Design and Analysis of Digital Notch Filter for Removal of Power Line Interference Signal

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Abstract
Power line interference can highly impair a biomedical recording. Notch filters have been propounded to repress power line interference. In this paper, a second order digital notch filter for the reduction of the fundamental power line interference component in electrocardiogram (ECG) recording is furnished. The digital notch filter is uniquely specified by two distinguished parameters, the notch filter frequency and 3dB rejection bandwidth. To this end, a real ECG waveform from MIT-BIH database which contains the normal and abnormal waveforms is depraved by an artificial power line interference. The filtered signal after applying digital notch filter is compared with the real ECG waveform. Results demonstrate that power line interference of ECG are eliminated effectively by using this digital notch filter. Interference deletion can be accomplished continuously and quickly even if the conditions of interference are varying with time or frequency. The results are obtained from the digital notch filter and calculated performance parameter like Means Square Error (MSE), power spectrums density and Signal to Noise Ratio (SNR) of the noisy and filtered electrocardiogram waveforms.

Keywords: ECG, Notch filter, MATLAB, SNR, MSE

1. Introduction
Digital filters are being increasingly by utilized in various regions, like picture processing, speech processing, sonar and radar systems, digital control systems, etc., as they show a number of advantages according to the accuracy, stability, reliability, and flexibility in comparison with the more normal analog filters. In many usages, a digital filter is patterned to have prescribed magnitude responses. Such helpful type of digital filters is the digital notch filter which attenuates highly a special frequency part in the input waveform while releasing close frequencies comparatively unattenuated.

Electrocardiogram is a technique which has been according to the recording the cardiac electrical activity. Electrocardiogram is a non-stationary waveform which contains valuable clinical data. But, this data is being frequently depraved by artifact. Moreover, electrocardiogram waveforms have interfered by power line noise (50/60 Hz). This may come from having erected a grounding protection, feeding lines of measurement devices, an amplifier and shielding wrong pattern \cite{1}. There only is a single prior step in order to remove this type of interference in electrocardiogram (ECG) signals. This step mentions to maintain the waveform specifications for diagnosis. When electrocardiogram starts recording, frequency is also being changed by power line circa 50/60 Hz. When this frequency starts modification, henceforth there will not be an exact result for this filter. Hence, the digital notch filter for removal of power line interference will be needed.

Digital notch filter is a band-rejection filter with a narrow stop band and it transits whole frequencies except power line interference (50/60Hz) in a stop band centered at a notch frequency. The amplitude of the digital notch filter is flat at every frequency except for the stop band at (50/60 Hz) both directions of the center frequency. The center frequency phase response of the digital notch filter demonstrates the best rate of variation at center frequency. The Q-factor is the size of stop band pertaining to center frequency and impacts of 3-dB points, bandwidth of stop band. At extremely high level Q-factor values, the digital filter circuit will commence overshoot and undershoot that will deprave the entirety of the notch in the filter.
In this paper, the expansion of an appropriate digital transfer function possessing a notch specification is first planned and the goal is to decrease the number of parameters specifying the transfer function. For this purpose, it is demonstrated that two parameters are adequate to specify a second-order digital notch filter, and as an outcome, such a filter can be achieved utilizing only two multipliers.

The 50/60Hz digital notch filter removes a narrow frequency band circa 50/60Hz. These techniques are simple and can be accomplished at a low cost. In addition, they produce an undesirable waveform correction. The interferences are being removed but some significant frequency components of electrocardiogram waveforms are also eliminated [2].

This article is organized as beneath. Section II summarizes the transfer function of a digital notch filter. Simulation results are announced in Section III. Finally, summary and conclusions are shown in Section IV.

2. Transfer Function of a Digital Notch Filter
The transfer function of an analog notch filter is shown as:

$$H(s) = \frac{s^2 + \lambda^2}{s^2 + bs + \lambda^2}$$  

(1)

The sketch of the magnitude function $|H(j\omega)|$ is designed in Fig. 2 and demonstrates that $\omega = \lambda$ is the notch frequency at which there is no transition via the filter. In the frequency band centered at $\omega = \lambda$ and of width $b$, whole waveform components are attenuated by more than 3-dB. Therefore $b$ is the 3-dB rejection bandwidth. Note that the DC gain is 0-dB.

Apply the bilinear transformation to the transfer function of an analog notch filter for sketching of the digital notch filter [3, 4].

$$S = \frac{z - 1}{z + 1}$$  

(2)

to $H(s)$.

This yields

$$G(z) = H(s)|_{s=\frac{z-1}{z+1}} = \frac{(1+\lambda^2) - 2(1-\lambda^2)z^{-1} + (1+\lambda^2)z^{-2}}{(1+\lambda^2 + b) - 2(1-\lambda^2)z^{-1} + (1+\lambda^2 - b)z^{-2}}$$  

(3)

The neglecting invariant $l/2$ which is tantamount to that of adding a flat 6-dB gain to the frequency response, can be rewritten (4) in the desired form as:

$$G(z) = \frac{2(1-\lambda^2)}{1+\lambda^2 + b}$$  

(4)

where

$$a_1 = \frac{2(1-\lambda^2)}{1+\lambda^2 + b}$$  

(5)

$$a_2 = \frac{1+\lambda^2 - b}{1+\lambda^2 + b}$$  

(6)

The neglecting invariant $l/2$ which is tantamount to that of adding a flat 6-dB gain to the frequency response, can be rewritten (4) in the desired form as:

$$G(z) = \frac{(z^{-2} + 1) - 2a_1z^{-1} + a_2(z^{-2} + 1)}{1-a_1z^{-1} + a_2z^{-2}}$$  

(7)

To decrease the number of coefficients for specifying the digital notch filter transfer function $G(z)$, the right side of (3) can be explained in another form as:

$$G(z) = \frac{1}{2} \left( \frac{1 + a_1}{1 - a_1} \right) z^{-1} + \frac{1 + a_2}{1 - a_2} z^{-2}$$  

(4)

The notching invariant $l/2$ which is tantamount to that of adding a flat 6-dB gain to the frequency response, can be rewritten (4) in the desired form as:

$$G(z) = \frac{2(1-\lambda^2)}{1+\lambda^2 + b}$$  

(5)

$$a_1 = \frac{2(1-\lambda^2)}{1+\lambda^2 + b}$$  

(6)

$$a_2 = \frac{1+\lambda^2 - b}{1+\lambda^2 + b}$$  

(6)

The DC gain achieved by letting $z = 1$ in (7) is 2, which is 6-dB, as envisaged. The notch filter transfer function is shown by (7) and it is specified by two distinguished parameters $a_1$ and $a_2$. It will be given in the next part that $G(z)$ as shown above can be achieved by a digital filter involving two multipliers of coefficients $a_1$ and $a_2$.

Relate the invariants $a_1$ and $a_2$ to the notch frequency $\omega_0$ and the 3-dB rejection band $\Omega$. From (3), the magnitude is shown as:

$$|G[\exp(j\omega T)]| = \frac{|(1+\lambda^2)\cos \omega T - (1-\lambda^2)|^2}{|1+(\lambda^2)\cos \omega T - (1-\lambda^2)|^2 + b^2 \sin^2 \omega T}$$  

(8)

The notch frequency $\omega_0$ is achieved by regulating the numerator of (8) equal to zero, which outputs

$$\cos \omega_0 T = \frac{1-\lambda^2}{1+\lambda^2}$$  

(9)

$$\omega_0 = \frac{1}{T} \cos^{-1} \frac{1-\lambda^2}{1+\lambda^2}$$  

(10)

From (8) note the frequencies $\omega_0$ and $\omega_2$ where the magnitude $|G[\exp(j\omega T)]|$ is down 3-dB from its value are achieved by solving,

$$|\{\lambda^2 + 1\cos \omega T - (1-\lambda^2)\}|^2 = b^2 \sin^2 \omega T$$  

(11)
which after algebra outputs

$$\cos(\Omega T) = \cos(\omega_2 - \omega_1)T = \frac{(1 + \lambda^2)^2 - b^2}{(1 + \lambda^2)^2 + b^2} \quad (12)$$

or

$$\Omega = \frac{1}{T} \cos^{-1} \left( \frac{(1 + \lambda^2)^2 - b^2}{(1 + \lambda^2)^2 + b^2} \right) \quad (13)$$

Equations (9) and (12) can be solved for $\lambda^2$ and $b$, which when replaced in (5) and (6) show the values of the invariants $a_1$ and $a_2$ as a function of $\omega_o$ and $\Omega$. From (9)

$$\lambda^2 = \tan^2(\omega_o T / 2) \quad (14)$$

and from (12)

$$b = (1 + \lambda^2) \tan(\Omega T / 2) \quad (15)$$

replacing (14) and (15) in (5) and (6) achieved the desired explanations:

$$a_1 = \frac{2 \cos \omega_o T}{1 + \tan(\Omega T / 2)} \quad (16)$$

$$a_2 = \frac{1 - \tan(\Omega T / 2)}{1 + \tan(\Omega T / 2)} \quad (17)$$

The notch frequency $\omega_0$ can be varied while maintaining the 3-dB rejection band and DC gain constants just by changing $a_1$. Similarly, the rejection bandwidth can be varied by changing only $a_2$. But, to maintain the notch frequency constant, $a_1$ must be regulated accordingly.

3. Simulation Results

To demonstrate that the digital notch filter is impressive in clinical situations, the procedure has been credited utilizing several electrocardiogram (ECG) recordings with widespread variety of waveform morphologies from MIT-BIH arrhythmia database. In this paper taking advantages of test abnormal ECG recordings as the reference for the research and real noise are obtained from MIT-BIH.

To assert omission of noises a real ECG signal has been taken which includes natural noises like baseline wander, power line interference, Electro-myography noise and muscle noises from data 105 to 203 and also added to artificial power line interference are applied to the digital notch filter. To study comparative efficiency and results are plotted in Fig. 3 to Fig. 8. To know performance of digital notch filters by calculation of SNR (Signal to Noise Ratio) and MSE (Mean Square Error) was done. The value are shown in tabular form.

<table>
<thead>
<tr>
<th>MIT-BIH ECG Signal</th>
<th>Signal to Noise Ratio (SNR)</th>
<th>Mean Square Error (MSE)</th>
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<tbody>
<tr>
<td></td>
<td>Noise ECG</td>
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<tr>
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</table>

Fig 3: Input ECG Signal of 105.m

Fig 4: 60 Hz power line Interference
4. Conclusion
In this paper, the digital notch filter designed to remove the power line interference from ECG signal was proposed. The results obtained from notch filter was applied to various real time ECG signals from MIT-BIH database samples compared on parameters such as Power Spectral Density (PSD), Mean Square Error (MSE) and Signal to Noise Ratio (SNR). The notch filter was designed by using signal processing Tool in MATLAB ®2014a software. By observing SNR of ECG before and after filtering are 8.7013, 14.4309 and MSE before and after filtering are 1.2303, 0.7290, Hence the digital notch filter gives more accurate results because amplitude response of filter is flat for all frequencies except for the stop band at 50/60 Hz. It has concluded that the digital notch filter performs better when compared to all digital filters for elimination of power line interference from ECG signals.

5. Reference