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Linear programming method for portfolio selection and optimal financial investment in a developing economy

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Abstract
Portfolio selection and optimal financial investment in a developing economy was studied using Linear Programming method. We particularly considered the Nigerian economy and got the values for liquidity, dividend and risk from five establishment we used from the record of the Central bank of Nigeria. We took the ratio of each company's dividend, risk and liquidity to the total for each of these parameters and then formulated the effectiveness function and then the constraint equations. Thus we obtained a linear programming problem which we solved from where we got the three best stocks to invest in and the capital required for such investment.

Keywords: Portfolio, Investment, linear programming, developing economy, optimal

Introduction
Efficient Diversification of Investment by Harry Markowitz (1959) [11, 13] is the pillar on which the concept of Modern Portfolio Theory (MPT) stands. This model has to do with choosing from a collection of different investment options especially on stocks. The main aim of constructing a portfolio is to maximize profit at a most minimal risk level, given that the right choices of assets are made and the right fraction of investment fund is allocated to each stock. In investment, actual returns of portfolios might vary significantly from their expected values. This is as a result of the risk factor involved. Financial risk is as a result of the deviation from the expected value composed of both below and above expected risks outcomes due to positive surprises or non-occurrence of anticipated negative events. Investment simply means buying shares, property or goods because one hopes that the value will increase and thus can make profit. One needs to properly manage his or her investment in order to make profit. Since early 1990, investment management process has become more industrial and scientific for some reasons which include:

1. Need for a structured process with documented steps and measurable results
2. Asset managers have to choose from several possible investment portfolios.
3. Pressure from regulatory bodies and the media
4. The large size of the market makes it important to adopt a safe and reputable method in investing someone's money.

The investment management process involves the following steps

1. Setting investment objective
2. Establishing an investment policy
3. Selecting an investment strategy
4. Selecting the specific assets
5. Measuring and evaluating investment performance.

Modern portfolio theory (MPT) was developed in the 1950 through the early 1970s by Harry Markowitz and was considered an important advance in the mathematical modeling of finance. Modern portfolio theory is a theory of finance which attempts to maximize portfolio's expected return for a given amount of portfolio risk or minimize risk for a given level of expected return by carefully choosing the proportions of various assets.

Modern portfolio theory models asset return as a normally distributed function, defines risk as the standard of deviation of return and models of portfolio as a weighted combination of asset so that the portfolio return is also weighted combination of asset returns. The main concept behind modern portfolio theory is that the assets in an investment portfolio should not be selected individually. It is important to check how each asset changes in price relative to how every other asset in the portfolio changes in price. Generally, asset with higher expected returns are riskier. Therefore modern Portfolio theory is a form of diversification. It explains how to find the best possible diversification method under certain assumptions and for specific quantitative definition of risk and return. Markowitz (1991), Elton and Gruber (1997) discussed the main issues that an investor faces when making a choice in an investment, for example the securities to invest in and how to allocate resources among the variety of different securities. These led to the discussion of portfolio theories especially the modern portfolio theory (MPT) which was developed by Nobel prize award economist Harry Markowitz. His theory is the opposite of traditional asset selection. In this work we are to find out how an investor can use MPT to achieve a higher return than investing in an index portfolio. The investor should diversify and also maximize expected return, that is, he should diversify his funds among all those securities which give maximum expected return. The law of large numbers will ensure that the actual yield of the portfolio will be almost the same as the expected yield. The law of large number states that there is a portfolio which gives both maximum expected return and minimum variance and this portfolio is recommended to the investor. An investor can reduce risk by holding a diversified portfolio of assets. Diversification may give the same expected portfolio return with reduced risk.

Optimal Model Construction

The fundamental premise of modern portfolio theory is that any stock has the probability to increase or decrease in any parameter depending on the market and thus their inclusion or exclusion in a portfolio does not matter individually. When these stocks are placed together the interaction between them reduce the overall price volatility which contributes to the stability of the portfolio. All portfolios are designed with return in mind and with the modern portfolio theory one can choose an expected return and work towards minimizing the risk in investing in a combination of stocks or maximizing return of the portfolio. We are tempted to invest more in a stock with a higher dividend with the hope of making profit which may not be achieved because there are other factors which we must consider to ensure we make profit, for example: risk and liquidity. It is known that when we take the mean of the dividend of the stocks we want to invest in, we look at the deviation from mean to enable us choose the right stock to invest in.

Risk is the force that ultimately drives the return of an investment up or down. An investor that takes on an investment is putting himself in a risky situation. This risk has to be managed and the long term potential of an investment depends on the investor being willing and prepared for the risk. One has to be ready for the risk and also has to decide the potential loss that can be afforded, (Litterman 2003). Successful investments are those assets with the best risk reward relationship. The concept of portfolio theory is to maximize the relationship between risk and reward (Bodie *et al*, 2004) [3]. The chance that an investments actual return will be different than expected is the portfolio risk. This is the possibility of loosing some or all of the original investment. The standard deviation is the measurement of uncertainty associated with the asset, a measure of the change or the movement of a set of data from its mean. The more spread the data is the higher the deviation. The major consequence of lower risk is that it allows investment manager to measure allocation to equities thereby increasing portfolio return. The frame of reference here is the classic Markowitz portfolio problem which requires return, risk and correlation prediction for successful implementation plus the liquidity factor consideration for each asset or the organization in question. On the liquidity level, we have that the higher the liquidity the more likely it is to invest in a stock. High liquidity implies low risk when one invests in shares of that organization and high return when sold at the right time.

Literatures reviewed, Nwodobie (2013) [14], on mean-variance analysis of efficient portfolio selection in the Nigerian capital market showed that the effect of the instrument - liquidity has not been used in investment on stocks. In calculating risk, the issue of liquidity has not been incorporated, therefore liquidity must come in on its own as a factor. In this work, we included the instrument - liquidity, knowing that most establishment in a growing economy like Nigeria lives on borrowed funds. We got our data from the annual report of the organizations involved which are zenith bank, fidelity bank, diamond bank, dangote cement and 7up bottling company. We values for the current asset, current liabilities, earnings per share and others for the establishment involved, from their annual reports. We used these data in calculating the values for dividend, risk and liquidity respectively for these organizations. The data used are shown as:

Table 1

	Zenith Bank (P)	Fidelity Bank (Q)	Diamond Bank (X)	Dangote Cement (Y)	7up Bottling (Z)
Dividend (per ₦)	29.00	4.06	4.34	6.00	2.20
Risk Factor (%)	4.20	1.19	1.67	20.83	6.03
Liquidity (₦ trillions)	1.85	1.23	1.12	2.00	1.39

Source: Annual report of the above establishment

For a developing economy like that of Nigeria, one needs to consider the liquidity of an organization before investing in it's shares. In Nigeria, most organizations are living on borrowed funds and large profit does not mean it must be declared as dividend since the organization is owing. Borrowed funds must be offset before reasonable dividend payment is made to shareholders. The ratio of each instrument (dividend, risk and liquidity) to the other is taken to establish the effect of each on the other and on the portfolio.

We took the ratios of each company's dividend, risk and liquidity to the total for each of the parameters, that is the

total dividend, risk and liquidity which is 45.60, 33.92 and 7.59 respectively. This is to establish the effect of each parameter on the other. To determine the values of the coefficients in the effectiveness function, we multiplied the return on investment for each company by its liquidity and then divided by the risk on investment for that company, respectively. This is also to establish the effect of each parameter on the other, hence we have the linear programing problem:

$$\text{Max } Z = 12.77P + 4.20Q + 2.91X + 0.58Y + 0.51Z$$
$$\text{St: } 0.64P + 0.09Q + 0.10X + 0.13Y + 0.05Z \leq 45.60$$
$$0.12P + 0.04Q + 0.05X + 0.61Y + 0.18Z \leq 33.92$$
$$0.24P + 0.16Q + 0.15X + 0.26Y + 0.18Z \leq 7.59$$
$$P,Q,X,Y,Z, S_1, S_2, S_3 \geq 0$$

which we can write in a tabular form and then solve by simplex method to obtain the stock to invest in and the amount to be invested on the said stock. Thus, we have

Solution

Table 2

	Cj		**12.77**	**4.20**	**2.91**	**0.58**	**0.51**	**0**	**0**	**0**	
Row		**Sol.**	**P**	**Q**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P0**
1	0	S_1	0.64	0.09	0.10	0.13	0.05	1	0	0	45.60
2	0	S_2	0.12	0.04	0.05	0.61	0.18	0	1	0	33.92
3	0	S_3	0.24	0.16	0.15	0.26	0.18	0	0	1	7.59
		Z	0	0	0	0	0	0	0	0	
	Cj	Zj	12.77	4.20	2.91	0.58	0.51	0	0	0	

We now have the next table as

Table 3

	Cj		**0.288**	**18.57**	**36.83**	**2.01**	**3.67**	**0**	**0**	**0**	
Row		**Soln**	**P**	**Q**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P0**
1	0	S_1	0	-0.34	-0.30	-0.56	-0.43	1	0	-2.67	25.34
2	0	S_2	0	-0.04	-0.03	0.48	0.09	0	1	-0.50	30.12
3	12.77	P	1	0.67	0.63	1.08	0.75	0	0	4.17	31.65
		Zj	12.77	8.56	8.05	13.79	9.58	0	0	53.25	404.17
		Cj-Zj	0	-4.36	-5.14	-13.21	-9.07	0	0	-53.25	

Max Z = = 404.17 for P = 31.65.

Table 4: Choosing the second best stock to invest in using the data below

	Fidelity Bank (Q)	**Diamond Bank (X)**	**Dangote Cement (Y)**	**7up Bottling (Z)**
Dividend (per ₦)	4.06	4.34	6.00	2.20
Risk Factor (%)	1.19	1.67	20.83	6.03
Liquidity (₦ trillions)	1.23	1.12	2.00	1.39

From the table above, we retake the ratios of the parameters respectively to the total. This will also give us a maximization linear programing problem which we will solve to enable us make the second choice of the stock to invest in. Hence we have the maximization problem

$$\text{Max } Z = 4.20Q + 2.91X + 0.58Y + 0.51Z$$
$$\text{St: } 0.25Q + 0.26X + 0.36Y + 0.13Z \leq 16.6$$
$$0.04Q + 0.06X + 0.70Y + 0.20Z \leq 29.72$$
$$0.21Q + 0.20X + 0.35Y + 0.24Z \leq 5.74$$
$$Q,X,Y,Z, S_1, S_2, S_3 \geq 0$$

Solution

Table 5

	Cj		**4.20**	**2.91**	**0.58**	**0.51**	**0**	**0**	**0**	
Row	**0**	**Soln**	**Q**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P0**
1	0	S_1	0.25	0.26	0.36	0.13	1	0	0	16.60
2	0	S_2	0.04	0.06	0.70	0.20	0	1	0	29.72
3	0	S_3	0.21	0.20	0.35	0.24	0	0	1	5.74
4	0	Zj	0	0	0	0	0	0	0	0
5		Cj-Zj	4.20	2.91	0.58	0.51	0	0	0	

We have the next table as follows

Table 6

	Cj		4.20	2.91	0.58	0.51	0	0	0	
Rows		**Soln**	**Q**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P**
1	0	S_1	0	0.02	-0.06	-0.16	1	0	-1.19	9.77
2	0	S_2	0	0.02	0.63	0.15	0	1	-0.19	28.63
3	4.20	Q	1	0.95	1.67	0.14	0	0	4.76	27.33
		Zj	4.20	3.99	7.01	4.79	0	0	19.99	114.79
		Cj-Zj	0	-1.08	-6.43	-4.28	0	0	-19.99	

Max Z = 114.79 for Q = 27.33

Table 7: Choosing the second best stock to invest in using the data below

	Diamond Bank (X)	Dangote Cement (Y)	7up Bottling (Z)
Dividend (per ₦)	4.34	6.00	2.20
Risk Factor (%)	1.67	20.83	6.03
Liquidity (₦ trillions)	1.12	2.00	1.39

From the table above, we retake the ratios of the parameters respectively to the total. This will also give us a maximization linear programing problem which we will solve to enable us make the third choice of the stock to invest in. Hence we have the maximization problem

$$\text{Max } Z = 2.91X + 0.58Y + 0.51Z$$
$$\text{St: } 0.35X + 0.48Y + 0.18Z \leq 12.54$$
$$0.06P + 0.73Y + 0.21Z \leq 28.53$$
$$0.25P + 0.44Y + 0.31Z \leq 4.51$$
$$X,Y,Z, S_1, S_2, S_3 \geq 0$$

Solution

Table 8

	Cj		2.91	0.58	0.51	0	0	0	
Row		**Soln**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P**
1	0	S_1	0.35	0.48	0.18	1	0	0	12.54
2	0	S_2	0.06	0.73	0.21	0	1	0	28.53
3	0	S_3	0.25	0.44	0.31	0	0	1	4.51
		Zj	0	0	0	0	0	0	
		Cj- Zj	2.91	0.58	0.51	0	0	0	

We have the next table as follows

Table 9

	Cj		2.91	0.58	0.51	0	0	0	
Row		**Soln**	**X**	**Y**	**Z**	**S_1**	**S_2**	**S_3**	**P**
1	0	S_1	0	-0.14	-0.25	1	0	-1.40	6.23
2	0	S_2	0	0.62	0.14	0	1	-0.24	27.45
3	2.91	X	1	1.76	1.24	0	0	4	18.04
		Zj	2.91	5.13	3.61	0	0	11.64	52.50
		Cj-Zj	0	-4.55	-3.10	0	0	-11.64	

Max Z = 52.50 for X = 18.04

Out of the above five stocks we have chosen the best three to invest in, they are stocks P, Q and X.

Results and Discussion

In this work, we took the ratios of the variables, that is return, risk and liquidity before forming the equation. This is to establish the effect of each variable on the other and ultimately on the optimal result.

We also used the ordinary simplex method because it will allow us choose the best three stocks to go for among the five stocks which we are to choose from. It also tells us how much we can invest in each to optimize profit.

The choice of the second and third stocks to invest in were not dependent on the first choice because the ratios were calculated afresh in order to choose the next stock to invest in. In making the first choice, we have that investing =N= 31.64 in stock P gives us a maximum yield of 404.17. In making the second choice, we have that investing =N= 27.33 of the total fund in stock Q will give a maximum yield of =N= 114.79. In making the third choice, we have that investing =N=18.04 of the total fund in stock X will give us a maximum yield of =N=52.50. This gives a good idea of what is expected from the investment should the stated amounts are actually invested. This method can therefore be extended to any number of selected stocks from where an investor wish to select the required portfolio that he actually wish to hold. This is actually a new method of portfolio selection which takes into consideration the liquidity of the organization and goes beyond only selecting the portfolio but goes ahead to equally state how much should be invested on each of the selected stock to have optimal yield from the portfolio. In this work, we can see that the Simplex method did not give the values required to be invested on each of the stocks at one time rather, we obtained the respective values separately. Reason for this behavior is not quite clear but we cannot conclude that only one stock can optimally give the best dividend coupled with the fact that diversification of investment is highly encouraged to reduce

loss. Equally, the financial leverage of the investor is another determinant of how many stocks to hold at a time. Thus, this is a good method for determination of the stocks to invest in and how much to invest at the same time.

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