A study on the significance of game theory in mergers & acquisitions pricing

Yonus Ahmad Dar and Dr. Yogesh Sharma

Abstract
Mergers and acquisitions pricing is usually a difficult task as it is hard to estimate the price to be paid to purchase the target company. Corporate Finance valuations provide some support, however, Baker, Pan and Wurgler (2009) explain that psychological pricing factors exist and the final price can be quite different from the initial valuations.

This paper develops a two-person Merger & Acquisition model that intends to provide a simulation model in order to understand if the final negotiated price of a Merger & Acquisition transaction equals the mid-point of the acquirer’s offer and target’s bid price. Also, it reviews behavioral factors that can be part of Merger & Acquisition valuation and uses the model to simulate results to identify, in this case, if it is better for the acquirer to be risk taking or risk averse and for the target to be optimistic or pessimistic.

Keywords: Mergers and acquisitions, game theory, prospect theory

Introduction
Mergers and Acquisitions (M&A) is a significant research area in the field of Corporate Finance. There is substantial finance literature on how M&A transactions can be valued. At present, the investment banking industry uses the concepts of balance sheet valuation models (book value, liquidity value and replacement cost), dividend discount model, price earnings ratio, discount cash flow analysis, transaction multiples and economic profit model to price such transactions.

M&A pricing has been a challenging task, primarily as different valuation methods can provide different results. Due to a biased system of beliefs and psychological anchoring in M&A negotiation these models fall short in providing suitable estimates of offer prices that will be acceptable for M&A transactions and usually under or over-value such transactions. Baker, Pan and Wurgler (2009) [3] and Baker, Ruback and Wurgler (2004) [2] indicate that behavioral factors can impact the pricing of targets in M&A transactions. M&A pricing can been seen as a negotiation between the acquirer and the target to agree on a price at which the acquirer can purchase the target.

As a result, we need to use game theory to review such negotiations. Previous research shows that game theory provides an insight into strategic human interaction in such financial negotiations. Contemporary game theory has two forms: non co-operative game theory (Nash 2001) [11] and co-operative game theory. Von Neumann and Morgenstern (2004) [15] and Nash (2000; 2001) [10, 11] have suggested two game theory approaches to resolve bargaining problems: axiomatic or strategic. The axiomatic approach (sometimes called co-operative theory) assists by providing a set of valuable axioms. On the other hand, the strategic approach models outcomes in a non co-operative game. Mergers and acquisitions would come under the premise of zero sum or non cooperative games.

The objective of this paper is to develop a M&A game theoretic model that will analyze if the offer price provided by an acquirer should be equal to the mid-point of the acquirer’s offer and the target’s bid price? It will also analyze if the acquirer will do better by being risk taking or risk averse and if the target will do better by optimistic or pessimistic in this model.
Merger & acquisition game theoretical model
The two-person M&A model is an incomplete information game between the acquirer and the target, where both players need to agree to a price that will be suitable to them for the sale of the target company to the acquirer. The extensive form for this game depicts a three-stage game between the acquirer (buyer) and target (seller). In this game, the acquirer can choose either the “Increase Bid” or “Reduce Bid” strategy and the target can choose the “Increase Bid” or “Stable Bid” strategy.

This game also has two chance nodes which are associated with probabilities related to the expected behavior of the acquirer if it is either “risk taking” or “risk averse” and if the target is “optimistic” or “pessimistic”. Here, probabilities are assigned based on the expected behavior (type) that the acquirer or target may show.

If these probabilities are changed then the equilibrium point in this game may change as well. Before we move further, we should note that the pay-offs for each strategy is relative to any other strategy in this game. So, the “Increase Bid” strategy has a pay-off of -1 (for the acquirer as it needs to pay more to purchase the target) and +1 (for the target if we consider a zero-sum game).

However, we use the concept called Prospect Theory (Khaneman and Tversky 2009) [5], which states that humans prefer positive events at least twice as much as negative events. Therefore, positive events are divided by 2.25 to equate them to negative events (Metzger and Reiger 2010 for application of prospect theory to game theory) [8]. As a result all positive events would have a pay-off equal to +0.44 instead of +1. On the other hand, the “Reduce Bid” strategy has a pay-off of +1 (for the acquirer or +0.44 after considering prospect theory) and -1 (for the target). The “Stable Bid” strategy will be associated with a pay-off equal to 0. The extensive form (diagrammatic form of the game providing the sequences of strategies played by each player) and normal form (tabular form of the game providing the final outcomes of each strategy pair assuming strategies are played simultaneously) of the two-person M&A model are provided here.

Research Study
Figure 1 provides the strategic form of the two-person M&A model, where the acquirer will make the initial offer and the target will decide on the same node, if he wants to accept that offer. If the offer is rejected, then the acquirer can either increase or reduce his bid in the future that is represented by the next node. An increase in bid occurs when the acquirer wants to purchase the target and is willing to pay more. However, it could be possible that between the time when the initial offer was provided and when the subsequent offer is provided, the target’s value could have decreased due to any number of reasons. As a result, the acquirer can either increase or reduce the offer based on the circumstance at that moment in time.

Similarly, the game will continue till the time where either the target accepts the offer or if the acquirer does not want to provide any further offers as it has reached his limit where he does not believe he would like to pay a higher amount to purchase the target firm.

The strategic form of this game can be translated into the normal form. Strategies in the normal form can be directly translated back to the strategic form for both the acquirer and target. For example, the strategy “III” for acquirer and “IS” for target would mean that the acquirer will initially choose the “Increase Bid” strategy in the first play.

In response to this action the target will choose the “Increase Bid” strategy as well because it follows the strategy “IS” which means it will “Increase Bid” if the acquirer chooses the “Increase Bid” and it will opt for the “Stable Bid” strategy if the acquirer chooses “Reduce Bid”. Similarly, the acquirer will choose the “Increase Bid” strategy as it follow the “III” strategy. This occurs as the acquirer will choose the “Increase Bid” strategy if the target had chosen the “Increase Bid” strategy or the “Stable Bid” strategy.

However, before we move on to develop the normal form of the game, it is important to explain how the pay-offs have been calculated in the strategic form. For each time, the bid has been increased (strategy: “I” Increase Bid) by the acquirer he receives a pay-off of +1 and every time the acquirer reduces the bid (strategy: “R” Reduce Bid) he receives a pay-off of -1.

Therefore, the strategies “RII” and “IRR” will receive a pay-off of 0 for the acquirer because in the first round he will increase the bid and then reduce the bid in the second round. Similarly, the target will receive the opposite of these pay-offs, i.e. +1 for Increase Bid “I” and 0 for keeping the bid Stable - Stable Bid “S”.

Therefore, the pay-off for “SI” and “IS” will be similar for the Target with the pay-off being +1
Based on the strategies and pay-offs provided in the strategic and normal forms of this game, you will notice that the acquirer is at a greater disadvantage than the target as he can lose 3 units if the price is increased by the acquirer and target (upper most point of the strategic form game) compared to when the price is reduced (lower most point of the strategic form game).

This occurs due to the strategies followed by the target. The target company will never reduce the price, but will opt to increase the price where possible. This causes an imbalance in the pay-off structure and it may result in a situation where equilibrium state is more suitable to the target than the acquirer.

If we now consider a two-person merger and acquisition game with incomplete information with the conditions that both target and acquirer, do not know the type of the other party (i.e. probability with which the acquirer will choose “Increase Bid” or “Reduce Bid” or the target will choose “Increase Bid” or “Stable Bid”). In everyday business practice, acquirer and target companies do not disclose their expected offer and bid prices because they do not want the opponent to know how much they are ready to pay or how much they want before they will accept the offer.

In the extensive form of the two-person M&A model, the base case is when the acquirer is risk taking and the target is pessimistic. Further, the pay-offs are decreased by 50 per cent from the base case, if the acquirer is risk averse and the target is pessimistic. On the contrary, the pay-offs are increased by 50 per cent, if the acquirer is risk averse, but the target is optimistic.

The pay-offs are increased 100 per cent from the base case when the acquirer is risk taking and the target is optimistic.

Also, some are optimistic and others are pessimistic. It is possible that not only these but other behavioral traits can be used and even combined to develop more realistic M&A valuation games. The pay-offs will change based on the behavioral characteristics and strategies that are chosen to represent the M&A valuation game. It is inevitable that game theory will be used more often for M&A valuation as it helps support the psychological pricing variances that are not available through traditional Corporate Finance valuation techniques.

Those techniques do provide some theoretical basis to the M&A valuation decision, however due to behavioral biases, it is not possible to obtain an understanding of what would be considered as the most optimal offer (Nash Equilibrium Point) that will provide the best result to the acquirer and the target. Further, the concept of prospect theory that has such a significant impact on pricing, cannot be considered by the traditional valuation model.

Therefore, the two-person M&A model has been developed to help provide a tool that could be used to simulate M&A pricing that can provide a platform for valuing potential M&A transaction that relate to real world situations.
Subdivide the domain into two sub-domains, one from \([0, \frac{1}{2}]\) and another from \([\frac{1}{2}, 1]\). In each of these two sub-domains define interpolating functions \(v_1(x)\) and \(v_2(x)\).

At the interface between these two sub-domains the following interface conditions shall be enforced:

\[ v_1 \left( \frac{1}{2} \right) = v_2 \left( \frac{1}{2} \right) \]
\[ v'_1 \left( \frac{1}{2} \right) = v'_2 \left( \frac{1}{2} \right) \]

Let the interpolating functions be defined as:

\[ v_1(x) = \sum_{n=0}^{N} u_n T_n(y_1(x)) \]
\[ v_2(x) = \sum_{n=0}^{N} u_{n+N} T_n(y_2(x)) \]
\[ y_1(x) = 4x - 1 \]
\[ y_2(x) = 4x - 3 \]

Where \(T_n(y)\) is the \(n\)th cardinal function of the Chebyshev polynomials of the first kind with input argument \(y\).

It is important to recall that we are dealing with an incomplete game, where the acquirer does not know if the target is optimistic or pessimistic and the target does not know if the acquirer is risk-taking or risk-averse. Due to this fact, we have information sets around similar node types. For example, if we need to calculate the pay-off for the acquirer when he uses the “IIII” strategy against the “II” strategy used by the target.

We will need to add the pay-offs for each instance where the acquirer will use the “IIII” strategy multiplied by the acquirer’s and target’s probabilities. So, this would be \((1.0) \times (0.1) \times (-6) + (1.0) \times (0.9) \times (-4.5) + (0.0) \times (0.1) \times (-3) + (0.0) \times (0.9) \times (-1.5)\), which would equal -4.65. Similarly, the pay-off for the target will be calculated being +2.07.

\[ \text{Fig 4: Target's pay-off profile} \]

\[ \text{Fig 5} \]
In the present text, we derive an integral representations for the functions $M_2$ and $M_{10}$ by using the technique of fractional integration by part as given below:

\[
M_2 \left[ a, a', b, b', c, c' ; \begin{array}{c} x, y \\ c + c' \\ d \end{array} \right] = \frac{\Gamma(c + c')}{\Gamma(c) \Gamma(c')} \int_0^1 u^{c'-1} (1 - u)^{c-1} M_1 \left[ \begin{array}{c} a, a', b, b', c, c' ; \\ d, c, c' ; \\ u x, (1 - u) y \end{array} \right] \]

where $Re(c') > 0$, $Re(c) > 0$, $|x| < 1$, $|y| < 1$.

Also,

\[
M_{10} \left[ \begin{array}{c} a, b, c ; \\ d, e, e' ; \end{array} \right] = \frac{\Gamma(d)}{\Gamma(c') \Gamma(d - c')} \int_0^1 u^{d-1} (1 - u)^{d-c'-1} M_{10} \left[ \begin{array}{c} a, b, c ; \\ c', e, e' ; \\ u x, u y \end{array} \right] du
\]

where $Re(c') > 0$, $Re(d - c') > 0$, $|x| < 1$, $|y| < 1$. 
Results show that the equilibrium price (Nash Equilibrium Point) is closer to the acquirer’s offer price. However, through the simulation of the behavioral factors it is seen that the most optimal outcome for the acquirer is to be a risk taker and for the target to be pessimistic. This seems like a suitable option because the acquirer will be willing to provide a higher offer and the target will be more open to accept a reasonable offer.

This can occur due to many reasons; one of them is that the target does not want to provide a low price because it may be able to obtain a higher price if the acquirer is ready to pay more. Similarly, the acquirer does not want to disclose his expected offer price because he does not want to pay too much for purchasing the target company. Both the acquirer and target company will try to find out approximately what the expected offer and bid price their opponent has in mind and then they will try to provide a price that suits them (i.e. acquirer will start off by providing a lower offer price) and then they will try to negotiate their way through to an equilibrium price.

In this game, we have added a probability that the seller could be optimistic (33.3% of the time) or pessimistic (66.6% of the time) and the target could be risk taking (33.3% of the time) and risk averse (66.6% of the time). These probabilities can be changed, but have been set to show that it is possible to find the stable equilibrium state even when the type (behavior) of the opponent is not known completely.

As discussed earlier, this occurs in reality where players guess the type (behavior) of their opponents, which helps them assess which strategies to choose. When we assess this incomplete information game, the value for the acquirer is +1 and for the target is -1. The acquirer (player 1) in this case will choose the strategy “R*RRR” and “R*RRR” (with the probability of 0.499 respectively) and target (player 2) will choose the strategy “II” (with the probability of 0.489) and “SI” (with the probability of 0.511).

This is similar to the strategies that the acquirer and target selected in the complete information. This shows that the incomplete information does not change the optimal strategies of either the acquirer or the target. This could occur due to the fact that the acquirer’s primary motive is to reduce the price (use strategy “R*RRR”) and the target’s motive is to increase the price (use strategy “II”).
However, both do not want to push the opponent too harshly to avoid the deal being broken, therefore they also use the strategies “R*RRI” and “SI”. This usually is seen to occur in reality as acquirers and target companies use optimal mixed strategies similar to what is proposed in order to obtain the best possible outcome.

Any of the other three options are less likely to provide such a suitable outcome. This paper has only undertaken preliminary research in this area, however it shows that there is a possibility to simulate behavioral factors in valuing two-person M&A & Acquisition transactions. This is specially the case as Merger & Acquisition valuations include a significant factor of psychological pricing that cannot be explained by traditional Corporate Finance models. Hopefully, this research will provide a platform for other models to be developed to simulate behavioral factors related to Merger & Acquisition valuations.

**Significance of the study**

The two-person M&A model has identified the behaviors (types) of the acquirer (risk taking – risk averse) and target (optimistic – pessimistic) that will change between players depending on the M&A transaction. It is also often hard to find the behaviors of opponents and the associate probabilities to these behaviors in order to calculate potential NEPs for such games. However, this model does provide us some significant insight on how such M&A transactions can be analyzed more accurately, which will help the acquirer or target, develop and simulate strategies that it can follow against its opponent. An example of such a simulation of the behavioral probabilities of the acquirer and target.

A simulation of the two-person M&A model will help us understand if the acquirer should choose to be risk taking or risk averse and the target will be able to gauge if he should act optimistically or pessimistically. The data from the simulation of the two-person M&A model has shown that the Nash Equilibrium Point for such a series of games lies at (2, -2), which relates to the acquirer being a risk taker and the target acting pessimistically.

In an everyday scenario such a result makes sense because if the acquirer is a risk taker, he will be ready to offer more for the sale. While, on the other hand if the target is pessimistic then he will be more likely to negotiate with the acquirer on a reasonable price. However, if we consider the opposite situation, where the acquirer is risk averse and the target is optimistic, then it is less likely that a successful negotiation will result.

As, the acquirer will be unwilling to negotiate a higher price for the merger and the target will be unlikely to accept a lower price for this transaction. Such a situation will most likely result in the merger to break down. The situation will not be much different when the acquirer is risk averse and the target is pessimistic – in this case, the M&A valuation will likely favor the acquirer compared to the target, which may result in the target discontinuing the transaction.

However, if the acquirer was risk taking and the target was optimistic, there still is a risk that the target may ask for more than the acquirer is willing to offer, which may result in the merger talks being stalled. In everyday business practice, we find all different varieties of people, some who are risk taking, while others are risk averse.

**Conclusion**

Results of the two-person M&A model show that the strategies used by the acquirer are “RRRR” or “RRIR” against the target’s strategies “II”. As a result, the pay-off to the acquirer is +0.68 and the target is -1.55. When this model is used to simulate the behavior of the acquirer and target, it is seen that it would be better for the acquirer to be risk taking and the target to be pessimistic.

This would help as the acquirer would be willing to pay more, while the target will be willing to accept less for the sale of the target company. It is to be noted that these are preliminary results from this research, as this model is developed further the results and discussion will possibly help provide greater insight into psychological pricing of M&A transactions.

**References**