On rgwα-Closed and rgwα-Open maps in topological spaces

RS Wali and Vijayalaxmi R Patil

Abstract
The aim of this paper is to introduce new type of closed maps rgwα-closed maps and rgwα-open maps, rgwα*-closed maps and rgwα*-open maps. We also obtain some properties of rgwα-closed maps and rgwα-open maps.

Keywords: rgwα-closed maps, rgwα*-closed maps and rgwα-open maps, rgwα*-open maps

Mathematical subject classification (2010): 54C10

1. Introduction
Mappings play an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mapping which are studied for different types of closed sets by various mathematicians for the past many years. Generalized closed mappings were introduced and studied by Malghan [17], wg-closed maps and rwg-closed maps were introduced and studied by Nagavani [21]. Regular closed maps, gpr-closed maps, rg-closed maps and rga-closed and rga-open maps have been introduced and studied by Long [24], Gnanambal [12], Arockiarani [13], A.Vaidivel and K.Vairamanickam [34] respectively. In this paper, a new class of maps called regular generalized weakly α-closed (briefly, rgwα-closed) maps, rgwα*-closed maps have been introduced and studied their relations with various generalized closed maps. Also we defined rgwα-open maps and rgwα*-open maps and studied some of its properties. Let us recall the following definitions which are used in our present study.

2. Preliminaries
Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or Ac denotes the complement of A in X.

Definition 2.1: A subset A of a topological space (X, τ) is called
1. Semi-open set [23] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.
2. Pre-open set [18] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
3. α-open set [14] if A ⊆ int(cl(int(A))) and α-closed set if cl(int(cl(A))) ⊆ A.
4. Semi-pre open set [1] (=β-open) if A ⊆ cl(int(cl(A))) and a semi-pre closed set (=β-closed) if int(cl(int(A))) ⊆ A.
5. Regular open set [32] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).
6. Regular semi open set [10] if there is a regular open set U such that U ⊆ A ⊆ cl(U).
7. Regular α-open set [34] (briefly, rα-open) if there is a regular open set U s.t U ⊆ A ⊆ αcl(U).

Definition 2.2: A subset A of a topological space (X, τ) is called
1. W-closed set [31] if cl(A) ⊆ A whenever A ⊆ U and U is semi-open in X.
2. Wα-closed set [8] if αcl(A) ⊆ A whenever A ⊆ U and U is w-open in X.
3. Generalized closed set(briefly g-closed) [22] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
4. Generalized semi-closed set (briefly gs-closed) \([4]\) if 
\(\text{sc}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
5. Generalized pre regular closed set (briefly gpr-closed) \([12]\) if 
\(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
6. Regular generalized \(\alpha\)-closed set (briefly, rg\(\alpha\)-closed) \([34]\) if 
\(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular \(\alpha\)-open in \(X\).
7. \(\alpha\)-generalized closed set (briefly \(\alpha g\)-closed) \([15]\) if 
\(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).
8. Generalized \(\alpha\)-closed set (briefly \(\alpha g\)-closed) if 
\(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).
9. Weakly generalized closed set (briefly, wg-closed) \([21]\) if 
\(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
10. Regular weakly generalized closed set (briefly, rwg-closed) \([21]\) if 
\(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
11. Generalized pre closed (briefly gp-closed) set \([12]\) if 
\(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
12. Regular w-closed (briefly rw-closed) set \([9]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular semi-open in \(X\).
13. Generalized regular closed (briefly gr-closed) set \([7]\) if 
\(\text{rc}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
14. Generalized regular weak (briefly grw-closed) set \([19]\) if 
\(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular semi-open in \(X\).
15. Generalized weak \(\alpha\)-closed (briefly gwa-closed) set \([30]\) if 
\(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).
16. Generalized star weakly \(\alpha\)-closed set (briefly gwa-closed) \([28]\) if 
\(\text{scr}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).
17. Regular generalized weakly \(\alpha\)-closed (briefly rgwa-closed) \([28]\) if 
\(\text{rgcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \(X\).

The compliment of the above mentioned closed sets are their open sets respectively.

**Definition 2.3:** A map \(f : (X, \tau) \to (Y, \sigma)\) is said to be
i) regular-continuous (r-continuous) \([3]\) if \(f^{-1}(V)\) is r-closed in \(X\) for every closed subset \(V\) of \(Y\).
ii) completely-continuous \([10]\) if \(f^{-1}(V)\) is regular closed in \(X\) for every closed subset \(V\) of \(Y\).
iii) strongly-continuous \([12]\) if \(f^{-1}(V)\) is clopen (both open and closed) in \(X\) for every subset \(V\) of \(Y\).
iv) g-continuous \([6]\) if \(f^{-1}(V)\) is g-closed in \(X\) for every closed subset \(V\) of \(Y\).
v) w-continuous \([6]\) if \(f^{-1}(V)\) is w-closed in \(X\) for every closed subset \(V\) of \(Y\).
vi) \(\alpha\)-continuous \([14]\) if \(f^{-1}(V)\) is \(\alpha\)-closed in \(X\) for every closed subset \(V\) of \(Y\).

\(\beta\)-quotient map if \(f\) is \(\beta\)-irresolute and \(f^{-1}(V)\) is an \(\beta\)-closed set in \(X\).

**Lemma 2.5:** see \([26]\)

1. Every \(\beta\)-closed (resp. regular-closed, w-closed, \(\alpha\)-closed and \(\beta\)-closed) set is rgwa-closed set in \(X\).
2. Every \(rg\alpha\)-closed set is \(\alpha\)-closed set.
3. The set \(g\)-closed (resp. rg-closed, gr-closed, gpr-closed, rgw-closed, gwα-closed and \(g\times\alpha\)-closed) set is independent with rgwa-closed set.

**Definition 2.7:** A topological space \((X, \tau)\) is called
1. an \(\alpha\)-space \([14]\) if every \(\alpha\)-closed subset of \(X\) is closed in \(X\).
2. \(\frac{1}{2}\alpha\)-space \([17]\) if every g-closed set is closed.
3. \(g\frac{1}{2}\alpha\)-space \([13]\) if every \(g\)-closed set is \(\alpha\)-closed.
4. \(\beta\frac{1}{2}\alpha\)-space \([33]\) if every \(\beta\)-closed and \(g\)-closed set is \(\beta\)-closed.
5. \(\tau\alpha\)-space \([36]\) if every \(\alpha\)-closed set is \(\alpha\)-closed.
6. \(\tau\text{rgwa}\)-space \([27]\) if every rgwa-closed set is closed.

**Definition 2.8:** A map \(f : (X, \tau) \to (Y, \sigma)\) is said to be
1. \(\alpha\)-closed \([14]\) if \(f(F)\) is \(\alpha\)-closed in \(Y\) for every closed subset \(F\) of \(X\).
2. αg-closed \[13\] if \( f(F) \) is αg-closed in \( Y \) for every closed subset \( F \) of \( X \).
3. wg-closed \[21\] if \( f(V) \) is wg-closed in \( Y \) for every closed subset \( V \) of \( X \).
4. rwg-closed \[21\] if \( f(V) \) is rwg-closed in \( Y \) for every closed subset \( V \) of \( X \).
5. gs-closed \[4\] if \( f(V) \) is gs-closed in \( Y \) for every closed subset \( V \) of \( X \).
6. gp-closed \[16\] if \( f(V) \) is gp-closed in \( Y \) for every closed subset \( V \) of \( X \).
7. gpr-closed \[12\] if \( f(V) \) is gpr-closed in \( Y \) for every closed subset \( V \) of \( X \).
8. wa-closed \[8\] if \( f(V) \) is wa-closed in \( Y \) for every closed subset \( V \) of \( X \).
9. g-closed \[6\] if \( f(V) \) is g-closed in \( Y \) for every closed subset \( V \) of \( X \).
10. w-closed \[31\] if \( f(V) \) is w-closed in \( Y \) for every closed subset \( V \) of \( X \).
11. rga-closed \[34\] if \( f(V) \) is rga-closed in \( Y \) for every closed subset \( V \) of \( X \).
12. gr-closed \[7\] if \( f(V) \) is gr-closed in \( Y \) for every closed subset \( V \) of \( X \).
13. rw-closed \[9\] if \( f(V) \) is rw-closed in \( Y \) for every closed subset \( V \) of \( X \).
14. rgw-closed \[19\] if \( f(V) \) is rgw-closed in \( Y \) for every closed subset \( V \) of \( X \).
15. regular-closed \[32\] if \( f(F) \) is closed in \( Y \) for every regular closed set \( F \) of \( X \).
16. Contra-closed \[5\] if \( f(F) \) is closed in \( Y \) for every open set \( F \) of \( X \).
17. Contra regular-closed \[32\] if \( f(F) \) is r-closed in \( Y \) for every open set \( F \) of \( X \).
18. Contra semi-closed \[20\] if \( f(F) \) is s-closed in \( Y \) for every open set \( F \) of \( X \).

**Definition 2.9:** A map \( f: (X, τ) → (Y, σ) \) is said to be
1. g-open \[6\] if \( f(U) \) is g-open in \( (Y, σ) \) for every open set \( U \) of \( (X, τ) \),
2. w-open \[31\] if \( f(U) \) is w-open in \( (Y, σ) \) for every open set \( U \) of \( (X, τ) \),
3. wg-open \[21\] if \( f(U) \) is wg-open in \( (Y, σ) \) for every open set \( U \) of \( (X, τ) \),
4. rwg-open \[21\] if \( f(U) \) is rwg-open in \( (Y, σ) \) for every open set \( U \) of \( (X, τ) \),
5. gpr-open \[12\] if \( f(U) \) is gpr-open in \( (Y, σ) \) for every open set \( U \) of \( (X, τ) \),
6. regular-open \[32\] if \( f(U) \) is open in \( (Y, σ) \) for every regular open set \( U \) of \( (X, τ) \).

3 **rgwa-Closed and rgwa-Open Maps**

**Definition 3.1:** A map \( f: (X, τ) → (Y, σ) \) is said to be regular generalized weakly \( α \)-closed (briefly, rgwa-closed) if the image of every closed set \( \text{int}(X, τ) \) is rgwa-closed in \( (Y, σ) \).

**Theorem 3.2:** Every closed map is rgwa-closed map, but not conversely.

**Proof:** The proof follows from the definitions and fact that every \( α \)-closed map is rgwa-closed.

The converse of the above Theorem need not be true, as seen from the following example.

**Example 3.4:** Let \( X=\{a,b,c,d\} \), \( τ=\{X, ϕ, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \) and \( σ=\{Y, ϕ, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\} \). Let map \( f: X→Y \) defined by \( f(a)=a, f(b)=c, f(c)=d, f(d)=a \), then \( f \) is rgwa-closed map but not closed map and not \( α \)-closed map, as image of closed set \( F=\{d\} \) in \( X \), then \( f(F)=\{a\} \) in \( Y \), which is not \( α \)-closed, not closed set in \( Y \).

**Theorem 3.5:** If \( f: (X, τ) → (Y, σ) \) is contra-w-closed and rgwa-closed map then \( f \) is \( α \)-closed map.

**Proof:** Let \( V \) be a closed set in \((X, τ)\). Then \( f(V) \) is weak-open and rgwa-closed By Lemma 2.6, \( f(V) \) is \( α \)-closed. Thus \( f \) is \( α \)-closed map.

**Theorem 3.6:** If a map \( f: X→Y \) is closed, then the following holds.

i) If \( f \) is w-closed map, then \( f \) is rgwa-closed map.

ii) If \( f \) is r-closed map (resp. \( β \)-closed, rw-closed, \( α \)-closed, wa-closed, gs-closed, rs-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and \( g^∗ω \)-closed map) then \( f \) is rgwa-closed map.

iii) If \( f \) is rgwa-closed map, then \( f \) is \( gβ \)-closed map.

**Proof:**

(i) The proof follows from the definitions and fact that every \( w \)-closed set is rgwa-closed.

(ii) Similarly we can prove (ii).

(iii) The proof follows from the definitions and fact that every rgwa-closed set is \( gβ \)-closed

The converse of the above Theorem need not be true, as seen from the following example.

**Example 3.7:** Let \( X=\{a,b,c,d,e\} \), \( τ=\{X, ϕ, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\} \) and \( σ=\{Y, ϕ, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\} \). Let map \( f: X→Y \) defined by \( f(a)=b, f(b)=c, f(c)=d, f(d)=a \), then \( f \) is rgwa-closed map, but not \( w \)-closed, \( r \)-closed, \( β \)-closed, rw-closed, rwg-closed, wa-closed, gs-closed, rs-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and \( g^∗ω \)-closed map, as image of closed set \( f(\{b,c\})=\{c,d\} \) which is not \( w \)-closed, \( r \)-closed, \( β \)-closed, rw-closed, rwg-closed, wa-closed, gs-closed, rs-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and \( g^∗ω \)-closed map, the image of closed set \( f(\{a,c,e\})=\{a,b,d\} \) is not \( w \)-closed and the image of closed set \( f(\{b,c,d,e\})=\{a,c,d,e\} \) is not \( r \)-closed set.

**Example 3.8:** Let \( X=\{a,b,e\} \), \( τ=\{X, ϕ, \{a\}, \{b\}, \{a,b\}\} \) and \( σ=\{Y, ϕ, \{a\}, \{b\}\} \). Let map \( f: X→Y \) defined by \( f(a)=b, f(b)=a, f(c)=c \), then \( f \) is \( gβ \)-closed map, but not rgwa-closed map, as the image of closed set \( f(\{b,c\})=\{a,c\} \) which is not rgwa-closed set.

**Remark 3.9:** The following examples show that rgwa-closed maps are independent of g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps.
Example 3.10: Let X = \{a, b, c\}, τ = \{X, \phi, \{a\}, \{b, c\}\} \sigma = \{Y, \phi, \{a\}\}. Let map f: X → Y defined by f(a) = b, f(b) = a, f(c) = c, then f is g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps, but not rgwα-closed map, as the image of closed set f{a} = \{b\} which is not rgwα-closed set.

Example 3.11: Let X = \{a, b, c, d\}, τ = \{X, \phi, \{a\}, \{b, c\}\} \text{ and } \sigma = \{Y, \phi, \{a\}, \{b, a, c\}\}. Let map f: X → Y defined by f(a) = a, f(b) = c, f(c) = d, f(d) = a then f is rgwα-closed map but not g-closed, wg-closed, αg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed and rgw-closed maps, as the image of closed set F = \{d\} = \{a\} which is rgwα-closed set but not g-closed, wg-closed, rg-closed, gr-closed, gpr-closed, rwg-closed, rgw-closed and αg-closed set.

\[\text{Fig 1: By } A \rightarrow B \text{ we mean } A \text{ implies } B \text{ but not conversely and } A \Rightarrow B \text{ means } A \text{ and } B \text{ are independent of each other.}\]

Theorem 3.13: If f: (X, τ) → (Y, σ) is contra closed and gβ-closed map then f is rgwα-closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) is open and gβ-closed. f(V) is rgwα-closed. Thus f is rgwα-closed map.

Theorem 3.14: If f: (X, τ) → (Y, σ) is αg-continuous and (Y, σ) is ταg-space then f is rgwα-closed map.

Proof: Let V be a closed set in (X, τ). Since f is αg-continuous f(V) is αg-closed set and f(V) is gα-closed hence rgwα-closed. Therefore f is rgwα-closed map.

Theorem 3.15: If a mapping f: (X, τ) → (Y, σ) is rgwα-closed, then rgwαcl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof: Suppose that f is rgwα-closed and A ⊆ X. Then cl(A) is closed in X and so f(cl(A)) is rgwα-closed in (Y, σ). We have f(cl(A)) ⊆ f(cl(A)), by Theorem 5.2(iv) in [28], rgwαcl(f(A)) ⊆ rgwαcl(f(cl(A))) → (i). Since cl(A) is rgwα-closed in (Y, σ), rgwαcl(f(cl(A))) = f(cl(A)) → (ii), by the Theorem 5.3 in [28]. From (i) and (ii), we have rgwαcl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Corollary 3.16: If a mapping f: (X, τ) → (Y, σ) is a rgwα-closed, then the image f(A) of closed set A in (X, τ) is τrgwα -closed in (Y, σ).

Proof: Let A be a closed set in (X, τ). Since f is rgwα-closed, by above Theorem 3.15, rgwαcl(f(A)) ⊆ f(cl(A)) → (i). Also cl(A) = A, as A is a closed set and so f(cl(A)) = f(A) → (ii). From (i) and (ii), we have rgwαcl(f(A)) ⊆ f(A). We know that f(A) ⊆ rgwαcl(f(A)) and so rgwαcl(f(A)) is α-cl. Therefore f(A) is τrgwα-closed in (Y, σ).

Theorem 3.17: Let (X, τ) be any topological spaces and (Y, σ) be a topological space where “rgwαcl(A) = αcl(A)” for every subset A of Y” and f: (X, τ) → (Y, σ) be a map, then the following are equivalent.

(i) f is rgwα-closed map.
(ii) rgwαcl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof: (i) ⇒ (ii) Follows from the Theorem 3.14.

(ii) ⇒ (i) Let A be any closed set of (X, τ). Then A = αcl(A) and so f(A) = f(αcl(A)) = rgwαcl(f(A)) by hypothesis. We have f(A) ⊆ rgwαcl(f(A)), by Theorem 5.2(ii) in [28]. Therefore f(A) = rgwαcl(f(A)). Also f(A) = rgwαcl(f(A)) = αcl(f(A)), by hypothesis. Thus f(A) is rgwα-closed set in (Y, σ) and hence f is rgwα-closed map.

Theorem 3.18: A map f: (X, τ) → (Y, σ) is rgwα-closed if and only if for each subset S of (Y, σ) and each open set U containing fα(S) ⊆ U, there is a rgwα-open set V of (Y, σ) such that S ⊆ V and fα(V) ⊆ U.

Proof: Suppose f is rgwα-closed. Let S ⊆ Y and U be an open set of (X, τ) such that fα(S) ⊆ U. Now X − U is closed set in (X, τ). Since f is rgwα-closed, f(X − U) is rgwα-closed set in (Y, σ). Then V = Y − f(X − U) is a rgwα-open set in (Y, σ). Note that fα(S) ⊆ U implies S ⊆ V and fα(V) = X − fα((X − U)) ⊆ X − (X − U) = U. That is fα(V) ⊆ U. For the converse, let F be a closed set of (X, τ). Then fα(f(F)) ⊆ F and F is an open in (X, τ). By hypothesis, there exists a
rgw*-open set V in (Y, σ) such that f(F) ⊆ V and f*(V) ⊆ F and so F ⊆ (f*(V))*. Hence V* ⊆ f*(F) ⊆ f((f*(V))*) ⊆ V* which implies f(F) = V*. Since V* is rgw*-closed, f(F) is rgw*-closed. Thus f(F) is rgw*-closed in (Y, σ) and therefore f is rgw*-closed map.

**Remark 3.19:** The composition of two rgw*-closed maps need not be rgw*-closed map in general and this is shown by the following example.

**Example 3.20:** Let X = Y = Z = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}, {b}, {c}, {a, c}} and η = {Z, ϕ, {a}, {b}, {a, b}}. Define f(X, τ) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c and g: (Y, σ) → (Z, η) by g(a) = b, g(b) = c, g(c) = a. Then f and g are rgw*-closed maps, but their composition g ◦ f: (X, τ) → (Z, η) is not rgw*-closed map, because F = {b, c} is closed in (X, τ), but g ◦ f(F) = g ◦ f({b, c}) = g({a, c}) = {a, b} which is not rgw*-closed in (Y, σ).

**Theorem 3.21:** If f: (X, τ) → (Y, σ) is closed map and g: (Y, σ) → (Z, η) is rgw*-closed map, then the composition g ◦ f: (X, τ) → (Z, η) is rgw*-closed map.

**Proof:** Let f be any closed set in (X, τ). Since f is closed map, f(F) is closed set in (Y, σ). Since g is rgw*-closed map, g(f(F)) is rgw*-closed set in (Z, η). That is g ◦ f(F) = g(f(F)) is rgw*-closed and hence g ◦ f is rgw*-closed map.

**Remark 3.22:** If f: (X, τ) → (Y, σ) is rgw*-closed map and g: (Y, σ) → (Z, η) is closed map, then the composition need not be rgw*-closed map as seen from the following example.

**Example 3.23:** Let X = Y = Z = {a, b, c}, τ = {X, ϕ, {a}, {a, b}, {a, c}, {b}, {c}, {a, c}} and η = {Z, ϕ, {a}, {b}, {a, b}}. Define f(X, τ) → (Y, σ) by f(a) = b, f(b) = a, f(c) = c and g: (Y, σ) → (Z, η) by g(a) = b, g(b) = c, g(c) = a. Then f is rgw*-closed map and g is closed map, but their composition g ◦ f: (X, τ) → (Z, η) is not rgw*-closed map, because F = {b, c} is closed in (X, τ), but g ◦ f(F) = g ◦ f({b, c}) = g({a, c}) = {a, b} which is not rgw*-closed in (Y, σ).

**Theorem 3.24:** If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is rgw*-closed maps and (Y, σ) be a τrgw*-space then g ◦ f: (X, τ) → (Z, η) is rgw*-closed map.

**Proof:** Let A be a closed set of (X, τ). Since f is rgw*-closed, f(A) is rgw*-closed in (Y, σ). Then by hypothesis, f(A) is closed set in (Y, σ). Since g is rgw*-closed, g(f(A)) is rgw*-closed in (Z, η) and g(f(A)) = g ◦ f(A) is rgw*-closed map.

**Theorem 3.25:** If f: (X, τ) → (Y, σ) is g-closed, g: (Y, σ) → (Z, η) be rgw*-closed and (Y, σ) be τ1/2-space then their composition g ◦ f: (X, τ) → (Z, η) is rgw*-closed map.

**Proof:** Let A be a closed set of (X, τ). Since f is g-closed, f(A) is g-closed in (Y, σ). Since (Y, σ) is τ1/2-space, f(A) is closed in (Y, σ). Since g is rgw*-closed, g(f(A)) is rgw*-closed in (Z, η) and g(f(A)) = g ◦ f(A). Therefore g ◦ f is rgw*-closed map.

**Theorem 3.26:** Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be two mappings such that their composition g ◦ f: (X, τ) → (Z, η) be rgw*-closed mapping. Then the following statements are true.

(i) If f is continuous and surjective, then g is rgw*-closed.
(ii) If g is rgw*-irresolute and surjective, then f is rgw*-closed.
(iii) If f is g-continuous and surjective and (X, τ) is a τ1/2-space, then g is rgw*-closed.
(iv) If g is strongly rgw*-continuous and injective, then f is rgw*-closed.

**Proof:**
(i) Let A be a closed set of (Y, σ). Since f is continuous, f(A) is closed in (X, τ) and since g ◦ f is rgw*-closed, (g ◦ f)(f(A)) is rgw*-closed in (Z, η). That is (g(A)) = rgw*-closed in (Z, η), since f is surjective. Therefore g is rgw*-closed.

(ii) Let B be a closed set of (X, τ). Since g ◦ f is rgw*-closed, g ◦ f(B) is rgw*-closed in (Z, η). Since g is rgw*-irresolute, g ◦ f(B) = rgw*-closed set in (Y, σ). That is f(B) is rgw*-closed in (Y, σ), since f is injective. Therefore f is rgw*-closed.

(iii) Let C be a closed set of (Y, σ). Since f is g-continuous, f(C) is a closed set in (X, τ). Since (X, τ) is a τ1/2-space, f(C) is closed set in (X, τ). Since g ◦ f is rgw*-closed, (g ◦ f)(C) is rgw*-closed in (Z, η). That is g(C) is rgw*-closed in (Z, η), since f is injective. Therefore g is rgw*-closed.

(iv) Let D be a closed set of (X, τ). Since g ◦ f is rgw*-closed, g ◦ f(D) is rgw*-closed in (Z, η). Since g is strongly rgw*-continuous, g ◦ f(D) is closed set in (Y, σ). That is f(D) is closed set in (Y, σ), since g is injective. Therefore f is closed.

**Theorem 3.27:** Let f: (X, τ) → (Y, σ) and cl(F) = F for every rgw*-closed set in (Y, σ), where X is regular, then Y is regular.

**Proof:** Let U be an open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈ X such that f(x) = p. Since X is regular and f is continuous, there is an open set V in X such that x ∈ V ⊆ cl(V) ⊆ f(U). Hence p ∈ f(V) ⊆ cl(f(V)) ⊆ U → (i). Since f is rgw*-closed, f(cl(V)) is rgw*-closed set contained in the open set U. By hypothesis, cl(cl(f(V))) = cl(fcl(V)) and cl(cl(f(V))) = cl(cl(fcl(V))) → (ii). From (i) and (ii), we have p ∈ f(V) ⊆ cl(f(V)) ⊆ U and f(V) is open, since f is open. Hence Y is regular.

**Theorem 3.28:** If a map f: (X, τ) → (Y, σ) is an open, continuous, rgw*-closed surjection and cl(F) = F for every rgw*-closed set in (Y, σ), where X is regular, then Y is regular.

**Proof:** Let U be an open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈ X such that f(x) = p. Since X is regular and f is continuous, there is an open set V in X such that x ∈ V ⊆ cl(V) ⊆ f(U). Here p ∈ f(V) ⊆ cl(f(V)) ⊆ U → (i). Since f is rgw*-closed, f(cl(V)) is rgw*-closed set contained in the open set U. By hypothesis, cl(cl(f(V))) = cl(fcl(V)) and cl(cl(f(V))) = cl(cl(fcl(V))) → (ii). Therefore f is open. Hence Y is regular.

**Theorem 3.29:** If a map f: (X, τ) → (Y, σ) is a continuous, rgw*-closed map from a normal space (X, τ) onto a space (Y, σ) then (Y, σ) is τ-normal.

**Proof:** Let A and B be two disjoint closed sets of (Y, σ). Then f(A) and f(B) are disjoint closed sets of (X, τ), since
\( f \) is continuous. Therefore there exists open sets \( U \) and \( V \) such that \( f^1(A) \subseteq U \) and \( f^1(B) \subseteq V \), since \( X \) is normal. Using theorem 3.18, there exists \( \text{rgw-} \)open sets \( C, D \) in \( (Y, \sigma) \) such that \( A \subseteq C, B \subseteq D, f^1(C) \subseteq U \) and \( f^1(D) \subseteq V \). Since \( A \) and \( B \) are closed, \( A \) and \( B \) are \( \alpha \)-closed and \( \text{wat} \)-closed. By the definition of \( \text{rgw-} \)open, \( C \) is \( \text{rgw-} \)open if and only if \( A \subseteq \text{rint}(C) \) whenever \( A \subseteq C \) and \( A \) is \( \text{wat} \)-closed, we get \( A \subseteq \text{rint}(C) \). Thus \( A \subseteq \text{rint}(C) \) and \( B \subseteq \text{rint}(D) \). Hence \( X \) is \( \alpha \)-normal.

**Theorem 3.30:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an \( \alpha \)-irresolute, \( (Y, \sigma) \) is a \( \alpha \)-normal space then \( f \) is an \( \text{rgw-} \)irresolute map.

**Proof:** Let \( U \) be \( \text{rgw-} \)closed in \( (Y, \sigma) \) and \( U \) is \( \alpha \)-closed and \( \beta \)-closed. Since \( (Y, \sigma) \) is a \( \beta \)-space, \( U \) is \( \beta \)-closed. Since \( f \) is \( \beta \)-irresolute, \( f^1(U) \) is \( \beta \)-closed. Hence \( f^1(U) \) is \( \text{rgw-} \)closed. Thus \( f \) is \( \text{rgw-} \)irresolute map.

**Theorem 3.31:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an \( \beta \)-irresolute, \( (Y, \sigma) \) is a \( \beta \)-normal space then \( f \) is an \( \text{rgw-} \)irresolute map.

**Proof:** Let \( U \) be \( \text{rgw-} \)closed in \( (Y, \sigma) \). Then \( U \) is \( \beta \)-closed. Since \( f \) is \( \beta \)-irresolute and \( (X, \tau) \) is a \( \beta \)-space, \( f^1(U) \) is \( \beta \)-closed and \( \alpha \)-open. Hence \( f^1(U) \) is \( \text{rgw-} \)closed. Thus \( f \) is \( \text{rgw-} \)irresolute map.

**Theorem 3.32:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a \( \beta^* \)-quotient map and \( (Y, \sigma) \) is a \( \beta \)-space then \( f \) is a \( \text{rgw-} \)irresolute map.

**Proof:** Let \( U \) be \( \text{rgw-} \)closed in \( (Y, \sigma) \). Then \( U \) is \( \beta \)-closed. Since \( f \) is \( \beta \)-irresolute and \( (X, \tau) \) is a \( \beta \)-space, \( f^1(U) \) is \( \beta \)-closed and \( \alpha \)-open. Hence \( f^1(U) \) is \( \text{rgw-} \)closed. Thus \( f \) is \( \text{rgw-} \)irresolute map.

**Definition 3.33:** A map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called a regular generalized weakly \( \alpha \)-open map (briefly \( \text{rgw-} \)open map) if the image \( f(A) \) is \( \text{rgw-} \)open in \( (Y, \sigma) \) for each open set \( A \) in \( (X, \tau) \).

From the definitions we have the following results.

**Theorem 3.34:**
(i) Every open map is \( \text{rgw-} \)open but not conversely.
(ii) Every \( \alpha \)-open (resp. \( r \)-open, \( w \)-open, \( \text{wat} \)-open, \( \text{wat} \)-open, \( \alpha \)-open, \( \beta \)-open, \( r \)-open, \( w \)-open, \( \text{rgw-} \)-open, \( \text{g}^* \text{wat} \)-open) map is \( \text{rgw-} \)open but not conversely.
(iii) Every \( \text{rgw-} \)open map is \( \beta \)-open but not conversely.
(iv) \( g \)-open (resp. \( \text{wg-} \)-open, \( r \)-open, \( g \)-open, \( \alpha \)-open, \( \text{rgw-} \)-open, \( \text{grw-} \)-open, \( \text{gpr-} \)-open) map is independent with \( \text{rgw-} \)closed map.

**Theorem 3.35:** For any bijection map \( f: (X, \tau) \rightarrow (Y, \sigma) \), the following statements are equivalent:
(i) \( f^1: (Y, \sigma) \rightarrow (X, \tau) \) is \( \text{rgw-} \)continuous.
(ii) \( f \) is \( \text{rgw-} \)open map and
(iii) \( f \) is \( \text{rgw-} \)closed map.

**Proof:**
(i) \( \Rightarrow \) (ii) Let \( U \) be an open set of \( (X, \tau) \). By assumption, \( f^1(U) = f(U) \) is \( \text{rgw-} \)open in \( (Y, \sigma) \) and so \( f \) is \( \text{rgw-} \)open.
We define another new class of maps called rgwa*-closed maps which are stronger than rgwa-closed maps.

**Definition 3.40:** A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be regular generalized weakly $\alpha^*$-closed map (briefly rgwa*-closed map) if the image $f(A)$ is rgwa- closed in $(Y, \sigma)$ for every rgwa-closed set $A$ in $(X, \tau)$.

**Theorem 3.41:** Every rgwa*-closed map is rgwa-closed map but not conversely.

**Proof:** The proof follows from the definitions and fact that every closed set is rgwa-closed. The converse of the above Theorem is not true in general as seen from the following example.

**Example 3.42:** Let $X = \{a, b, c, d, e\}$, $Y = \{a, b, c, d\}$ $\tau = \{X, \phi \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{a,d,e\}\}$ and $\sigma = \{Y, \phi \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a)=b$, $f(b)=c$, $f(c)=d$, $f(d)=a$, $f(e)=d$. Then $f$ is rgwa-closed map but not rgwa*-closed map, since for the rgwa-closed set $\{a,b,d\}$ in $(X, \tau)$, $f(\{a,b,d\}) = \{a,b,c\}$ which is not rgwa-closed set in $(Y, \sigma)$.

**Theorem 43:** If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are rgwa*-closed maps, then their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is also rgwa*-closed.

**Proof:** Let $F$ be a rgwa-closed set in $(X, \tau)$. Since $f$ is rgwa*-closed map, $f(F)$ is rgwa-closed set in $(Y, \sigma)$. Since $g$ is rgwa*-closed map, $g(f(F))$ is rgwa-closed set in $(Z, \eta)$. Therefore $g \circ f$ is rgwa*-closed map.

Analogous to rgwa*-closed map, we define another new class of maps called rgwa*-open maps which are stronger than rgwa-open maps.

**Definition 3.44:** A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be regular generalized weakly $\alpha^*$-open map (briefly rgwa*-open map) if the image $f(A)$ is rgwa-open set in $(Y, \sigma)$ for every rgwa-open set $A$ in $(X, \tau)$.

**Remark 45:** Since every open set is a rgwa-open set, we have every rgwa*-open map is rgwa-open map. The converse is not true in general as seen from the following example.

**Example 3.46:** Let $X = \{a, b, c, d, e\}$, $Y = \{a, b, c, d\}$ $\tau = \{X, \phi \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{a,d,e\}\}$ and $\sigma = \{Y, \phi \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as $f(a)=b$, $f(b)=c$, $f(c)=d$, $f(d)=a$, $f(e)=d$. Then $f$ is rgwa-open map but not rgwa*-open map, since for the rgwa-open set $\{c,e\}$ in $(X, \tau)$, $f(\{c,e\}) = \{d\}$ which is not rgwa-open set in $(Y, \sigma)$.

**Theorem 47:** If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are rgwa*-open maps, then their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is also rgwa*-open.

**Proof:** Proof is similar to the Theorem 3.43.

**Theorem 3.48:** For any bijection map $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:

i) $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ is rgwa-irresolute.

ii) $f$ is rgwa*-open map.

iii) $f$ is rgwa*-closed map.

**Proof:** Proof is similar to that of Theorem 3.35.

**4. Conclusion**

In this paper we have introduced and studied the properties of rgwa-Closed and rgwa-Open maps. Our future extension is rgwa- Closed and rgwa-Open maps in Fuzzy Topological Spaces.

**5. Acknowledgement**

The Authors would like to thank the referees for useful comments and suggestions.

**6. References**

17. Maki H, Umehara J, Noiri T. Every Topological space is pre $T\frac{1}{2}$mem Fac sci, Kochi univ, Math, 1996; 17:33-42.