{\tau}_1 = \{ \widetilde{X}, F_1, F_{E_1}, F_{E_2}, \}$, $\tilde{\tau}_2 = \{ \widetilde{X}, F_\phi, F_{E_2}, F_{E_4}, F_6, F_8, F_{E_6}, F_{E_8}, F_{E_9} \}$. Then $\tilde{\tau}_{1,2}$-soft open set are $\{ \widetilde{X}, F_\phi, F_{E_1}, F_{E_2}, F_{E_4}, F_6, F_8, F_{E_6}, F_{E_8}, F_{E_9} \}$

(i) $F_{E_1}$ is a $(1, 2)^*$-soft open but not $(1, 2)^*$-soft regular open.

(ii) $F_{E_2}$ is a $(1, 2)^*$-soft α-open but not $(1, 2)^*$-soft open.

(iii) $F_{E_3}$ is a $(1, 2)^*$-soft semi open but not $(1, 2)^*$-soft α open.

(iv) $F_{E_4}$ is a $(1, 2)^*$-soft β-open set but not $(1, 2)^*$-soft preopen.

**Example 2.24:** Let us consider the soft subsets of $\tilde{X}$ that are given in Example 2.8. Let $(\widetilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space, where $\tilde{\tau}_1 = \{ \widetilde{X}, F_\phi, F_{E_1}, F_{E_2}, \}$, $\tilde{\tau}_2 = \{ \widetilde{X}, F_\phi, F_{E_2}, F_{E_4}, F_6, F_8, F_{E_6}, F_{E_8}, F_{E_9} \}$. Then $\tilde{\tau}_{1,2}$-soft open set are $\{ \widetilde{X}, F_\phi, F_{E_1}, F_{E_2}, F_{E_4}, F_6, F_8, F_{E_6}, F_{E_8}, F_{E_9} \}$

(v) The soft subset $F_{E_1}$ is $(1, 2)^*$-soft β-open set but not $(1, 2)^*$-soft semi open.

**Example 2.25:** Let $X = \{ x_1, x_2, x_3 \}$, $E = \{ e_1, e_2, e_3 \}$ and $\tilde{X} = \{ ( e_1, \{ x_1, x_2, x_3 \}, (e_2, \{ x_1, x_2, x_3 \}, (e_3, \{ x_1, x_2, x_3 \}). Then $\tilde{\tau}_1 = \{ \widetilde{X}, F_\phi, F_{E_1}, F_{E_2}, \}$, $\tilde{\tau}_2 = \{ \widetilde{X}, F_\phi, F_{E_2}, F_{E_4}, F_6, F_8, F_{E_6}, F_{E_8} \}$. Where $F_{E_1} = \{ ( e_1, \{ x_1 \}), (e_2, \{ x_2 \}), (e_3, \{ x_3 \}) \}$, $F_{E_2} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_2, x_3 \}), (e_3, \{ x_1, x_2 \}), \}$. $F_{E_4} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_1, x_3 \}), (e_3, \{ x_2, x_3 \}) \}$, $F_{E_6} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_2, x_3 \}), (e_3, \{ x_1, x_3 \}) \}$, $F_{E_8} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_2, x_3 \}), (e_3, \{ x_1, x_3 \}) \}$. $F_{E_9} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_1, x_3 \}), (e_3, \{ x_2, x_3 \}) \}$, $F_{E_9} = \{ ( e_1, \{ x_1, x_2 \}), (e_2, \{ x_1, x_3 \}), (e_3, \{ x_2, x_3 \}) \}$. Then $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is a soft bitopological space. (vi) Consider $F_{E_1}$, the soft subset of $\tilde{X}$. Where $F_{E_1} = \{ ( e_1, \{ x_1 \}), (e_2, \{ x_1, x_2 \}), (e_3, \{ x_1, x_3 \}) \}$, $\tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_{E_1})) = \tilde{X}$ and $F_{E_1} \subseteq \tilde{X}$. But $\tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_{E_1})) = F_{E_1}$ Hence $F_{E_1}$ is $(1, 2)^*$-soft preopen set but not $(1, 2)^*$-soft α-open.
3. (1,2)*-soft b-open sets: In this section, we introduce (1,2)*-soft b-open sets in soft bitopological spaces and study some of their properties.

Definition 3.1: Let $\left(X, \tau_1, \tau_2\right)$ be a soft bitopological space and $F_a \subseteq X$. Then $F_a$ is called a (1,2)*-soft b-open set (briefly (1,2)*-sb-open) if $F_a \subseteq \overline{\tau_{1,2}} \cap \left(\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)}\right) \cap \left(\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)}\right)$.

Example 3.2: In Example 2.17, the (1,2)*-soft b-open sets are $\left\{\pi, F_{E_1}, F_{E_2}, F_{E_3}, F_{E_4}, F_{E_5}, F_{E_6}, F_{E_7}, F_{E_8}, F_{E_9}\right\}$.

Theorem 3.3: Let $\left(X, \tau_1, \tau_2\right)$ be a soft bitopological space. Then (i) Every (1,2)*-soft pre-open set is (1,2)*-soft b-open set. (ii) Every (1,2)*-soft b-open set is (1,2)*-soft b-open set. (iii) Every (1,2)*-soft semi open set is (1,2)*-soft b-open set.

Proof: Let $\left(X, \tau_1, \tau_2\right)$ be a soft bitopological space and $F_a \subseteq X$. Let $F_a$ be a (1,2)*-soft pre-open set. Then $F_a \subseteq \overline{\tau_{1,2}} \cap (\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)}) \cap (\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)})$. Thus (i) proved.

Remark 3.4: The converse of the above lemma is need not be true as seen in the following example.

Example 3.5: Let us consider the soft subsets of $X$ that are given in Example 2.8. Let $\left(X, \tau_1, \tau_2\right)$ be a soft bitopological space, where $\tau_1 = \left\{\pi, F_{E_1}, F_{E_2}\right\}$, $\tau_2 = \left\{\pi, F_{E_3}, F_{E_4}\right\}$. Then $\tau_1, \tau_2$ - soft open set are $\left\{\pi, F_{E_1}, F_{E_2}, F_{E_3}, F_{E_4}\right\}$. $\tau_1, \tau_2$ - soft closed set are $\left\{\pi, F_{E_1}, F_{E_2}, F_{E_3}, F_{E_4}\right\}$.

Theorem 3.7: An arbitrary union of (1,2)*-soft b-open sets are (1,2)*-soft b-open set.

Proof: Let $\left\{(F_a)_a\right\}$ be a collection of (1,2)*-soft b-open sets. Then for each $\alpha$, $(F_a)_a \subseteq \overline{\tau_{1,2}} \cap (\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)_a}) \cap (\overline{\tau_{1,2}} \cap \overline{\text{cl}(F_a)_a})$. Thus (i) proved.
Proof: Let \( F_A, F_B \in (X, \tau_1, \tau_2) \) be a \((1,2)^*\)-soft open set and \((1,2)^*\)-sb-open set respectively then \( F_A \cap F_B \subseteq F_A \cap \overline{F_B} \subseteq F_A \cap \overline{\overline{F_B}} \subseteq F_A \cap \overline{\overline{\overline{F_B}}} \). Also, \( \overline{\overline{F_B}} = \overline{\overline{\overline{\overline{F_B}}}} \) and \( \overline{\overline{F_B}} = \overline{\overline{F_A \cap \overline{F_B}}} \). Therefore, \( \overline{\overline{F_B}} \subseteq \overline{\overline{\overline{F_A \cap \overline{F_B}}}} \). So, \( \overline{\overline{F_B}} \) is a \((1,2)^*\)-sr-closed set.

Theorem 3.9: Let \((X, \tau_1, \tau_2)\) be a soft bitopological space. (i) The intersection of \((1,2)^*\)-soft open set and \((1,2)^*\)-sb-open set is \((1,2)^*\)-sb-open set and \((1,2)^*\)-sb-open set is \((1,2)^*\)-sb-open set.

Proof: Let \( F_A \), \( F_B \in (X, \tau_1, \tau_2) \) be a \((1,2)^*\)-soft open set and \((1,2)^*\)-sb-open set respectively, then \( F_A \cap F_B \subseteq F_A \cap \overline{F_B} \subseteq F_A \cap \overline{\overline{F_B}} \subseteq F_A \cap \overline{\overline{\overline{F_B}}} \). Thus (i) proved.

(ii) Let \( F_A \), \( F_B \in (X, \tau_1, \tau_2) \) be a \((1,2)^*\)-soft open set and \((1,2)^*\)-sb-open set respectively, then \( F_A \cap F_B \subseteq \overline{F_A} \cap \overline{F_B} \subseteq \overline{\overline{F_A}} \cap \overline{\overline{F_B}} \subseteq \overline{\overline{\overline{F_A}} \cap \overline{\overline{F_B}}} \). Thus (ii) proved.
4. \((1,2)^*\)-soft b-closed sets
In this section we introduce \((1,2)^*\)-soft b-closed sets in soft bitopological spaces and study some of their properties.

**Definition 4.1:** Let \( (\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2) \) be a soft bitopological space and \( F_A \subseteq \tilde{X} \). Then \( F_A \) is called \((1,2)^*\)-soft b-closed set (briefly \((1,2)^*\)-sb-closed) if \( \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \bigcup \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))) \subseteq F_A. \) The complement of \((1,2)^*\)-soft b-closed set is \((1,2)^*\)-soft b-open set.

**Example 4.2:** In Example 2.17, the \((1,2)^*\)-soft b-closed sets are \( \{ \tilde{X}, F_\emptyset, F_{E_2}, F_{E_4}, F_{E_5}, F_{E_6}, F_{E_7}, F_{E_8}, F_{E_9} \} \)

**Theorem 4.3:** An arbitrary intersection of \((1,2)^*\)-soft b-closed sets is \((1,2)^*\)-soft b-closed set.

**Proof:** Let \( \{ (F_B)_{\alpha} \} \) be a collection of \((1,2)^*\)-soft b-closed sets in \( \tilde{X} \). Then for each \( \alpha, \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_B)_{\alpha} \)) \bigcup \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_B)_{\alpha} \)) \subseteq F_B. Since \( \{ (F_B)_{\alpha} \} \) is an arbitrary family of \((1,2)^*\)-sb-open sets in \( \tilde{X} \). Hence, by theorem 3.7, \( \bigcap_{\alpha} (F_B)_{\alpha}^C \) is a \((1,2)^*\)-sb-open set. But \( \bigcup_{\alpha} (F_B)_{\alpha}^C = \left[ \bigcap_{\alpha} (F_B)_{\alpha} \right]^C \). Therefore \( \bigcap_{\alpha} (F_B)_{\alpha} \) is a \((1,2)^*\)-sb-closed set.

**Result 4.4:** The union of two \((1,2)^*\)-soft b-closed sets need not be \((1,2)^*\)-soft b-closed set. In example 3.5, \( F_{E_1} \) and \( F_{E_2} \) are \((1,2)^*\)-soft b-closed sets. But the union \( F_{E_1} \bigcup F_{E_2} = F_{E_3} \), which is not \((1,2)^*\)-soft b-closed set.

**Theorem 4.5:** Let \( F_A \) be a \((1,2)^*\)-sb-closed set in soft bitopological space \( \tilde{X} \). (i) If \( F_A \) is a \((1,2)^*\)-sr-closed set then \( F_A \) is a \((1,2)^*\)-ss-closed set. (ii) If \( F_A \) is a \((1,2)^*\)-sr-open set then \( F_A \) is a \((1,2)^*\)-sp-closed set.

**Proof:** Since \( F_A \) is a \((1,2)^*\)-sb-closed set, \( \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))) \bigcap \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \subseteq F_A. Since \( F_A \) is \((1,2)^*\)-sr-closed set, \( F_A = \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))). Therefore, \( F_A \bigcap \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \subseteq F_A. Thus, \( \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \subseteq F_A. Thus (i) proved.

Since \( F_A \) is \((1,2)^*\)-sr-open set, \( F_A = \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))). Therefore, \( F_A \bigcap \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))) \subseteq F_A. This implies, \( \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))) \subseteq F_A. Thus (ii) proved.

**Theorem 4.6:** Any \((1,2)^*\)-sb-closed set in soft bitopological space \( \tilde{X} \) is \( \tilde{\tau}_{1,2} \)-int(\( F_A \)) is \((1,2)^*\)-sr-closed set.

**Proof:** Let \( F_A \) be a \((1,2)^*\)-sb-closed set. This implies \( \tilde{\tau}_{1,2} \)-cl(\( \tilde{\tau}_{1,2} \)-int(\( F_A \))) \bigcap \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \subseteq F_A. Also, \( \tilde{\tau}_{1,2} \)-int(\( F_A \)) \subseteq \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \bigcap \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))). Thus \( \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))) \subseteq \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))). Therefore, \( \tilde{\tau}_{1,2} \)-int(\( F_A \)) = \( \tilde{\tau}_{1,2} \)-int(\( F_A \)) \bigcup \tilde{\tau}_{1,2} \)-int(\( \tilde{\tau}_{1,2} \)-cl(\( F_A \))). So, \( \tilde{\tau}_{1,2} \)-int(\( F_A \)) is \((1,2)^*\)-sr-closed set.

**Theorem 4.7:** \( F_A \) be a \((1,2)^*\)-sb-closed set in \( \tilde{X} \) if and only if \( F_A \) is the intersection of \((1,2)^*\)-ss-closed set and \((1,2)^*\)-sp-closed set.

**Proof:** Follows from the definition 4.1.

5. \((1,2)^*\)-soft b-interior and \((1,2)^*\)-soft b-closure

**Definition 5.1:** Let \( (\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2) \) be a soft bitopological space and \( F_A \) be a soft set over \( \tilde{X} \). (i) \((1,2)^*\)-soft b-closure (briefly \((1,2)^*\)-sbcl(\( F_A \))) of a set \( F_A \) in \( \tilde{X} \) defined by \((1,2)^*\)-sbcl(\( F_A \)) = \( \bigcap \{ F_B \subseteq F_A : F_B \) is a \((1,2)^*\)-soft b-closed set in \( \tilde{X} \}) \). (ii) \((1,2)^*\)-soft b-interior (briefly \((1,2)^*\)-sbint(\( F_A \))) of a set \( F_A \) in \( \tilde{X} \) defined by \((1,2)^*\)-sbint(\( F_A \)) = \( \bigcup \{ F_B \subseteq F_A : F_B \) is a \((1,2)^*\)-soft b-open set in \( \tilde{X} \}) \).
Let \( F_A \) be a soft set in a soft bitopological space \( \tilde{X} \). Then(i) \((1, 2)^*- \text{sscl}(F_A) = F_A \cup \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \) (ii) \((1, 2)^*- \text{ssint}(F_A) = F_A \cap \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iii) \((1, 2)^*- \text{spcl}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iv) \((1, 2)^*- \text{spint}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(F_A)) \)

**Theorem 5.5:** Let \( F_A \) be a soft set in a soft bitopological space \( \tilde{X} \). Then(i) \((1, 2)^*- \text{sbcl}(F_A) = F_A \cup [ \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \cap \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A))] \) (ii) \((1, 2)^*- \text{sbint}(F_A) = F_A \cap [ \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \cap \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A))] \)

**Proof:** Proof is obvious.

**Theorem 5.6:** Let \( F_A \) be a soft set in a soft bitopological space \( \tilde{X} \). Then(i) \((1, 2)^*- \text{sbcl}(F_A) \subseteq F_A \cup F_B \) or \((1, 2)^*- \text{sbint}(F_A) \subseteq F_A \cap F_B \) Then \((1, 2)^*- \text{sbcl}(F_A) \cup F_B \) or \((1, 2)^*- \text{sbint}(F_A) \cap F_B \) This implies, \((1, 2)^*- \text{sbcl}(F_A) \subseteq (1, 2)^*- \text{sbcl}(F_A) \cup F_B \)

**Proof:** Since \( F_A \subseteq F_A \cup F_B \) or \( F_A \subseteq F_A \cap F_B \) Thus, \((1, 2)^*- \text{sbcl}(F_A) \subseteq (1, 2)^*- \text{sbcl}(F_A) \cup F_B \)

**Theorem 5.7:** Let \( F_A \) be a soft set in a soft bitopological space \( \tilde{X} \). Then(i) \((1, 2)^*- \text{sbcl}(F_A) \subseteq (1, 2)^*- \text{sscl}(F_A) \cap (1, 2)^*- \text{spcl}(F_A) \) (ii) \((1, 2)^*- \text{sbint}(F_A) \supseteq (1, 2)^*- \text{ssint}(F_A) \cup (1, 2)^*- \text{spint}(F_A) \)

**Proof:** (i) Suppose \( F_A = (1, 2)^*- \text{sbcl}(F_A) = \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \)

**Proof:** (i) Suppose \( F_A = (1, 2)^*- \text{sbcl}(F_A) = \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \)

\( F_A \) is the smallest \((1, 2)^*- \text{soft b-open set in} \ X \) which contains \( F_A \) and \((1, 2)^*- \text{soft b-closed set in} \ X \) which is contained in \( F_A \).

The following lemma is used in the sequel.

**Lemma 5.2:** Let \( F_A \) be a soft set in a soft bitopological space \( X \). Then(i) \((1, 2)^*- \text{sscl}(F_A) = F_A \cup \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \) (ii) \((1, 2)^*- \text{ssint}(F_A) = F_A \cap \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iii) \((1, 2)^*- \text{spcl}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iv) \((1, 2)^*- \text{spint}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(F_A)) \)

**Proof:** Proof is obvious.

**Theorem 5.3:** Let \( F_A \) be a soft set in a soft bitopological space \( X \). Then(i) \((1, 2)^*- \text{sc}(F_A) = F_A \cup \tilde{\tau}_{1,2} - \text{int}(\tilde{\tau}_{1,2} - \text{cl}(F_A)) \) (ii) \((1, 2)^*- \text{scint}(F_A) = F_A \cap \tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iii) \((1, 2)^*- \text{sp}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(\tilde{\tau}_{1,2} - \text{int}(F_A)) \) (iv) \((1, 2)^*- \text{spint}(F_A) = F_A \cap (\tilde{\tau}_{1,2} - \text{cl}(F_A)) \)

**Proof:** Proof is obvious.
Conclusion
In this paper, we introduce the concept of \((1,2)^*\)-soft b-open sets and \((1,2)^*\)-soft b-closed sets in soft bitopological spaces. Also we study the properties of \((1,2)^*\)-soft b-interior and \((1,2)^*\)-soft b-closure. In future, using this concept various type of open sets on soft bitopological spaces shall be studied and developed.

References