On RGWA-locally closed sets in Topological Spaces

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Abstract
In this paper, we introduce three weaker forms of locally closed sets called RGWα-LC sets, RGWα-LC* set and RGWα-LC** sets each of which is weaker than locally closed set and study some of their properties and their relationships with w-lc, θg-lc, lδc, π-g-lc and rg-lc etc sets.

Keywords: RGWα-closed sets, rgwα-open sets, locally closed sets, rgwα-locally closed sets

Introduction
Kuratowski and Sierpinski [8] introduced the notion of locally closed sets in topological spaces. According to Bourbaki [4], a subset of a topological space (X, τ) is locally closed in (X, τ) if it is the intersection of an open set and a closed set in (X, τ). Stone [12] has used the term FG for locally closed set. Ganster and Reilly [5] have introduced locally closed sets, which are weaker forms of both closed and open sets. After that Balachandran et al. [3], Gnanambal [6], Arockiarani et al. [1], Pusphalatha [10] and Sheik John [11] have introduced α-locally closed, generalized α-locally closed, semi locally closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed and w-locally closed sets and their continuous maps in topological space respectively. Recently rgwα-closed sets and continuous maps were introduced and studied by Wali et al. [13, 15].

2. Preliminaries:

1. Locally closed (briefly LC) set [5] if A=U∩F, where U is open and F is closed in X.
2. regular generalized weakly α-closed set [13] (briefly rgwα-closed set) if rαcl(A)⊆U whenever A⊆U and U is wα-open in (X, τ).
4. 0g-lec set [2] if A=U∩F, where U is 0g-open and F is 0g-closed in X.
5. 0g-lec* set [2] if A=U∩F, where U is 0g-open and F closed in X.
6. 0g-lec** set [2] if A=U∩F, where U is open and F 0g-closed in X.
7. g-lec set [3] if A=U∩F, where U is g-open and F is g-closed in X.
8. g-lec* set [3] if A=U∩F, where U is g-open and F closed in X.
9. g-lec** set [3] if A=U∩F, where U is open and F is g-closed in X.
10. w-lec set if A=U∩F [11] where U is w-open and F is w-closed in X.
11. w-lec* set if A=U∩F [11] where U is w-open and F closed in X.
12. w-lec** set if A=U∩F [11] where U is open and F is w-closed in X.
14. rg-lec* set if A=U∩F [11] where U is g-open and F closed in X.
15. rg-lec** set if A=U∩F [11] where U is open and F is rg-closed in X.
16. log-lec set if A=U∩F [9] where U is log-open and F is log-closed in X.
17. log-lec* set if A=U∩F [9] where U is log-open and F closed in X.
18. log-lec** set if A=U∩F [9] where U is open and F is log-closed in X.
19. rg-log-lec set if A=U∩F [13] where U is rg-open and F is rg-closed in X.
20. rg-log-lec* set if A=U∩F [13] where U is rg-open and F closed in X.
21. rg-log-lec** set if A=U∩F [13] where U is open and F is rg-closed in X.

Example 2.1: Let X = {a, b, c} and τ = {X, Φ, {a}, {b}, {a, b}}. Here
Closed sets: {X, Φ, {c}, {a, c}, {b, c}}
θ-closed sets: {X, Φ}
Example 2.2: Let X = \{a, b, c\} and \tau = \{X, \emptyset, \{a\}\}. Here
Closed sets: \{X, \emptyset, \{a\}\}
\emptyset -closed sets: \{X, \emptyset\}
\emptyset g\text{-closed sets:} \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}
\emptyset \delta\text{-closed sets:} \{X, \emptyset\}
\emptyset ng\text{-closed sets:} \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}
\emptyset \pi\text{-closed sets:} \{X, \emptyset\}
g\text{-closed sets:} \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}
g\text{-closed sets:} \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}
g\text{-closed sets:} \{X, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}
rg\text{-closed sets:} \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}
rg\alpha\text{-closed sets:} \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}

Lemma 2.3: By seeing above example we can say
(i) Every closed set is rg\alpha\text{-closed set.}
(ii) Every w\text{-closed set is rg\alpha\text{-closed set.}
(iii) Every \emptyset\text{-closed set is rg\alpha\text{-closed set.}
(iv) Every \emptyset\text{-closed set is rg\alpha\text{-closed set.}
(v) Every \emptyset\text{-closed set is rg\alpha\text{-closed set.}
(vi) rg\alpha\text{-closed set and }\emptyset g\text{-closed, }\emptyset g\text{-closed, } ng\text{-closed } g\text{-closed and } rg\text{-closed sets are independent of each other.}

Lemma 2.4: The space (X, \tau) is }\tau_{rg\alpha}\text{-space if every }rg\alpha\text{-closed set is closed set.}

3. \text{rg\alpha-Locally Closed Sets in Topological Spaces}

Definition 3.1: A Subset A of a Topological space (X, \tau) is called regular generalized weakly }\alpha\text{-locally closed (briefly }rg\alpha\text{-locally closed) if }A=U\cup F \text{ where } U \text{ is } rg\alpha\text{-open in } (X, \tau) \text{ and } F \text{ is } rg\alpha\text{-closed in } (X, \tau).
The set of all }rg\alpha\text{-locally closed sets of } (X, \tau) \text{ is denoted by } RG\alpha\text{-LC}(X, \tau).

Example 3.2: Let X = \{a, b, c, d, e\} and \tau = \{X, \emptyset, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}\}
RG\alpha\text{-C}(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}
RG\alpha\text{-O}(X, \tau) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}
RG\alpha\text{-LC Set} = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{a, b, d, e\}, \{b, c, d, e\}

Remark 3.3: The following are well known
(i) A Subset A of (X, \tau) is }RG\alpha\text{-LC set iff its complement X-A is the union of a }rg\alpha\text{-open set and a }rg\alpha\text{-closed set.}
(ii) Every }rg\alpha\text{-open (resp. }rg\alpha\text{-closed) subset of (X, }\tau) \text{ is a }RG\alpha\text{-LC set.

Theorem 3.4: Every locally closed set is a }RG\alpha\text{-LC set but not conversely.
Proof: The proof follows from definition and fact that every closed (resp. open) set is }rg\alpha\text{-closed (rw-open).

Example 3.5: Let X = \{a, b, c, d, e\} and \tau = \{X, \emptyset, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}\}
LC\text{-Set} = \{X, \emptyset, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{b, c, d\}, \{b, c, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{b, c, d\}, \{b, c, e\}, \{a, b, c, d\}, \{a, b, c, e\}, \{b, c, d\}, \{b, c, e\}\}. Here the set \{a, b\} is }RG\alpha\text{-LC-Set but not LC-set.

Theorem 3.6: Every w\text{-lc set is a }RG\alpha\text{-LC set but not conversely.
Proof: The proof follows from definition and fact that every w\text{-closed (resp. w\text{-open) set is }rg\alpha\text{-closed (rg\alpha\text{-open).}
Example 3.7: Let \( X=\{a, b, c, d\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) then \( \{a, d\} \) is RGW\( \alpha \)-LC set but not a w-locally closed set in \((X, \tau)\).

Theorem 3.8: Every \( \theta \)-lc set is a RGW\( \alpha \)-LC set but not conversely.
Proof: The proof follows from definition and fact that every \( \theta \)-closed (resp. \( \theta \)-open) set is rgw\( \alpha \)-closed (rgw\( \alpha \)-open).

Example 3.9: Let \( X=\{a, b, c, d\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) then \( \{a\} \) is RGW\( \alpha \)-LC set but not a \( \theta \)-locally closed set in \((X, \tau)\).

Theorem 3.10: Every \( \theta \)-lc set is a RGW\( \alpha \)-LC set but not conversely.
Proof: The proof follows from definition and fact that every \( \delta \)-closed (resp. \( \delta \)-open) set is rgw\( \alpha \)-closed (rgw\( \alpha \)-open).

Example 3.11: Let \( X=\{a, b, c\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}, \{a, b\}\} \) then \( \{a, b\} \) is RGW\( \alpha \)-LC set but not a \( l \delta \)-closed set in \((X, \tau)\).

Theorem 3.12: Every \( \pi \)-lc set is a RGW\( \alpha \)-LC set but not conversely.
Proof: The proof follows from definition and fact that every \( \pi \)-closed (resp. \( \pi \)-open) set is rgw\( \alpha \)-closed (rgw\( \alpha \)-open).

Example 3.13: Let \( X=\{a, b, c, d\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\} \) then \( \{b, c\} \) is RGW\( \alpha \)-LC set but not a \( \pi \)-locally closed set in \((X, \tau)\).

Remark 3.14: lgc set, \( \theta \)-g lc set, \( l \delta \)gc set, \( \pi \)g-lc set, rg-lc sets and RGW\( \alpha \)-LC sets are independent of each other as seen from the following example.

Example 3.15: i) Let \( X = \{a, b, c\} \) and \( \tau=\{X, \emptyset, \{a\}\} \) then \( \{a, c\} \) is lgc set, \( \theta \)-g lc set, \( l \delta \)gc set, rg-lc set, but not RGW\( \alpha \)-LC set in \((X, \tau)\).

ii) Let \( X = \{a, b, c\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}\} \) then \( \{b\} \) is RGW\( \alpha \)-LC set, but not lgc set, \( \theta \)-g lc set, \( l \delta \)gc set, \( \pi \)g-lc set rg-lc set in \((X, \tau)\).

Definition 3.16: A subset \( A \) of \((X, \tau)\) is called a RGW\( \alpha \)-LC* set if there exists a rgw\( \alpha \)-open set \( G \) and a closed \( F \) of \((X, \tau)\) s.t \( A=G \cap F \) the collection of all RGW\( \alpha \)-LC* sets of \((X, \tau)\) will be denoted by RGW\( \alpha \)-LC*(\( X, \tau)\).

Definition 3.17: A subset \( B \) of \((X, \tau)\) is called a RGW\( \alpha \)-LC** set if there exists an open set \( G \) and rgw\( \alpha \)-closed set \( F \) of \((X, \tau)\) s.t \( B=G \cap F \) the collection of all RGW\( \alpha \)-LC** sets of \((X, \tau)\) will be denoted by RGW\( \alpha \)-LC**(\( X, \tau)\).

Theorem 3.18: 1. Every locally closed set is a RGW\( \alpha \)-LC* set.
2. Every locally closed set is a RGW\( \alpha \)-LC** set.
3. Every w-lc* set is RGW\( \alpha \)-LC* set.
4. Every w-lc** set is RGW\( \alpha \)-LC** set.
5. Every \( \theta \)-lc* set is a RW-LC* set.
6. Every \( \theta \)-lc** set is a RW-LC** set.
7. Every \( \delta \)-lc* set is a RW-LC* set.
8. Every \( \delta \)-lc** set is a RW-LC** set.
9. Every \( \pi \)-lc* set is a RW-LC* set.
10. Every \( \pi \)-lc** set is a RW-LC** set.
11. Every RGW\( \alpha \)-LC* set is RGW\( \alpha \)-LC set.
12. Every RGW\( \alpha \)-LC** set is RGW\( \alpha \)-LC set.
13. Every RGW\( \alpha \)-LC* set is rg-lc* set.
14. Every RGW\( \alpha \)-LC** set is rg-lc** set.
15. Every RGW\( \alpha \)-LC* set is rg-lc set.
16. Every RGW\( \alpha \)-LC** set is rg-lc set.

Proof: The proofs are obvious from the definitions and the relation between the sets. However the converses of the above results are not true as seen from the following examples.

Example 3.19: Let \( X = \{a, b, c, d\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) then \( \{a, d\} \) is RGW\( \alpha \)-LC set but not a \( \theta \)-locally closed set in \((X, \tau)\).

Example 3.20: Let \( X = \{a, b, c\} \) and \( \tau=\{X, \emptyset, \{a\}, \{b\}\} \) then \( \{a\} \) is RGW\( \alpha \)-LC set but not a \( \theta \)-lc* set in \((X, \tau)\).
(ii) The set \( \{b\} \) is RW-LC** set but not a \( \theta\)-lc** set in \((X, \tau)\).
(iii) The set \( \{c\} \) is RW-LC* set but not a \( \delta\)-lc* set in \((X, \tau)\).
(iv) The set \( \{a, b\} \) is RW-LC** set but not a \( \delta\)-lc** set in \((X, \tau)\).

Example 3.21: Let \( X = \{a, b, c\} \) and \( \tau = \{X, \phi, \{a\}\} \)
(i) The set \( \{a\} \) is RW-LC* set but not a \( \pi\)-lc* set in \((X, \tau)\).
(ii) The set \( \{b, c\} \) is RW-LC** set but not a \( \pi\)-lc** set in \((X, \tau)\).

Example 3.22: Let \( X = \{a, b, c, d\} \) and \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\} \)
(i) The set \( \{a\} \) is RGW\(\alpha\)-LC* set but not a RGW\(\alpha\)-LC* set in \((X, \tau)\).
(ii) The set \( \{b, c\} \) is RGW\(\alpha\)-LC** set but not a RGW\(\alpha\)-LC** set in \((X, \tau)\).

Example 3.23: Let \( X = \{a, b, c\} \) and \( \tau = \{X, \phi, \{a\}\} \)
(ii) The set \( \{a, b\} \) is RGW-LC set but not a RGW-LC* set in \((X, \tau)\).
(iii) The set \( \{a, c\} \) is RGW-LC* set but not a RGW-LC** set in \((X, \tau)\).
(iv) The set \( \{b\} \) is RGW-LC set but not a RGW-LC* set in \((X, \tau)\).
(v) The set \( \{c\} \) is RGW-LC set but not a RGW-LC** set in \((X, \tau)\).

Example 3.24: \( lgc* \) set, \( \theta\)-g lc* set, \( \delta\) gc* set, \( \pi\)g-lc* sets and RGW\(\alpha\)-LC* sets are independent of each other and \( lgc** \) set, \( \theta\)-g lc** set, \( \delta\) gc** set, \( \pi\)g-lc** sets and RGW\(\alpha\)-LC** sets are independent of each other as seen from the following examples.

Remark 3.25: Let \( X = \{a, b, c, d\} \) and \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b\}, \{a, b, c\}\} \) Here
(i) \( \{a, d\} \) is RGW\(\alpha\)-LC* but not \( \theta\)-g lc* set and \( \delta\) gc* set in \((X, \tau)\).
(ii) \( \{a, b, c\} \) is RGW\(\alpha\)-LC* but not \( \pi\)g-lc* set in \((X, \tau)\).
(iii) \( \{c\} \) is RGW\(\alpha\)-LC** but not \( \theta\)-g lc** set and \( \delta\) gc** set in \((X, \tau)\).
(iv) \( \{a, b, d\} \) is RGW\(\alpha\)-LC** but not \( \pi\)g-lc** set in \((X, \tau)\).

Example 3.26: Let \( X = \{a, b, c\} \) and \( \tau = \{X, \phi, \{a\}\} \) Here
(i) \( \{a, b\} \) is \( lgc* \) set, but not RGW\(\alpha\)-LC* set in \((X, \tau)\).
(ii) \( \{a, c\} \) is \( lgc** \) set, but not RGW\(\alpha\)-LC** set in \((X, \tau)\).

Remark 3.27: RGW\(\alpha\)-LC* sets and RGW\(\alpha\)-LC** sets are independent of each other as seen from the examples.

Example 3.28: (i) Let \( X = \{a, b, c, d\} \) and \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) then set \( \{a, b, d\} \) is RGW\(\alpha\)-LC** set but not a RGW\(\alpha\)-LC** set in \((X, \tau)\).
(ii) Let \( X = \{a, b, c, d\} \) and \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) then set \( \{c\} \) is RGW\(\alpha\)-LC* set but not a RGW\(\alpha\)-LC** set in \((X, \tau)\).

Remark 3.29: From the above discussion and known results we have the following implications in the diagram.

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Fig 1
Example 3.33: Let X = {a, b, c} with the topology τ = {X, φ, {a}, {b}, {a, b}} then RGWα-LC*(X, τ) = RGWα-LC*(X, τ) = RGWα-LC(X, τ) = P(X). However RGWαO(X, τ) = {X, φ, {a}, {b}, {a, b}, {a, b, c}} ≠ τ.

Theorem 3.34: If GO(X, τ) = τ, then GLC(X, τ) ⊆ RGWα-LC(X, τ).

Proof: For any space (X, τ) w.k.t LC(X, τ) ⊆ GLC(X, τ) and LC(X, τ) ⊆ RGWα-LC**(X, τ) ⊆ RGWα-LC(X, τ) since RGWαO(X, τ) = τ, RGWα-LC(X, τ) = LC(X, τ) by theorem 3.30, it follows that LC(X, τ) = RGWα-LC**(X, τ) = RGWα-LC(X, τ).

Remark 3.36: The converse of the theorem 3.44 need not be true in general as seen from the following example.

Example 3.37: Consider X = {a, b, c, d} with the topology τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}} then RGWα-LC(X, τ) = RGWα-LC*(X, τ) = P(X). But RGWαC(X, τ) = {X, φ, {a}, {b}, {c}, {d}, {a, c}, {b, c}, {a, c, b}, {a, c, d}, {b, c, d}} and LC(X, τ) = {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {b, c}, {a, c, b}, {a, c, d}, {b, c, d}}. That is RGWαC(X, τ) ⊆ LC(X, τ).

Theorem 3.38: For a subset A of (X, τ) if A ∈ RGWα-LC(X, τ) then A = U∪(rgwαcl (A)) for some open set U.

Proof: Let, A ∈ RGWα-LC(X, τ) then there exist a rgwα-open U and a rgwα-closed set F s.t. A = U∪F. Since A ⊆ F, rgwαcl(A)= rgwαcl(F) =F. Now U∪(rgwαcl(A))∪ U∪F = A, that is U∪(rgwαcl(A))= A.

Remark 3.39: The converse of the theorem 3.38 need not be true in general as seen from the following example.

Example 3.40: Consider X = {a, b, c, d} with the topology τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}} then Take A = {a, d}, rgwαcl(A) = {a, d} now, A = X∩(rgwαcl(A)) for some rgwα-open set X but {a, d} ∉ RGWα-LC(X, τ).

Theorem 3.41: For a subset A of (X, τ), the following are equivalent.

(i) A ∈ RGWα-LC**(X, τ).
(ii) A = U∪ (cl(A)) for some rgwα-open set U.
(iii) cl(A)-A is rgwα-closed.
(iv) AU (cl(A)) is rgwα-open.
Proof: (i) If A is a rgwα-open set and cl(A) is a closed set, then cl(A) = A = U ∩ f(A). This shows that A = U ∩ cl(A) for some rgwα-open set U. Hence A = RGWα-LC*(X, τ).

(ii) If A is a rgwα-open set and A ∩ cl(A) = ∅, then cl(A) is a closed set, and cl(A) = A = U ∩ f(A). This shows that A = U ∩ cl(A) for some rgwα-open set U. Hence A = RGWα-LC*(X, τ).

(iii) If A is a rgwα-open set and A = cl(A), then cl(A) = A = U ∩ f(A). This shows that A = U ∩ cl(A) for some rgwα-open set U. Hence A = RGWα-LC*(X, τ).

Example 3.44: Let X = {a, b, c, d} with the topology τ = {∅, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}. Take A = {a, b, d}. Then rgwαcl(A) = rgwαcl({a, b, d} = {a, b, d}. Also A = X ∩ rgwαcl(A) = A = {a, b, d} for some open set X in {a, b, d} ∈ RGWα-LC**(X, τ).

Theorem 3.42: For a subset A of (X, τ) if AERGWA-LC**(X, τ), then there exists an open set U such that A = U ∩ rgwαcl(A).

Proof: Let AERGWA-LC**(X, τ), then there exists an open set U and a rgwα-closed set A = U ∩ f(A) Since A = U ∩ A. Hence A = U ∩ f(A) for some rgwα-open set U. Now consider U ∩ cl(A) = A = U ∩ (cl(A))c. This shows that A = (cl(A))c is rgwα-open.

Remark 3.43: The converse of the theorem 3.42 need not be true in general as seen from the following example.

Example 3.44: Let X = {a, b, c, d} with the topology τ = {∅, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}. Take A = {a, b, d}. Then rgwαcl(A) = rgwαcl({a, b, d} = {a, b, d}. Also A = X ∩ rgwαcl(A) = A = {a, b, d} for some open set X in {a, b, d} ∈ RGWα-LC**(X, τ).

Theorem 3.45: For A and B in (X, τ) the following are true.

(i) If AERGWA-LC*(X, τ) and BERGWA-LC*(X, τ), then A ∩ B = ERGWA-LC*(X, τ).

(ii) If AERGWA-LC**(X, τ) and B is open, then A ∩ B = ERGWA-LC**(X, τ).

(iii) If AERGWA-LC*(X, τ) and B is rgwα-open, then A ∩ B = ERGWA-LC*(X, τ).

(iv) If AERGWA-LC**(X, τ) and B is rgwα-open, then A ∩ B = ERGWA-LC**(X, τ).

(v) If AERGWA-LC**(X, τ) and B is closed, then A ∩ B = ERGWA-LC**(X, τ).

Definition 3.46: A topological space (X, τ) is called RGWα-submaximal if every dense set in it is RGWα-open.

Theorem 3.47: A Topological Space (X, τ) is rgwα-submaximal if and only if P(X) = RGWα-LC*(X, τ).

Proof: Let (X, τ) be rgwα-submaximal, AEP(X) and V = AU(X-cl(A)). Then cl(V) = cl(AU(X-cl(A))) = cl(AU(X-cl(A))) = X. That is cl(V) = X. It follows that V is dense in (X, τ). By assumption, V is rgwα-open. By Theorem 3.41, AERGWA-LC*(X, τ). Therefore P(X) = RGWα-LC*(X, τ). Conversely, let A be dense in (X, τ) and P(X) = RGWα-LC*(X, τ). Then A = AU(X-cl(A)). Since AERGWA-LC*(X, τ), A = AU(X-cl(A)) is rgwα-open by Theorem 3.41. Hence (X, τ) is rgwα-submaximal.

Theorem 3.48: If (X, τ) is a submaximal space then it is RGWα-submaximal but converse need not be true in general.

Proof: Let (X, τ) be submaximal space and A be a dense subset of (X, τ). Then A is open. But every open set is rgwα-open and so A is rgwα-open. Therefore (X, τ) is a RGWα-submaximal space.
Example 3.49: Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}\}$. Then Topological space $(X, \tau)$ is RGW$\alpha$-submaximal but set $A = \{b, c\}$ is dense in $(X, \tau)$ but not open therefore $(X, \tau)$ is not submaximal.

Theorem 3.50: A topological space $(X, \tau)$ is RGW$\alpha$-submaximal if and only if $P(X) = RGW\alpha-LC^\tau(X, \tau)$.

Proof

Necessity: Let $A \in P(X)$ and $U = \cup(X-cl(A))$. Then it follows $cl(U) = cl(\cup(X-cl(A))) = cl(A) \cup (X-cl(A)) = X$. Since $(X, \tau)$ is RGW$\alpha$-sub maximal, $U$ is RGW$\alpha$-open, so $A, \alpha \in \text{RGW}^\alpha-LC^\tau(X, \tau)$ from the Theorem 3.41 Hence $P(X) = RGW\alpha-LC^\tau(X, \tau)$.

Sufficiency: Let $A$ be dense sub set of $(X, \tau)$. Then by assumption and Theorem 3.41 that $A \cup (X-cl(A)) = \Lambda$ holds, $A, \alpha \in \text{RGW}^\alpha-LC^\tau(X, \tau)$ and $A$ is RGW$\alpha$-open. Hence $(X, \tau)$ RGW$\alpha$-sub maximal.

Theorem 3.51: If $(X, \tau)$ is RGW$\alpha$-space then $RGW\alpha-LC(X, \tau) = LC(X, \tau)$.

Proof: Straight Forward.

Theorem 3.52: Let $(X, \tau)$ and $(Y, \sigma)$ be topological spaces.

i) If $A \in \text{RGW}^\alpha-LC(X, \tau)$ and $B \in \text{RGW}^\alpha-LC(Y, \sigma)$ then $A \times B \in \text{RGW}^\alpha-LC(X \times Y, \tau \times \sigma)$.

ii) If $A \in \text{RGW}^\alpha-LC^\tau(X, \tau)$ and $B \in \text{RGW}^\alpha-LC^\sigma(Y, \sigma)$ then $A \times B \in \text{RGW}^\alpha-LC^\tau(X \times Y, \tau \times \sigma)$.

iii) If $A \in \text{RGW}^\alpha-LC^{**}(X, \tau)$ and $B \in \text{RGW}^\alpha-LC^{**}(Y, \sigma)$ then $A \times B \in \text{RGW}^\alpha-LC^{**}(X \times Y, \tau \times \sigma)$.

Proof: i) If $A \in \text{RGW}^\alpha-LC(X, \tau)$ and $B \in \text{RGW}^\alpha-LC(Y, \sigma)$. Then there exist RGW$\alpha$-open sets $U$ and $V$ of $(X, \tau)$ and $(Y, \sigma)$ and RGW$\alpha$-closed sets $G$ and $F$ of $X$ and $Y$ respectively such that $A = U \cap G$ and $B = V \cap F$. Then $A \times B = (U \times V) \cap (G \times F)$ holds. Hence $A \times B \in \text{RGW}^\alpha-LC(X \times Y, \tau \times \sigma)$.

ii) and iii) Similarly the follow from the definition.

4. Conclusion

In this paper we have introduced and studied the properties of RGW$\alpha$-locally closed sets and RGW$\alpha$*- locally closed sets and RGW$\alpha$**-locally closed sets. Our future extension is to study RGW$\alpha$-locally continuous, closed, open maps in Topological Spaces.

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6. References

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