V₄ Cordial Labeling of Fan and Globe

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Abstract

Let $<A,*>$ be any abelian group. A graph $G = (V(G), E(G))$ is said to be $A$-cordial if there is a mapping $f: V(G) \rightarrow A$ which satisfies the following two conditions with each edge $uv$ labeled as $f(u) * f(v)$.

(i) $|v_f(a) - v_f(b)| \leq 1, \forall a,b \in A$

(ii) $|e_f(a) - e_f(b)| \leq 1, \forall a,b \in A$

where $v_f(a)$= the number of vertices with label $a$.
$v_f(b)$= the number of vertices with label $b$.
$e_f(a)$= the number of edges with label $a$.
$e_f(b)$= the number of edges with label $b$.

We note that if $A = <V_4,*>$ is a multiplicative group. Then the labeling is known as $V_4$ Cordial Labeling. A graph is called a $V_4$ Cordial graph if it admits a $V_4$ Cordial Labeling. In this paper, we proved that $F_n= P_n+K_1$ and Globe $(G(l))$ are $V_4$ Cordial graphs.

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1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary[4]. For labeling of graphs, we referred Gallian[1].

A vertex labeling of a graph $G$ is an assignment of labels to the vertices of $G$ that induces for each edge $uv$ a label depending on the vertex labels of $u$ and $v$.

A graph $G$ is said to be labeled if the $n$ vertices are distinguished from one another by symbols such as $v_1, v_2,...,v_n$. In a labeling of a particular type, the vertices are assigned values from a given set, the edges have a prescribed induced labeling must satisfy certain properties. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2].

2. Preliminaries

Definition 2.1: Let $G = (V,E)$ be a simple graph. Let $f:V(G)\rightarrow \{0,1\}$ and for each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1 differ by atmost 1. A graph is called Cordial if it has a cordial labeling.

Definition 2.2: Let $<A,*>$ be any abelian group. A graph $G = (V(G), E(G))$ is said to be $A$-cordial if there is a mapping $f: V(G) \rightarrow A$ which satisfies the following two conditions with each edge $e= uv$ is labeled as $f(u)*f(v)$.

(i) $|v_f(a) - v_f(b)| \leq 1, \forall a,b \in A$

(ii) $|e_f(a) - e_f(b)| \leq 1, \forall a,b \in A$

where $v_f(a)$= the number of vertices with label $a$.
$v_f(b)$= the number of vertices with label $b$.
$e_f(a)$= the number of edges with label $a$.
$e_f(b)$= the number of edges with label $b$.

We note that if $A = <V_4,*>$ is a multiplicative group. Then the labeling is known as
V₄ Cordial Labeling. A graph is called a V₄ Cordial graph if it admits a V₄ Cordial Labeling.

**Definition 2.3**
Fan $F_n = P_n + K_1$ is obtained from the Path $P_n$ by joining each vertex of $P_n$ to a vertex $u$.

**Definition 2.4**
Globe is a graph obtained from two isolated vertex are joined by $n$ paths of length 2. It is denoted by (Gl(n)).

3. Main Results

**Theorem 3.1**
$F_n = P_n + K_1$ is a V₄Cordial graph.

**Proof:** Let $V = \{1, -1, i, -i\}$.

Let $E (P_n + K_1) = \{(u_i u_j) : 1 \leq i \leq n\} \cup \{(u_i u_{i+1}) : 1 \leq i \leq n - 1\}.$

Define $f : V \rightarrow V_4.$

The vertex labeling are,

$$f(u_i) = \begin{cases} 
1 & \text{if } i \equiv 0 (\text{mod } 8) \\
-1 & \text{if } i \equiv 1, 6 (\text{mod } 8) \\
i & \text{if } i \equiv 2, 5 (\text{mod } 8), 1 \leq i \leq n \\
-1 & \text{if } i \equiv 3, 7 (\text{mod } 8)
\end{cases}$$

The edge labeling are,

$$f(u_i u_{i+1}) = \begin{cases} 
-1 & \text{if } i \equiv 0, 2 (\text{mod } 8) \\
i & \text{if } i \equiv 1, 5 (\text{mod } 8) \\
-1 & \text{if } i \equiv 3, 7 (\text{mod } 8), 1 \leq i \leq n - 1 \\
i & \text{if } i \equiv 4, 6 (\text{mod } 8)
\end{cases}$$

**Vertex Conditions**
(i) $v_f(1) = \frac{n}{4} + 1$ and $v_f(i) = v_f(-i) = v_f(-1) = \frac{n}{4}$ when $n \equiv 0, 4 \pmod{8}$
(ii) $v_f(1) = v_f(-i) = \frac{n-1}{4} + 1$ and $v_f(i) = v_f(-1) = \frac{n-1}{4}$ when $n \equiv 1 \pmod{8}$
(iii) $v_f(1) = v_f(i) = v_f(-i) = \frac{n-2}{4} + 1$ and $v_f(-1) = \frac{n-2}{4}$ when $n \equiv 2, 6 \pmod{8}$
(iv) $v_f(1) = v_f(-i) = v_f(-1) = \frac{n+1}{4}$ when $n \equiv 3, 7 \pmod{8}$
(v) $v_f(1) = v_f(i) = \frac{n-1}{4} + 1$, and $v_f(-i) = v_f(-1) = \frac{n-1}{4}$ when $n \equiv 5 \pmod{8}$

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1, \forall a, b \in V_4$

**Edge Conditions**
(i) $e_f(i) = e_f(-i) = e_f(-1) = \frac{n}{2}$, and $e_f(-i) = \frac{n-2}{2}$, when $n \equiv 0 \pmod{8}$
(ii) $e_f(1) = e_f(-1) = e_f(i) = \frac{n-1}{2}$ and $e_f(-i) = \frac{n+1}{2}$ when $n \equiv 1 \pmod{8}$
(iii) $e_f(1) = e_f(i) = e_f(-i) = \frac{n}{2}$ and $e_f(-1) = \frac{n-2}{2}$, when $n \equiv 2, 6 \pmod{8}$
(iv) $e_f(1) = e_f(i) = e_f(-i) = \frac{n-1}{2}$ and $e_f(-1) = \frac{n+1}{2}$ when $n \equiv 3 \pmod{8}$
(v) $e_f(1) = e_f(-i) = e_f(-1) = \frac{n-1}{2}$ and $e_f(i) = \frac{n+1}{2}$, when $n \equiv 5, 7 \pmod{8}$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1, \forall a, b \in V_4$

Hence, $F_n = P_n + K_1$ is a V₄Cordial Graph.

For example, the V₄Cordial Labeling of $P_4, P_5, P_6, P_{11}, P_{13}$ and $P_7$ is shown in below figure 3.21-3.27.
When $n \equiv 0, 4 \pmod{8}$

Fig 3.21

When $n \equiv 1 \pmod{8}$

Fig 3.22

When $n \equiv 2, 6 \pmod{8}$

Fig 3.23

Fig 3.24
whenn≡ 3,7 (mod 8)

Fig 3.25

whenn≡ 5 (mod 8)

Fig 3.26

whenn≡ 5 (mod 8)

Fig 3.27

Theorem 3.3
Globe (Gl(n)) is a is a V₄ Cordial graph, when n≡0,1,3(mod 4).
Proof: Let \( V₄ = \{1, -1, i, -i\} \).
Let \( V(Gl(n)) = \{u, v, w_i; 1 \leq i \leq n\} \).
Let \( E(Gl(n)) = \{(uw_i) : 1 \leq i \leq n\} \cup \{(vw_i) : 1 \leq i \leq n\} \).
Define \( f: V(Gl(n)) \to V₄ \).

Case(I)
When \( n≡0 \) (mod 4)
The vertex labeling are,
Let \( f(u) = 1, f(v) = -1 \).
The edge labeling are,
\[
f(w_i) = \begin{cases} 
1 & \text{if } i \equiv 0 \pmod{4} \\
-1 & \text{if } i \equiv 1 \pmod{4} \\
i & \text{if } i \equiv 2 \pmod{4}, \ 1 \leq i \leq n \\
-i & \text{if } i \equiv 3 \pmod{4}
\end{cases}
\]

Vertex Conditions
(i) \( v_f(1) = v_f(-1) = \lfloor \frac{n}{4} \rfloor + 1 \) and \( v_f(i) = v_f(-i) = \lfloor \frac{n}{4} \rfloor \)
Hence, it satisfies the condition of \( |v_f(a) - v_f(b)| \leq 1, \forall \ a,b \in V_4 \)

Edge Conditions
(i) \( e_f(1) = e_f(-1) = \frac{n}{2} \)
Hence, it satisfies the condition of \( |e_f(a) - e_f(b)| \leq 1, \forall \ a,b \in V_4 \)
Hence, \( (G_l(n)) \) is a \( V_4 \)-Cordial Graph.
For example, the \( V_4 \)-Cordial Labeling of \( (G_l(n)) \) is shown in below figure 3.41

![Figure 3.41](image)

Case(II)
when \( n \equiv 1 \pmod{4} \)
The vertex labeling are,
Let \( f(u) = 1, f(v) = i \)
\[
f(w_i) = \begin{cases} 
1 & \text{if } i \equiv 0 \pmod{4} \\
-1 & \text{if } i \equiv 1 \pmod{4} \\
i & \text{if } i \equiv 2 \pmod{4}, \ 1 \leq i \leq n \\
-i & \text{if } i \equiv 3 \pmod{4}
\end{cases}
\]

The edge labeling are,
\[
f(uw_i) = \begin{cases} 
1 & \text{if } i \equiv 0 \pmod{4} \\
-1 & \text{if } i \equiv 1 \pmod{4} \\
i & \text{if } i \equiv 2 \pmod{4}, \ 1 \leq i \leq n \\
-i & \text{if } i \equiv 3 \pmod{4}
\end{cases}
\]

Fig 3.41
Case(III) when $n \equiv 3 \pmod{4}$

The vertex labeling are,

Let $f(u) = 1, f(v) = -i$ and $f(w_i) = 1$.

\[
f(w_i) = \begin{cases} 
1 & \text{if } i \equiv 0 \pmod{4} \\
-1 & \text{if } i \equiv 1 \pmod{4} \\
i & \text{if } i \equiv 2 \pmod{4}, 1 \leq i \leq n-1 \\
-1 & \text{if } i \equiv 3 \pmod{4} 
\end{cases}
\]

The edge labeling are,

Let $f(uw_i) = 1$ and $f(vw_i) = -i$.

\[
f(uw_i) = \begin{cases} 
1 & \text{if } i \equiv 0 \pmod{4} \\
-1 & \text{if } i \equiv 1 \pmod{4} \\
i & \text{if } i \equiv 2 \pmod{4}, 1 \leq i \leq n-1 \\
-1 & \text{if } i \equiv 3 \pmod{4} 
\end{cases}
\]

Vertex Conditions (i)$v_f(1) = v_f(-1) = v_f(i) = \left[\frac{n}{4}\right]+1$and $v_f(-i) = \left\lceil\frac{n}{4}\right\rceil + 1$.

Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1, \forall a,b \in V_4$.
Hence, it satisfies the condition of $|v_f(a) - v_f(b)| \leq 1, \forall a,b \in V_4$

**Edge Conditions**

(i)$e_f(1) = e_f(i) = \lfloor \frac{n}{2} \rfloor + 1$ and $e_f(-1) = e_f(-i) = \lfloor \frac{n}{2} \rfloor$

Hence, it satisfies the condition of $|e_f(a) - e_f(b)| \leq 1, \forall a,b \in V_4$

Hence, $(G_l(n))$ is a $V_4$Cordial Graph.

For example, the $V_4$Cordial Labeling of $(G_l(n))$ is shown in below figure 3.43

**Fig 3.43**

4. **References**


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