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A mathematical model of two phase hepatic blood flow in arterioles during liver abscess

Niharika Tiwari, V Upadhyay, AK Agrawal and Amita Arjariya

Abstract

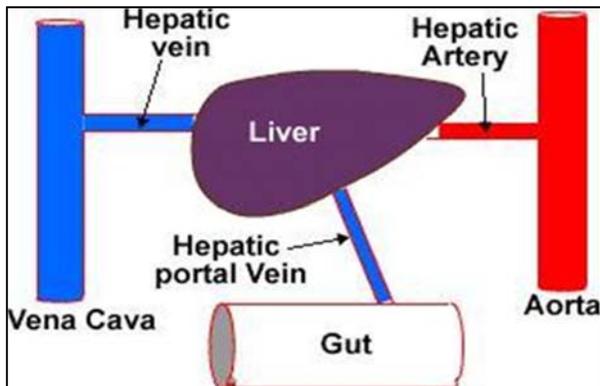
In this mathematically modeling, we formulate the Hepatic blood flow in Liver. Keeping in the view the nature of hepatic circulatory system in human body. The viscosity increase in the arterioles are remote from the heart and proximate to the Liver. P.N. Pandey and V. Upadhyay have considered the blood flow as two phased, one of which is that of red blood cells and other is Plasma. We have also applied the Herschel Bulkley Non-Newtonian Model in bio-fluid mechanical set-up. We have collected a clinical data in case of Liver Abscess for Hematocrit v/s blood pressure. The graphical presentation for particular parametric values are much closer to the clinical observation. The overall presentation is in tensorial form and solution technique adopted is analytical as well as numerical. The role of Hematocrit is explicit in the determination of blood pressure in case of Hepatic blood flow in arterioles due to Liver Abscess.

Keywords: Hepatic blood flow, RBC and Plasma, Pressure drop and Pressure gradient

1 Introduction

1.1 Structure and function of liver

Liver is vulnerable to a variety of metabolic, toxic, microbial and circulatory insults. In some instances, the disease is primary while in others the hepatic involvement is secondary, which can be due to cardiac de-compensation, The liver is a reddish brown wedge shaped organ with four lobes of unequal size and shape. A human liver normally weighs 1.44 – 1.66 kg (3.2–3.7 lb). The liver is a dark, reddish - brown triangle - shaped organ that weighs about 3 pounds. It is both the largest internal organ and the largest gland in the human body. Located in the right upper quadrant of the abdominal cavity, it rests just below the diaphragm, to the right of the stomach and overlies the gallbladder. The upper surface of the liver showing two lobes visceral surface showing four lobes and the impression. The liver has the most complicated circulation of any organ. According to the anatomical peculiarity of the double afferent blood supply of the liver, 75%80% of the blood entering the liver is partially deoxygenated venous blood supplied by the portal vein, which collects all the blood that leaves the spleen, stomach, small and large intestine, gallbladder and pancrea. The hepatic artery accounts for the remaining 25% with well oxygenated blood. Total hepatic blood flow ranges between 800 and 1200 mL/min, which is equivalent to approximately 100 mL/min per 100 g liver wet weight. Although the liver mass constitutes only 2.5% of the total body weight, the liver receives nearly 25% of the cardiac output. The valve less portal vein is a low pressure/low resistance circuit, while the hepatic artery supplies the liver with arterial blood in a high pressure/high resistance system. The mean pressure in the hepatic artery is similar to that in the aorta, while portal vein pressure has been reported to range between 6 and 10 mmHg in humans.



Functions of the Liver

The focuses on the activities of the liver with respect to bile and plasma proteins, and the process of detoxification, it should be pointed out that the liver has other functions as well. These functions are as follows:

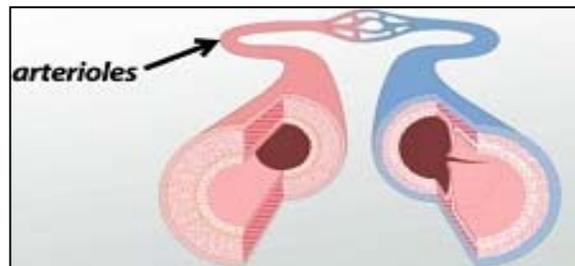
1. carbohydrate storage
2. amino acid metabolism
3. metabolism of steroidal hormones
4. metabolism of fat

Bile is a complex solution secreted by the cells of the liver into the bile duct. Approximately 250-1000 ml/day are secreted. It is golden yellow in color due to the presence of bile pigments (bilirubin and biliverdin). These pigments, it should be noted, are the breakdown products of hemoglobin. Also found in bile are the bile salts which are sodium and potassium salts of bile acids.

1.2 Structure and Function of Arterioles-

Arterioles is a small diameter blood vessel in the microcirculation that extends and branches out from an artery and leads to capillaries. Arterioles have muscular walls (usually only one to two layers of smooth muscle) and are the primary site of vascular resistance. The greatest change in blood pressure and velocity of blood flow occurs at the transition of arterioles to capillaries. The decreased velocity of flow in the capillaries increases the blood pressure, due to Bernoulli's principle. This induces gas and nutrients to move from the blood to the cells, due to the lower osmotic pressure outside of the capillary. The opposite process occurs when the blood leaves the capillaries and enters the venules, where the blood pressure drops due to an increase in flow rate. Arterioles receive autonomic nervous system innervation and respond to various circulating hormones in order to regulate their diameter. Retinal vessels lack a functional sympathetic innervations.

The up and down fluctuation of the arterial blood pressure is due to the pulsatile nature of the cardiac output and determined by the interaction of the stroke volume versus the volume and elasticity of the major arteries. In a healthy vascular system the endothelium lines all blood contacting surfaces, including arterioles, arteries, veins, capillaries, and heart chambers. Arteriole diameters decrease with age and with exposure to air pollution. Any pathology which constricts blood flow, such as stenosis, will increase total peripheral resistance and lead to hypertension.

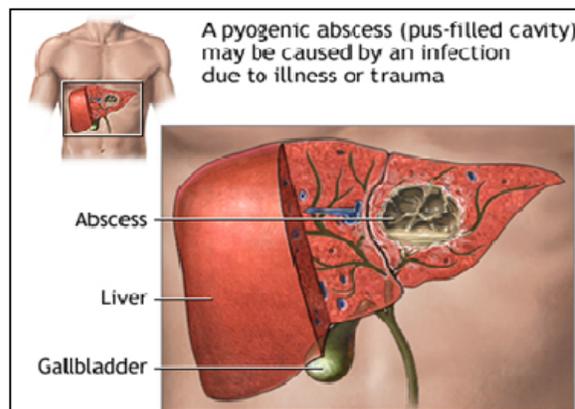


1.3 Liver Abscess

An abscess is an enclosed collection of liquefied tissue known as pus. Signs and symptoms of abscess include redness warmth and swelling. They are usually caused by a bacterial infection. There are two types of abscess- septic, sterile. Most abscess are Septic, which means that they are result of an infection. A liver abscess is a sore inside your liver that contains a thick fluid (pus). Pus in the liver is usually caused by bacteria Infection with a parasitic called amoeba. Liver Abscess occurs equally in men and women in those over age 50.

A current assessment of liver abscesses should allow for better understanding of the pathogenesis of the disease and improve the effectiveness of diagnosis and treatment. Amebic liver abscess occurs more commonly than pyogenic liver abscess on a worldwide basis. However, in the United States, pyogenic liver abscess predominates. The purpose of our study was to evaluate the etiology, management, morbidity, and mortality of all patients admitted to our medical center with diagnoses of pyogenic liver abscess. Liver abscesses are relatively uncommon in the Western countries, but in India and the Asian countries, they continue to present in the outpatient with regularity. Though the mortality has considerably decreased, the morbidity and associated loss of man hours are a concern.

Liver abscesses are relatively uncommon in the Western countries, but in India and the Asian countries, they continue to present in the outpatient with regularity. Though the mortality has considerably decreased, the morbidity and associated loss of man hours are a concern. Liver abscess (LA) is defined as collection of purulent material in liver parenchyma which can be due to bacterial, parasitic, fungal, or mixed infection. It is a common condition across the globe. Amoebiasis is presently the third most common cause of death from parasitic disease.



1.4 Blood and its composition

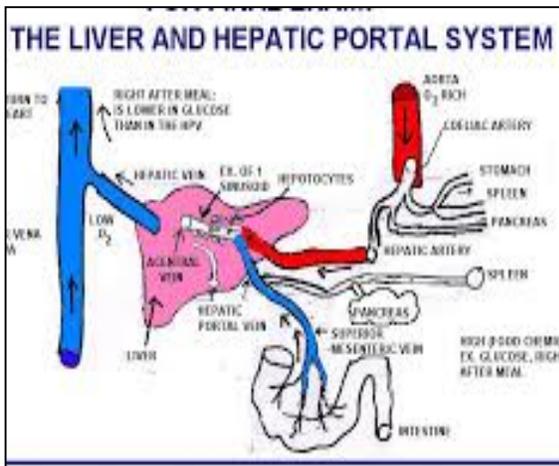
Blood is composed of plasma and formed elements. The plasma contains 91.5% water, 7% proteins and 1.5% other

solutes. The formed elements are platelets, white blood cells and red blood cells, the presence of these formed elements and their interaction with plasma molecules are the main reasons why blood differs so much from ideal Newtonian fluids.

Viscosity of plasma- Normal blood plasma behaves like a Newtonian fluid at physiological rates of shear. Typical values for the viscosity of normal human plasma at 37 °C is 1.4 m N·s/m². The viscosity of normal plasma varies with temperature in the same way as does that of its solvent water a 5 °C increase of temperature in the physiological range reduces plasma viscosity by about 10%. The red blood cell is highly flexible and biconcave in shape.

1.5 Structure & Functions of Hepatic circulatory System

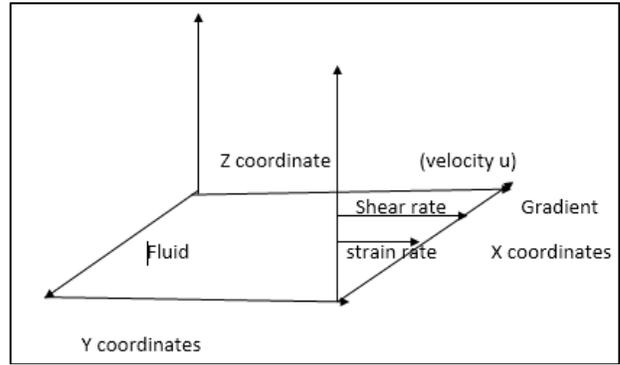
In hepatic circulatory system, the liver receives a dual blood supply from the hepatic portal vein and hepatic arteries. Supplying approximately 75% of the liver's blood supply, the hepatic portal vein carries venous blood drained from the spleen, gastrointestinal tract and its associated organs. The hepatic arteries supply arterial blood to the liver, accounting for the remainder of its blood flow. Oxygen is provided from both sources approximately half of the liver's oxygen demand is met by the hepatic portal vein, and half is met by the hepatic arteries. Blood flows through the liver tissue and empties into the central vein of each lobule.



2. Real Model

2.1 Choice of frame of reference

This graph represented a three dimensional cartesian coordinate system. Three dimensional system is a geometric setting in which three values called parameters are required to determine the position of an element. The position of that point in three dimensional space is given by an ordered triple of real numbers each number giving the distance of that point from the we select origin measured along the given axis which is equal to the distance of that point from the plane determined by the other two axis. The frame of reference for mathematical model of the moving blood keeping in view the difficulty and generality of the problem of blood flow. We select three dimensional orthogonal curvilinear co-ordinate system, prescribed as E³ called as 3-dimensional Euclidean space. We interpret the quantities related to blood flow in tensorial form which is more realistic. Let the co-ordinate axes be OXⁱ where O is origin and i=1,2,3. The mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity v^k = v^k(xⁱ,t).



2.2 Choice of Known Parameters: we have apply the only five known parameters and they are as follows-

- η_c = viscosity coefficient of blood cells
- η_m = viscosity coefficient of mixture of two phases
- η_p = viscosity coefficient of plasma
- Q = value of flow flux
- V_p = pressure gradient

Blood is the Non- Newtonian fluids, then using this constitutive equation for fluids.

$$\tau = \eta e^n$$

If n = 1 then the nature of fluid is Newtonian and if n ≠ 1 then the nature of fluid is Non-Newtonian fluids. Where, τ is denoted by stress, e is denoted by strain rate this constitutive equation is called Herschel - Bulkily Non Newtonian law, and n is denoted by the parameter, these equation uses equation of motion.

2.3 Constitution of Blood

Blood consists of a suspension of cells in an aqueous solution called plasma. There are about 5 × 10⁹ mili liter of healthy human body which about 95% are red cells or erythrocyts whose main function is transport oxygen from the lungs to all the cells of the body. Blood is concentrated suspension of several formed cellular elements, red blood cells (RBCs or erythrocytes), white blood cells (WBCs or leukocytes) and platelets (thrombocytes), in an aqueous polymeric and ionic solution, the plasma, composed of 93% water and 3% particles.

Plasma's central physiological function is to transport these dissolved substance, nutrients, wastes and the formed cellular elements throughout the circulatory system. Normal erythrocytes are biconcave discs with a mean diameter of 6 to 8 μm and a maximal thickness of 1.9 μm. The average volume of an erythrocyte is 90% μm. their number per cubic millimeter of blood is approximately 5 to 6 × 10⁶ and they represent approximately 40 to 45% by volume of the normal human blood and more than 99% of all blood cells. The first percentage is called hematocrit. Thrombocytes are a vital component of the blood clotting mechanism. The total volume concentration of leukocytes and thrombocytes is only about 1% (N. Bessonov *et al.*, 2016). Then we have considered only two phases of blood. Which is one of red blood cells and other phase is plasma. Plasma is the fluid part of blood and makes up the bulk of the volume. It contains substances that can be used to treat a number of different conditions.

Blood is made up of four separate components, which each perform a different function. They are:

Red blood cells: carry oxygen around the body and remove carbon dioxide

White blood cells: help the body fight infection

Platelets: tiny cells that trigger the process that causes the blood to clot (thicken)

Plasma: yellow fluid that transports blood cells and platelets around the body and contains a number of substances, including proteins

Plasma is the largest component of blood, making up about 55% of its overall content. It's mainly made of water and surrounds the blood cells, carrying them around the body.

2.4 Description of Two phase Blood Flow

In Blood Flow in small vessels the rheological properties of blood depend on the vessel diameter, in small vessels it is necessary to take in to near wall effects and aggregation of eearthroytes. In this paper a unified two phase model of blood is proposed which can describe the blood flow in both large and small blood vessels. The model describe the integral characteristic o the flow such as the hematocrit, viscosity and velocity o blood based on this model the well-known specific effects of the blood flow in blood vessel are explained. The analytical solution dependence of the blood velocity, blood viscosity and hematocrit on the blood vessel diameter rare derived. Comparison with experimental data are performed.

3. Mathematical Formulation

3.1 Equation of Continuity for two phase blood flow

The continuity equation is state that the amount of blood flow through one cardiac chamber is the same as the blood flow through the other chambers, it is based on the principle of conservation of mass. If the mass ratio of blood cells to plasma is r then clearly, $r = \frac{x\rho_c}{(1-x)\rho_p} \dots \dots \dots (2.1)$

ρ_p and ρ_c are densities of blood cells and blood plasma. Campbell and Pitcher has presented a model for two phase of blood (1958). equation of continuity for two phases according to the principle of conservation of mass defined by J.N and Gupta R.C.as follows-

$$\frac{\partial x\rho_c}{\partial t} + (x\rho_c v^i)_i = 0 \dots \dots \dots (2.2)$$

$$\frac{\partial (1-x)\rho_p}{\partial z} + ((1-x)\rho_p v^i)_i = 0 \dots \dots \dots (2.3)$$

Where v is the velocity of two phase blood flow and plasma.

If we define the uniform density of the blood ρ_m as follow

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \dots \dots \dots (2.4)$$

Then solve the equation 1.2 and 1.3

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i)_i = 0 \dots \dots \dots (2.5)$$

Equation of Motion for two phase blood flow: we taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum in liver abscess.

$$\frac{dy}{dx} + p - \tau = 0$$

$$\frac{dy}{dx} = -p + \tau$$

Where p = pressure, and τ = viscous force

We get the equation of motion for the phase of blood cells as follows;

$$x_{\rho_c} \frac{\partial v^i}{\partial t} + (x_{\rho_c} v^i) v_j^i = x_{\rho_c} g^{ij} + x_{\eta_c} (g^{jk} v_k^i) \dots \dots (2.6)$$

Similarly taking the viscosity coefficient of plasma to be the equation of motion for plasma will be as follows

$$(1-x)_{\rho_p} \frac{\partial v^i}{\partial t} + \{(1-x)_{\rho_p} v^i\} v_j^i = (1-x)_p g^{ij} + (1-x)_{\eta_c} (g^{jk} v_k^i) \dots (2.7)$$

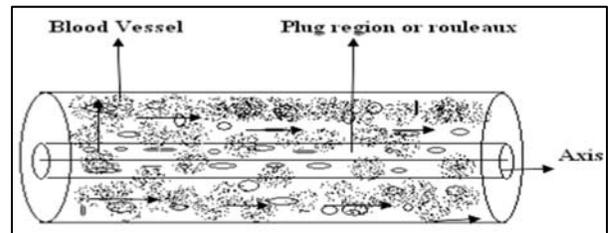
Now adding equation (2.6) & (2.7) and using relation the equation of motion for blood flow with the both phases will be as follows;

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v_j^i = -p_j + \eta_m (g^{ik} v_k^i)$$

where $\eta_m = x\eta_c + (1-x)\eta_p$ is the viscosity coefficient of blood as a mixture of two phase. The velocity of blood flow decreases the viscosity of blood increase. The Herschel Bulkley law holds good on the two phase blood flow through the veins arterioles, whose constitutive equation as follows-

$$T' = \eta_m e^n + T_p (T' > T_p) \text{ and } e = 0 (T' < T_p)$$

T_p is the yield stress.



When strain rate $e = 0 (T' < T_p)$ a core region is formed which flows just like a plug. Let the radius of the plug be $e = r_p$. The stress acting on the surface of plug will be T_p .

Equating the forces acting on the plug we get,

$$\pi r_p^2 = T_p 2\pi r_p$$

$$r_p = 2 \frac{T_p}{p} \dots \dots \dots (3.1)$$

The constitutive equation for test part of the blood vessel is $T' = \eta_m e^n + T_p$ or $T' - T_p = \eta_m e^n = T_e$

Where T_e =effective stress, whose generalized form will be as follows

$$T^{ij} = -p g^{ij} + T_e^{ij} \text{ where, } T_e^{ij} = \eta_m (e^{ij})^n$$

$$e^{ij} = g^{jk} v_k^i$$

Where the symbols have their usual meanings.

$$\text{Equation of Continuity - } \frac{1}{\sqrt{g} \sqrt{(g v^i)_i}} = 0$$

$$\text{Equation of Motion- } \rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v_j^i = -T_{e,j}^{ij} \dots \dots (3.2)$$

Where all the symbols have their usual meaning the blood vessels are cylindrical the governing equation have to be transformed into cylindrical co-ordinates. We know earlier. $x^1 = r, x^2 = \theta, x^3 = z$, the metric tensor in cylindrical co-ordinates is $[g_{ij}]$ and matrix of conjugate metric tensor is $[g^{ij}]$ where as the chritofel's symbols of 2nd kind are as follows;

$$\left\{ \begin{matrix} 1 \\ 2,2 \end{matrix} \right\} = -r, \left\{ \begin{matrix} 1 \\ 2,2 \end{matrix} \right\} = \left\{ \begin{matrix} 1 \\ 2,2 \end{matrix} \right\} = \frac{1}{r}$$

The tensorial equations can be transformed into cylindrical forms which are follows-

$$\text{Equation of continuity; } \frac{\partial v}{\partial z} = 0$$

$$\text{R-component; - } \frac{\partial v}{\partial z} = 0, \theta = 0$$

z-component; $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} [r (\frac{\partial v_z}{\partial r})^2]$
 the blood flow is axially symmetric in arteries concerned
 $v_\theta = 0$.

We get $v_z = v(r)$ and $dp = p(z)$
 and $0 = -\frac{dp}{dz} + \frac{\eta_m}{r} [r (\frac{dv}{dz})^n] \dots \dots \dots (3.3)$

let pressure gradient ; $-\frac{dp}{dz} = p$

$$r \left(\frac{dv}{dz}\right)^n = \frac{pr^2}{2\eta_m} + A \quad r = 0, v = v_0 \text{ then } -\frac{dv}{dr} = \left(\frac{pr}{2\eta_m}\right)^{1/n}$$

r replace from $r - r_p - \frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}pr_p}{\eta_m}\right)^{1/n}$

$$\frac{dv}{dr} = -\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \dots \dots \dots (3.4)$$

Above equation under no slip boundary condition $v = 0, r = R$

We get the equation-

$$V = \left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1} [R - r_p]^{\frac{n+1}{n}} - (r - r_p)^{\frac{n+1}{n}} \dots \dots \dots (3.5)$$

This equation for velocity of blood flow in arterioles.

If we put $r = r_p$ to get the velocity v_p of plug flow as define the equation.

$$v_p = \frac{n}{n+1} \left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{n+1}{n}} \dots \dots \dots (3.6)$$

4. Result and Discussion

Clinical data for diagnosis of Liver Abscess

s.n	B.P.	B.P. drop $\left(\frac{S+D/2+D}{3} - \frac{S+D}{2}\right)$	B.P in pascal	Hemoglobin	Hematocrit (3×hb)
1	130/90	-44	-5857.808	13.2	39.6
2	110/70	-30	-3993.96	13.4	40.2
3	110/60	-37	-4925.884	13.7	41.1
4	130/80	-44	-5857.808	13.1	39.6

The value of flow flux $Q = 900\text{ml}/\text{min} \Rightarrow 0.015\text{m}^3/\text{sec}$

If $r = 1, r_p \Rightarrow -1/3$

We take the value of $H = 40.2$ then pressure drop $P = -3993.96$

$x_p = 0.0015$

$x_m = 0.035$

Apply the formula $x_m = x_c X + x_p(1 - X)$ and find the value of x_c .

Let $X = \frac{H}{100}, 40.2/100 = 0.402$

Put the value in equation-

$$0.035 = x_c(0.402) + 0.0015(1 - 0.402)$$

$x_c = 0.0848333$ pascal / second

$z =$ length of arterioles $= 5 \times 10^{-5}$

$$p = -\int \frac{dp}{dz} = \frac{-0.3993.96}{5 \times 10^{-5}} = 79879200 \text{ pascal-sec.}$$

Where $dz = z_f - z_i$ is a length of arterioles and p is pressure drop.

$\frac{27Q}{2\pi} = \left(\frac{p_f - p_i}{z_f - z_i 3\eta_m}\right)^{1/n}$ solve by trial method we find the value of n .

$$0.0644578 = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] (7607.54286 \times 10^{-5})^{1/n}$$

Solve this equation the value of $n = -4.1$

Again we apply the formula-

$$p_f - p_i = 3\eta_m(z_f - z_i) \left(\frac{27Q}{2\pi A}\right)^n$$

Where $A = \frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}$

$$p_f - p_i = 3[\eta_c X + \eta_p(1 - X)](z_f - z_i) \left(\frac{27Q}{2\pi A}\right)^n = 3[0.00084833H + 0.00087] (5 \times 10^{-5})$$

$$(0.0644578)^{4.1} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right]^{-4.1}$$

$$= 3[0.00084833H + 0.00087] (939.583864)$$

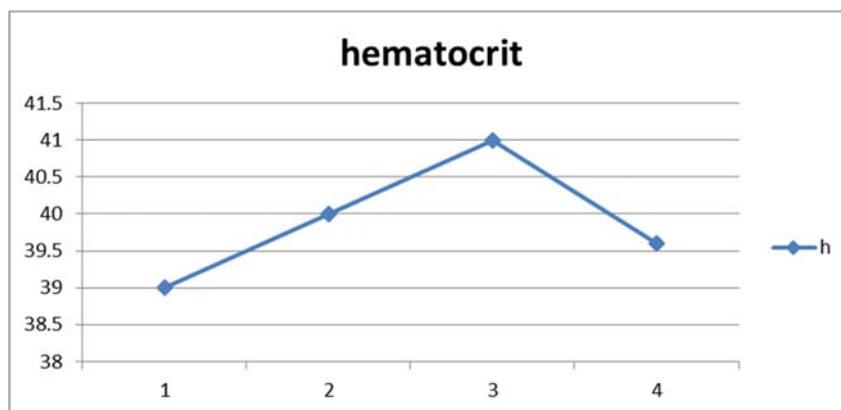
$$= [0.00254499H + 0.00261] (939.583864)$$

In this equation we contain the different values of Hematocrit we find the different values of pressure drop.

Table for Hemoglobin v/s Pressure drop

H Hematocrit)	39.6	40.2	41.1	39.6
BP (Blood Pressure drop)	97.145079	98.5798236	100.7319	96.4277

Graphical presentation of Clinical Data



5. Conclusion

A simple investigation of the graph is down convex on 39.6 and 40.2 and 41.1 graph is upper convex and again on point 39.9 graph is down convex blood pressure drop and hematocrit in suffering liver abscess patient and confirmations that when hematocrit increase then blood pressure drop also increased and shows a non-linear graph.

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