Introduction to fuzzy Logic

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Abstract
The paper presents a brief introduction to fuzzy logic concepts. The concepts of fuzzy logic can be applied to real world problems to get required solutions. The concept of fuzziness resembles human thought process and is therefore popular in practical life to give optimistic solutions to real world problems.

Keywords: Fuzzy logic, real world, human thoughts, practical life.

1. Introduction
As we know that human brain thinks in an imprecise and qualitative manner (non-numeric) like the milk is hot. Here temperature of hotness is not specified. Likewise there can be many more examples where humans thought in a fuzzy manner. To deal with imprecise, ambiguous thought process the concept of fuzzy logic was introduced. The basic concepts and application of fuzzy logic can be studied by using the research references (Lau et al., 2003 [1]; Wang and Lin, 2003 [3]; Cochran and Chen, 2005 [3]; Garg et al., 2007 [4]; Khatatnech and Mustafa, 2009 [5]; Bailador and Trivino, 2010 [6]; Rafik et al., 2014 [7]; Meimei and Zeshui, 2014 [8]; Zadeh, 1965 [9]). The next section presents brief introduction to fuzzy concepts.

2. Basics
The dictionary meaning of fuzzy is blurred, indistinct. The concept of fuzziness deals with uncertainty, imprecision, ambiguity, inconsistency, vagueness of situations. Fuzzy sets have no precise boundaries (Zadeh [9]). The membership of fuzzy set is measured using degree of membership. The fuzzy sets show gradual transition from membership to non-membership and vice-versa. The range of membership functions is the interval [0, 1]. The membership function of a fuzzy set A is denoted by \( \mu_A \),

\[ \mu_A: X \rightarrow [0,1], \text{ where } X \text{ is a universal set} \]

Degree of membership is 0 when the element is not in set, degree of membership is 1 when the element is in the set. Various functions and operations like union, intersection, addition, subtraction, multiplication, division can be applied to fuzzy sets.

2.1 Fuzzy membership functions
There exist multiple fuzzy membership functions like \( \Gamma \)-functions (increasing membership functions with straight lines), \( L \)-functions (decreasing membership functions with straight lines), \( S \)-functions, Bell shaped functions, \( \lambda \)-functions (triangular functions). The use and application of the membership functions depends situation of application. The most commonly used function is the triangular function. The triangular fuzzy membership function [9], denoted as \( \lambda: X \rightarrow [0, 1] \) is defined as follows:

\[
\lambda(x;a,b,c) = \begin{cases} 
(x - a)/(b - a), & a \leq x \leq b \\
(c - x)/(c - b), & b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
\]
Where a, b, c are real numbers $a \leq b \leq c$. The triangular fuzzy function can be represented graphically as shown in Figure 1:

![Fig 1: Triangular fuzzy membership function graph](image)

### 2.2 Fuzzy Operations

All the basic operations like addition, subtraction, multiplication, division can be applied to fuzzy sets. The basic example of addition and multiplication is discussed here. Given two fuzzy sets $A_1$ and $A_2$, defined as triangular membership functions as $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$. The addition and multiplication operation can be expressed as:

**Addition:** Let $\oplus$ denotes addition

$$A_1 \oplus A_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

**Multiplication:** Let $\otimes$ denotes multiplication

$$A_1 \otimes A_2 = (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2)$$

### 2.3 Fuzzy variables

Fuzzy theory deals with uncertainty and ambiguity in human thought process and quantify it in linguistic terms. The natural linguistic terms used in common usage are closer to human perceptions and thoughts than crisp numeric values. A linguistic variable is some non-numeric syllable/term used in natural usage. The linguistic variables can be defined as high, low, medium or good, bad, worse and there can be many more possibilities. Then fuzzy memberships are given to each of the variables. Let us define linguistic variables like High (H), Medium (M), Low (L). The membership values can be associated with each of the linguistic variables based on certain criteria as shown in Table 1.

<table>
<thead>
<tr>
<th>Linguistic Variable</th>
<th>High (H)</th>
<th>Medium (M)</th>
<th>Low (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy membership</td>
<td>(0.5,0.8,1)</td>
<td>(0.3,0.5,0.8)</td>
<td>(0,0.3,0.5)</td>
</tr>
</tbody>
</table>

Likewise membership functions can be plotted via graph as shown in Figure 2 for variables define in Table 1.

![Fig 2: Fuzzy membership graph for weighting criteria](image)

### 2.4 Applications

Fuzzy Logic finds applications in each and every area be it engineering, medical, geography, weather forecasting, sciences, cosmology etc. Also fuzzy logic finds application in multi-criteria analysis of certain objects. The applications of fuzzy logic are so vast that it cannot be enumerated in simple terms. A general multi-criteria problem that can be solved using fuzzy logic is defined as follows:

A team of $n$ experts, has to analyze and grant weights to $k$ criteria and the ratings to $m$ quality metrics for each of the $k$ criteria.

### 3. Conclusion

The paper presents the overview of basic fuzzy concepts that can be applied to practical world in order to give optimistic solutions to certain problems. The concept of fuzzy membership functions, fuzzy linguistic variables, membership plots, operations on fuzzy sets are discussed. The work of the paper can be extended by giving real world applications of fuzzy logic to give optimistic solutions to certain problems.

### 4. References