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# A study on some roman dominating results in the field on graph theory

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**Abstract**
A Roman dominating function on a graph G is a function $f: V \rightarrow \{0,1,2\}$ satisfying the condition that every vertex u$\in V$ for which $f(u) = 0$ is adjacent to at least one vertex v $\in V$ for which$f(v) = 2$. The weight of a roman dominating function is the value$f(V) = \sum_{v \in V} f(v)$. The roman domination number $\gamma_R(G)$ of G is the minimum weight of a Roman dominating function on G. A Roman dominating function on G is connected roman dominating function of G if either $\langle V_1 \cup V_2 \rangle$ or $\langle V_2 \rangle$ is connected. The connected roman domination number $\gamma_{RC}(G)$ of G is the minimum weight of a connected roman dominating function on G. A Roman dominating function on a block graph. A roman dominating function on a block graph B(G)=H is a function. The minimum weight of a roman dominating function on a block graph H is called the roman block domination number of G and is denoted by $\gamma_{RB}(G)$. In this paper the roman domination number of block graph H and obtain some results on $\gamma_{RB}(G)$ in terms of elements of G, but not in terms of H.

**Keywords:** Roman dominating function, weight, roman block domination, graph theory

## 1. Introduction
In this paper, G = (V, E) finite, simple, undirected $(p, q)$ graphs with $p = |V|$ and$q = |E|$. We denote open neighborhood of a vertex v of G by $N(v)$ and its closed neighborhood by$N[v]$. For a vertex set $S \subseteq V(G), N(S) = \cup_{v \in S} N(v)$ and$N(S) = \cup_{v \in S} N(v)$. The degree of a vertex x denotes the number of neighbors of x in G and $\Delta(G)$ is the maximum degree of G. Also $\delta(G)$ is the minimum degree of G. A set of vertices in G is a dominating set, if$N[S] = V(G)$. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. If S is a subset of V(G), then we denote by <S> the subgraph induced by S. A subset S of vertices is independent, if <S> has no edge. Let S be a set of vertices and u $\in S$. We say that a vertex v is a private neighbor of u with respect to S if $N[v] \cap S = \{u\}$. The private neighbor set of u with respect to S is the set $pn[u, S] = \{v; N[v] \cap S = \{u\}\}$.

## 2. Main Results
### 2.1 Theorem
For any non-trivial tree T, $\gamma_{RC}(T) = 2\gamma(T)$ if and only if every non end vertex of T is adjacent to at least one end vertex.

### Proof
Let $H_1 = \{v_i; 1 \leq i \leq p\}$ and $H_2 = \{v_j; 1 \leq j \leq p\}$ be the set of non-end vertices adjacent to at least one end vertex and the set of non-end vertices which are not adjacent to end vertex respectively. Let $f = (v_0, v_1, v_2\}$ be a $\gamma_{RC}$function of G. Suppose $H_2 \neq \emptyset$. Let D and $D_c$ be a $\gamma$-set and $\gamma_C$ set of G respectively. Then we have the following cases.
Case 1: Suppose$H_2 = 1$ or 2. Then we have two subcases.
Subcases1.1: Assume$H_2 = 1$. Let {u} $\in H_2$ such that {u} $\in N(H_2)$. Then {u} $\in V_1$ but {u} € $D$ which gives that$\gamma_{RC}(T) > 2\gamma(T)$, a Contradiction.
Subcase 1.2: Assume $H_2 = 2$ and $\{v_1, v_2\} \in H_2$ such that $\{v_1, v_2\} \in (H_1)$. Then $\{v_1, v_2\} \in V_1$.
But $\{v_1, v_2\}$€ $D$, which gives$\gamma_{RC}(T) > 2\gamma(T)$, a contradiction.

Case 2: Suppose $H_1 = k$ and $\{v_k; 3 \le k \le n\} \in H_2$. Then $\forall\{v_1; 1 \le l \le n\} \subseteq \{v_k\}, \{v_l\} \in V_1$. But $\{v_{3l}\} \in D$ and $\{v_1 - \{v_{3l}\}\} \notin D$ which gives, $\gamma_{RC}(T) > \gamma(T)$ again a contradiction.
For the converse to the above all cases, let $H_2 = \emptyset$. Then $|vi| = |V_2| = |D| = |D_2|$. Hence$\gamma_{RC}(T) = 2|V_2||V_1| = 2|D_c| + \emptyset = 2|D| = 2\gamma(T)$.

**2.2 Theorem**
For any nontrivial connected graph G, $\gamma(G) + \gamma_{RC}(G) \le p$

**Proof**
Let G be any nontrivial graph $f = \{v_0, v_1, v_2\}$ be a $\gamma_R$ function in B(G). Then $\{b_1, b_2, \dots . b_n\}$ be the number of vertices of B(G) corresponding to the blocks $\{B_1, B_2, \dots . B_n\}$ in G. we prove the result by induction on the number of blocks n of G.
Assume G is a graph with n=1 then p≥2, $\gamma(G) \ge 1$ and $\gamma_{RB}(G) = 1$.
We consider the following cases.
Case 1: Suppose $\gamma(G) = 1$ with n=1, then p≥ 2 and $\gamma\ (G) + \gamma_{RB}(G) = 1 + 1 = 2 \le p$.
Case 2: Suppose $\gamma(G) = 2$ with n=1, then p≥ 3 and $\gamma\ (G) + \gamma_{RB}(G) = 2 + 1 = 2 \le p$.
By above two cases and $\gamma\ (G) + \gamma_{RB}(G) \le p$
Assume the result is true for all graphs with n=k blocks, then $\gamma\ (G) + \gamma_{RB}(G) \le p$
Let $D_{RB}$=$\{b_1, b_2, \dots . b_j\}$ with j<n be the minimal roman dominating set of B(G) such that $|D_{RB}| = \gamma_{RB}(G)$. Suppose G has (k+1) blocks with p>p vertices. Then one vertex will be increased in B(G) such that $\gamma(G) \le \gamma(G)$ and $\gamma_{RB}(G) \le \gamma_{RB}(G)$. Clearly that $\gamma(G) + \gamma_{RB}(G) \le p$ by induction the result is true for n=k+1. Hence $\gamma(G) + \gamma_{RB}(G) \le p$

**3. Conclusion**
In this paper we showed the connected roman domination functions and roman block domination functions. Some theorem on both function are explained with their proofs. In this work finite, simple, undirected graph is considered for theorem proof.

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