Elegant labeling of some special line graph

S Lakshmi and S Priya

Abstract
An elegant labeling g of graph G with v edges is an injective function from the vertices of G to the set {0,1,2,.....v} such that when each edge (e=uv) is assigned the label [(g(x)+g(y)) mod (v+1)] the resulting edge labels are distinct and non-zero. In this paper it is shown to be certain families of line graphs are elegant graphs.

Keywords: Path graph, p2n, comb graph, Hnn, Bnn

1. Introduction
We consider all graphs are finite, simple and undirected. The G has v vertex and e edges. A Graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition. We refer to survey on graph labelling by Gallian [5]. Chang, Rogers and Hsu introduced elegant labeling (1981). The elegant labeling is a variation of harmonious labeling, Balakrishnan and Sampathkumar Gallian, Lee et al are found harmonious labeling and felicitious labeling of graphs.

The elegantness is also possible if n value is even found by Balakrishnan selvan, Yengnanaryan. They are apply this result on Hnn, Bnn. Recently Pn2, PnK have shown also elegant graphs by V Laxmi, Alias Gomathi, N murugan and A. Nagrajan.

2. Main Results
Theorem 2.1
The line graph of (P²n-e) is an elegant graph if n=1 (mod 2) n ≥3 and e = n-4

Proof:
Let G=(P²n) be a graph. The graph contains u vertices and v edges. The line graph of Pn² is denoted by L(P²n) [x1,x2,x3,.............xn;y1,y2,y3,.............yn] be the (e1,e2,e3,........en} We define the function g: V(G)→{0,1,2,........v}
g(x1)=0 g(x2)=1
g(x2i+1)=5i for 1≤i≤(v-u) / 2 g(x2i+2)=5i+1 for 1≤i≤(v-u) / 2 g(y1)=2
g(y2)=4i+j for 1≤i≤(v-u)/2 and 0≤j≤((v-u)/2)-1 g(y2i+1)=7i-j for 1≤i≤(v-u)/2 and j=0,2,4,.....(n-5)
In this labeling easy to verify that all the vertex labels are different. We get the edge label in the form {1,2,.....V}. No zero integers and edge labels are not repeated. Hence L(P²n-e) is a elegant labeling graph

Example 1
L(P24) is a elegant labeling graphs Here n=4 e=n-4

Correspondence
S Lakshmi
Assistant Professor,
Department of Mathematical
(PG&SF) PSGR Krishnammal
College for Women
Coimbatore, Tamil Nadu, India
Theorem 2.2
The line graph of $(P_{2n} - e)$ is an elegant graph if $n = 0 \pmod{2}$, $n \geq 4$ and $(e = n - 4)$

Proof
Let $G = (P_{2n})$ be a graph with $u$ vertices and $v$ edges and its line. The line graph of $P_{2n}$ is denoted by $L(P_{2n})$. The vertices are $\{x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n\}$ and the edge set is $\{e_1, e_2, \ldots, e_n\}$

The labeling function is defined by

$$g = \{0, 1, 2, \ldots, v\}$$

In this defined labeling we can easy to verify that all the vertex labels are distinct values. We get the edge label in the form $\{1, 2, 3, \ldots, V\}$ non zero integers and the edge labels are not repeated Hence $L(P_{2n} - e)$ is a elegant labeling graph

Example 1.
$L(p^5)$ is a elegant labeling graph Here $n=5, e=n-4, e=1$
Example 2

L(p²7) is an elegant labeling graph. Here n=7, e=n-4, e=3

Example 3

L(p²8) is an elegant labeling graph. Here n=8, e=n-4, e=4

Theorem 2.3

The line graph of (PnK1) is an elegant graph if n ≤ 5

Proof:
Let G=(Pn K1) be a comb graph with u vertices and v edges.
The line graph of Pn K1 is denoted by L(Pn K1).
\{x0, x1, x2, ..., xn, y0, y1, ..., yn\} and edge set \{e1, e2, ..., en\}
The labeling function is as follows g: V(G) → \{0, 1, 2, 3, ..., q\}
Case 1: \( n=2 \)
\[ g(x_1)=0 \quad g(y_1)=1 \quad g(y_2)=2 \]

Case 2: \( n=3 \)
\[ g(x_j)=j \quad \text{for} \quad 0 \leq j \leq n-2 \quad g(y_i)=j+1 \quad \text{for} \quad 0 \leq j \leq n \]

Case 3: \( n=4 \)
\[ g(x_1+2j)=5i+1 \quad \text{for} \quad j=0,1 \quad g(x_2)=2 \]
\[ g(y_1+2j)=5j \quad \text{for} \quad j=0,1 \quad g(y_2+2j)=3j+4 \quad \text{for} \quad j=0,1 \]

Case 4: \( n=5 \)
\[ g(x_j)=j \quad \text{for} \quad 0 \leq j \leq n-2 \]
\[ g(y_1+2j)=(n-1)+j(j+1)+i \quad \text{for} \quad 0 \leq j \leq (n-1)/2 \quad g(y_2j)=(n-1)+2j \quad \text{for} \quad j=1,2 \]

The vertex labeling pattern above which covers all the vertices with distinct values. The edge label in the form \( \{1,2,3,\ldots,V\} \) non-zero + integers and the edge labels are not repeated. Hence \( L(P_nK_1) \) is an elegant labeling.

**Example 1**

\( L(P_4 K_1) \) is an elegant labeling graph. Here \( n=4 \) P4 K1

![Graph 2.11](image)

\( L(P_4 K_1) \)

![Graph 2.12](image)

**Example 2**

\( L(P_3 K_1) \) is an elegant labeling graph. Here \( n=3 \) P3 K1

![Graph 2.13](image)

\( L(P_3 K_1) \)
Theorem 2.4
The line graph of (Hn,n) is an elegant graph if n ≤ 3

Proof:
Let G be a (Hn,n) graph
The graph G have u vertices and v edges The line graph of (Hn,n) is denoted by L(Hn,n)
The vertex set is \{x1,x2,x3,...,xn\} vertices set and edge set is \{e1,e2,...,en\}
The labeling function is defined as follows g:V(G)→{0,1,2,...,n-1}
Case 1: n=2
g(x1)=0 g(x2)=1 g(x3)=2
Case 2: n=3
g(x1)=6 g(x2)=0
g(x2+j)=j for j=1,2,3,4
Such that the vertex labels are distinct
We get edge label in the form \{1,2,3,...,V\} non zero integers and the edge label is not repeated
Hence L(Hn,n) is an elegant labeling graph

Example: 1
L(H2,2) is a elegant labeling graph Here n=2
(H2,2)→

Theorem 2.4
The line graph of Pn is an elegant graph if n=0 (mod 2)

Proof:
The line graph of (Pn) be is denoted by L(Pn) The line graph L(Pn) has u vertices and (V-1)edges We define the labeling function g=V(G)→{0,1,2,...,V} as follows
g(xj)=(u-1/2 +j) nod (v+1) for 1≤j≤u
The vertex labeling pattern defined above which covers all the vertices with distinct numbers.
The label vertices from right to left side of the path The edge label in the form of (1,2,3,...,v)
Example 1

The line graph of \( B_{n,n} \) is a elegant graph if \( n=2 \)

Proof:
Let \( G \) be a bistargraph with \( x_1, x_2, \ldots, x_n \) be the vertices and edges \( e_1, e_2, \ldots, e_n \). The line graph of \( G \) is denoted by \( L(B_{n,n}) \). It has \( V \) vertices and \( E \) edges. The labeling function is defined as:

\[
g: V(G) \rightarrow \{0, 1, 2, \ldots, v\}
\]

\[
g(x_j) = 0, \quad g(v_{j+1}) = 3j + 1 \quad \text{for } j=0, 1
\]

We can label the vertices with different values and we get the edge label in the form \( \{1, 2, 3, \ldots, v\} \) non zero integers and the edge label is not repeated.

Hence \( L(B_{n,n}) \) is a elegant labeling graph

Example 2

Theorem 2.5
The line graph of \( B_{n,n} \) is a elegant graph if \( n=2 \)

Proof:
Let \( G \) be a bistargraph with \( x_1, x_2, \ldots, x_n \) be the vertices and edges \( e_1, e_2, \ldots, e_n \). The line graph of \( G \) is denoted by \( L(B_{n,n}) \). It has \( V \) vertices and \( E \) edges. The labeling function is defined as:

\[
g: V(G) \rightarrow \{0, 1, 2, \ldots, v\}
\]

\[
g(x_j) = 0, \quad g(v_{j+1}) = 3j + 1 \quad \text{for } j=0, 1
\]

We can label the vertices with different values and we get the edge label in the form \( \{1, 2, 3, \ldots, v\} \) non zero integers and the edge label is not repeated.

Hence \( L(B_{n,n}) \) is a elegant labeling graph

Example 1
3. Conclusion
In this paper we have shown that line graphs of Path graph, p2n, comb graph, Hnn, Bnn are elegant graphs.

4. Reference