Relationship between mathematical thinking and creativity in mathematics among secondary school students

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Abstract
In the present study, the investigators tried to find out the relationship between Mathematical Thinking and Creativity in Mathematics among Secondary School Students. The investigators also tried to find the relationship between the fundamental processes of Mathematical Thinking with Creativity in Mathematics and the components of Creativity in Mathematics with Mathematical Thinking among Secondary School Students. The investigators adopted Correlation method for the study on a sample of 500 Secondary School Students selected using Random Sampling Technique. Test of Mathematical Thinking and Test of Creativity in Mathematics were used to collect data from the sample. The study revealed that there exists significant positive relationship between Mathematical Thinking and Creativity in Mathematics among Secondary School Students. Similarly there exist a significant relationship between Mathematical Thinking and the components of Creativity in Mathematics.

Keywords: Relationship, mathematical thinking, specifying, generalizing, conjecturing, convincing, creativity in mathematics, fluency, flexibility, originality

1. Introduction
The career projects and tasks taken by adults are so complex in the twenty first century, that they require the collaboration of many skilled people to reach creative solutions. According to Costa [25], there are hundreds of skills and attributes that could be identified as contributing factors to 21st century success. Some of them are

- Use real world digital and other research tools to access, evaluate and effectively apply information for authentic tasks.
- Work independently and collaboratively to solve problems and accomplish goals.
- Communicate information clearly and effectively using a variety of tools / media in varied contents for a variety of purposes.
- Demonstrate innovation, flexibility and adaptability in thinking patterns a, work habits and working conditions
- Effectively apply that analysis, synthesis and evaluate processes that enable productive problem solving

Mathematical way of seeing, thinking, and interpreting the world is an important aim of today’s system of Education [4]. Teaching is a human activity performed by complex human organism in a complex situation. Teaching Mathematics in schools is highly creative and complex endeavor. According to Schoenfield [26], Mathematics is an inherently social activity in which a community of trained practitioners (Mathematical Scientists) engages in the Science of Patterns – Systematic attempts based on observation, study and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically (“Pure Mathematics”) or models of Systems abstracted from real world objects (“Applied Mathematics”). The tools of Mathematics are abstraction, symbolic representation and symbolic manipulation. Learning to think mathematically means

(a) Developing a mathematical point of view – valuing the processes of mathematization and abstraction and having the prediction to apply them.
(b) Developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure – Mathematical Sense Making
The ability to think mathematically and to use Mathematical Thinking to solve problems is an important goal of schooling. In this respect, Mathematical Thinking will support Science, Technology, Economic Life and Development in an Economy. Stacey [17] identified four fundamental processes and showed how thinking mathematically and they are (1) Specifying (trying special cases, looking at examples) (2) Generalizing (looking for patterns and relationships) (3) Conjecturing (predicting relationships and results) and (4) Convincing (finding and communicating reasons why something is true). If students are to become good Mathematical thinkers, then Mathematical Thinking needs to be a prominent part of their education. Also, however students who have an understanding of the components of Mathematical Thinking will be able to use these abilities independently to make sense of Mathematics that they are learning [17]. According to Katagiri [11], Mathematical Thinking is like an attitude, as it can be expressed as a state of “attempting to do” or “working to do” something. It means that when one encounters a problem, one decides which set, or psychological set, to use to solve that problem. ‘Mathematical Thinking’ concept is a method of connotative as well as denotative meaning [11]. It acts as a guiding force that elicits knowledge and skills for solving the problem [11].


- Combining understanding, experiences, imagination and reasoning to construct new knowledge.
- Using existing Mathematical knowledge to create solutions to unfamiliar problems
- Posing questions and developing convincing arguments

Creative activities in Mathematics should give learners an opportunity to explore the possibilities, working in ways that motivate and engage their interests [27]. Mathematical Creativity is a complex phenomenon in which there is some commonalities to define (1) it involves divergent and convergent thinking (2) it has mainly three components that are fluency, flexibility and originality and (3) it is related to problem solving and problem posing [19].

There are investigations that find the ways of characterizing, identifying and prompting Mathematical Creativity. Poly [14] defined Mathematical Creativity as the ability to solve problems requiring independence, judgement, originality and creativity. Haylock [3] and Kwon, Park and Park [12] assessed student’s Creativity in Mathematics by employing open-ended problems and measuring divergent thinking skills. Sheffield [16] suggests that a key area to develop Creativity in Mathematics is the questioning that is used in teaching and learning. Leikin [13] explored the use of multiple solution tasks in evaluating student’s Creativity in Mathematics.

Creativity in Mathematics is defined as “the ability to generate information or ideas from given information or ideas in a Mathematical situation” [2]. It is the ability to go beyond the commonplace and ordinary in Mathematics, an ability to combine Mathematical information or experience in a unique and insightful manner.

1.1 Need and Significance of the Study

Mathematical Thinking is an ultimate goal of teaching that students will be able to conduct Mathematical investigations by themselves and that they will be able to identify where the Mathematics they have learned is applicable in real world situations [17]. It is clear that Mathematical Thinking is important in large measure to equip students with the ability to use Mathematics. Stephen [18] suggested that it is the responsibility of teachers to attempt to induce Mathematical Thinking in our students. The study of Tom and Tracy [22] highlighted the fact that one of the ways for developing Mathematical Thinking is that the opportunities are provided in schools for engaging the realistic activities to ensure that mathematical concepts and ideas can be taught and expressed in contents closer to student’s own experiences. The study of Binder [1] also found that the students who are characterized by mathematical accuracy and fluency are more able to present creative thinking in new Mathematical tasks. Talya and Miriam [20] show that the students who engaged in real life mathematical situations can stimulate their mathematical creative thinking. Thomas [21]; John [10]; Jaleel [6]; Jaleel and Titus [8][9]; Vale and Barbosa[23] and Yafitian [24] opined that Creativity of Mathematicians plays a vital role when new mathematical ideas are formulated.

The statement of Mathematics Science Education Board and National Research Council [29], ‘more than ever before, we need to think for a living; more than ever before, we need to think mathematically’ clearly suggests the importance of Mathematical Thinking in the present context. This implies that Mathematical Thinking will help the students to enable them to solve the problems they are facing in daily life situations and to use the Mathematical applications in the day to day processes. Again a study of Jaleel [7] revealed that there exists significant positive relationship between Mathematical Thinking and Achievement in Mathematics. Jaleel [6] studied the effectiveness of e – content on developing Mathematical Thinking among students. This reveals the importance of Mathematical thinking in the present Mathematics curriculum of School Education, for developing our younger ones as good Mathematical thinkers. The above discussions revealed that no studies are yet conducted for finding out the relationship between Mathematical thinking and Creativity in Mathematics, even though their individual influences were studied. Hence the investigators tried to find out the relationship between Mathematical Thinking and Creativity in Mathematics among Secondary School Students.

1.2 Objective of the Study

- To find out the relationship between Mathematical Thinking and Creativity in Mathematics among Secondary School Students
- To find out the relationship between Creativity in Mathematics and the fundamental processes of Mathematical thinking among secondary school students
- To find out the relationship between Mathematical thinking and components of Creativity in Mathematics among secondary school students
2. Methodology in Brief
2.1 Method Adopted
According to Best & Kahn [32], Quantitative research uses quantitative methods to describe what is, describing, recording, analyzing and interpreting conditions that exist. It involves some types of comparison or contrast and attempts to discover relationships between existing non manipulated variables. The investigators followed Correlation design for the present study. According to Cohen, Manion & Morrison [31], correlational (or causal) study as its name suggests involves the collection of two sets of data, one of which will be retrospective, with a view to determining the relationship between them. The basic design of such an experiment may be represented as:

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X O
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Where a strong relationship was found between the independent and dependent variables, three possible interpretations are open to the researchers: the variable X has caused O; the variable O has caused X; and that some third unidentified, and therefore unmeasured variable has caused X and O.

2.2 Sample selected for the study
The sample constitutes 500 Secondary School Students using Random Sampling Technique.

2.3 Tools used in the Study
The major tools used in the study involve;
- Test of Mathematical Thinking (Jaleel, 2015)
- Test of Creativity in Mathematics (Jaleel & Titus, 2015)

2.3.1 Test of Mathematical Thinking
The test of Mathematical thinking consists of 15 objective type test items. The reliability of the test was ensured using test-retest reliability and the reliability coefficient was found to be 0.82. The test was constructed giving due weightage to the content and the fundamental processes of Mathematical thinking. Content validity was established for the test. Norms were also established for the test. One example for the test items in Test of Mathematical thinking is as shown below:
Eg: Which shape did it receive when we increase the number of sides of a polygon infinitely?

<table>
<thead>
<tr>
<th>Students</th>
<th>Papers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>59</td>
</tr>
<tr>
<td>C</td>
<td>74</td>
</tr>
<tr>
<td>D</td>
<td>72</td>
</tr>
</tbody>
</table>

2.3.2 Test of Creativity in Mathematics
The test of Creativity in Mathematics consists of 10 items, with discriminating power greater than 2.58 and internal consistency greater than 0.40. Reliability of the test was established through Chronbach’s alpha coefficient and is found to be 0.96 and Inter score reliability (0.69). The validity of the test was established by criterion-related validity and the validity coefficient was found to be 0.67. Norms were also established. One example for the items in test of Creativity in Mathematics is described as below:
Eg: Which shape did it receive when we increase the number of sides of a polygon infinitely?

a. Star  b. Point  c. Circled. cannot determine the shape

2.4 Statistical Techniques Used in the Study
The various statistical techniques used in the study for Analysis includes:
1. Spearman’s Product moment correlation
2. Test of Significance of correlation

3. Analysis and Discussion
The following Table shows the relationship between: Mathematical Thinking and Creativity in Mathematics; Creativity in Mathematics and Fundamental processes of Mathematical Thinking and Mathematical Thinking and components of Creativity in Mathematics. Correlations between the variables were found out by Spearman’s product moment of correlation.

<table>
<thead>
<tr>
<th>Dependent &amp; Independent Variables</th>
<th>r</th>
<th>t</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Thinking and</td>
<td>0.17</td>
<td>3.835</td>
<td>P&lt;0.01</td>
</tr>
<tr>
<td>Creativity in Mathematics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creativity in Mathematics and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fundamental Processes of</td>
<td>(i) Specialization 0.16</td>
<td>3.54</td>
<td>P&lt;0.01</td>
</tr>
<tr>
<td>Mathematical Thinking</td>
<td>(ii) Generalization 0.19</td>
<td>4.3</td>
<td>P&lt;0.01</td>
</tr>
<tr>
<td>(iii) Conjecturing 0.11</td>
<td>2.4</td>
<td>P&lt;0.05</td>
<td></td>
</tr>
<tr>
<td>(iv) Convincing 0.07</td>
<td>1.67</td>
<td>P&gt;0.01</td>
<td></td>
</tr>
<tr>
<td>Mathematical Thinking and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Components of Creativity in</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics</td>
<td>(i) Fluency 0.14</td>
<td>3.25</td>
<td>P&lt;0.01</td>
</tr>
<tr>
<td>(ii) Flexibility 0.17</td>
<td>3.743</td>
<td>P&lt;0.01</td>
<td></td>
</tr>
<tr>
<td>(ii) Originality 0.13</td>
<td>2.82</td>
<td>P&lt;0.01</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 reveals that there exists significant positive relationship between Mathematical thinking and Creativity in Mathematics among secondary school students at 0.01 level of significance with df 498, since the obtained value exceeds the table value 2.57. This implies that Creativity in Mathematics is related to Mathematical Thinking among students at secondary level.

It is also clear from the Table 2 that Creativity in Mathematics is significantly related to three fundamental processes of Mathematical thinking, i.e., with Generalization and Specialization at 0.01 level and with conjecturing at 0.05 level of significance with df value 498. This implies that the fundamental processes Specialization, Generalization and Conjecturing of Mathematical Thinking are significantly related to Creativity in Mathematics. Thus the study revealed that there is significant positive relationship between Mathematical thinking and Creativity in Mathematics. This implies that Mathematical thinking (X) has caused Creativity in Mathematics (O) and the variable Creativity in Mathematics (O) has caused Mathematical Thinking (X). According to Cohen, Manion & Morrison [33], there is possibility of other variables that may
cause Mathematical Thinking (X) and Creativity in Mathematics (O). Again the study by Jaleel [8] revealed that Mathematical thinking is significantly related to Achievement in Mathematics (r = 0.22; t = 2.36, P<0.05 with df =238). Again it revealed that the fundamental process Convincing of Mathematical Thinking is significantly related to Achievement in Mathematics (r = 0.32; t = 3.54, P<0.01 with df =238). So it is concluded that Achievement in Mathematics is caused by Mathematical Thinking in General and by the fundamental process convincing of Mathematical Thinking.

The Table 2 shows that Mathematical thinking is significantly related to all the three components of Creativity in mathematics namely Fluency, Flexibility and Originality at. 01 level of significance with df 498. This implies that Mathematical thinking is significantly related to all the components of Creativity in mathematics. This implies that Mathematical Thinking (X) is caused by components of creativity in Mathematics (O) and components of Creativity in Mathematics (O) have caused Mathematical Thinking (X). Again from the study of Cohen, Manion & Morrison [33], there is a possibility that other unidentified or unmeasured variables that may cause creativity in Mathematics and components of creativity in Mathematics.

The study of Jaleel and Titus [8] shows that Achievement in Mathematics is significantly related to Creativity in Mathematics (r = 0.61; t = 11.88, P<0.01 with df =238). The study also revealed that Achievement in Mathematics is significantly related to the components of Creativity in Mathematics namely Fluency (r = 0.58; t = 10.98, P<0.01 with df =238), Flexibility (r = 0.58; t = 10.98, P<0.01 with df =238) and Originality (r = 0.55; t = 10.16, P<0.01 with df =238). This implies that the Achievement in Mathematics is caused by Creativity in Mathematics and the components of Creativity in Mathematics namely Fluency, Flexibility and Originality. These results lead to the major conclusion that Mathematical thinking and Creativity in Mathematics have an influence on Achievement in Mathematics. Otherwise it can be said that Achievement in Mathematics is caused by both Mathematical Thinking and Creativity in Mathematics. From the above discussions it is evident about the importance of Mathematical thinking and Creativity in Mathematics on Achievement in Mathematics. These findings suggest that Instructions giving proper importance to Mathematical Thinking and Creativity in Mathematics will surely benefit students in their Achievement in Mathematics.

4. Findings and Conclusion
The study revealed that there exists significant positive relationship between Mathematical Thinking and Creativity in Mathematics among Secondary School Students. The present study also revealed that there exists significant positive relationship between Creativity in Mathematics and fundamental processes – Specializing and Generalizing of Mathematical Thinking. Similarly there exist a significant relationship between Mathematical Thinking and the components of Creativity in Mathematics. The findings suggest that Mathematical Thinking has caused Creativity in Mathematics and vice versa. The study also revealed that proper Mathematical Thinking and Creativity in Mathematics may lead to better Achievement in Mathematics.

5. References
16. Sheffield L. Developing Mathematical Creativity- Questions may be the answer. In R. Leikin, A. Berman, & B. Koichu (Eds.), Creativity in Mathematics and the