Ubat graphs

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Abstract
Koshy (2010) introduced and investigated the concept Complementary Distance Pattern Uniform (CDPU) sets in a connected graph. In this paper, introduces one variety of Complementary Distance Pattern Uniform (CDPU) graphs which is ubat graphs. A graph $G$ is an ubat graph if there exists $M \subseteq V(G)$ such that for any vertex $u \in V(G) \setminus M$, there exists a vertex $v \in M$ with $d(u, v) = e(u)$, where $e(u)$ denotes eccentricity of $u$. The set $M$ is called an ubat set of $G$. An ubat set of $G$ of minimum cardinality is called an ubat set of $G$. The cardinality of an ubat set of $G$ is called the ubat number of $G$, denoted by $\alpha(G)$. A couple of results are generated in this study. Some of which are the following: $\alpha(G + H) \leq \min \{|V(G)|, |V(H)|\}$ where $G$ and $H$ be connected graphs. Let $G$ be a self-centered graph and $H$ be any graph. Then $\alpha_u(G+H) = 2$. Let $K_{1,n}$ and $K_{m,n}$ be a star of order $n + 1$ and a complete bipartite graph of order $n + m$, respectively.

Keywords: ubat graphs, distance, disjoint, join, corona, eccentricity

Introduction
Graph theory deals with the study of networks. It is studied in different directions and is applied in diverse applications. Some would study domination in graphs, others would study dimension in graphs and others would study geodetics covers. Koshy et al. (2010) [6, 7, 8] introduced and investigated the concept Complementary Distance Pattern Uniform (CDPU) sets in a connected graph. Koshy stressed that there are strong indications in the literature of Natarajan (2009) [9] that the notion of Complementary Distance Pattern Uniform (CDPU) sets could be used to design a class of Topological indexes that represent certain stereo chemical properties of the molecule. This paper introduces one varieties of Complementary Distance Pattern Uniform (CDPU) graphs which is called an ubat graphs.

A graph $G$ is an ubat graph if there exists $M \subseteq V(G)$ such that for any vertex $u \in V(G) \setminus M$, there exists a vertex $v \in M$ with $d(u, v) = e(u)$. The set $M$ is called an ubat set of $G$. An ubat set of $G$ of minimum cardinality is called an ubat set of $G$. The cardinality of an ubat set of $G$ is called the ubat number of $G$, denoted by $\alpha_u(G)$. This paper introduces and investigated ubat graphs. In particular, the researcher determined the ubat number of the join of graphs. Furthermore, we characterized all graphs with $ucdpu$ number equal to 1. The concept ubat graphs may be used to find a point in a place or network that is farthest from most of the other points. This point maybe utilized by putting on its undesirable things. For example, a nuclear plant may be situated on such place.

Objectives of the study
This study has the following objectives:
1. To introduced ubat graphs.
2. To obtain bounds or exact values of the ubat number of the join of graphs.
3. To determine relationships, if there are any, between the ubat number of a connected graph and its order, and other graph invariants.

Research methodology
The following steps were followed in conducting this study. First, the concepts ubat set, ubat number and ubat graph were introduced. Second, these concepts were discussed thoroughly by giving examples. Third, conjectures regarding the properties of concepts were presented. Finally, these conjectures were shown whether they are true or not.
Basic concepts

Graph/network
A graph or a network $G$ is an ordered pair $G = (V(G); E(G))$, where $V(G)$ is a nonempty set whose elements are called vertices, and $E(G)$ is a set of 2-element subsets of $V(G)$ called edges. The order of $G$ is the number of vertices, that is, the cardinality of $V(G)$. The size of $G$, denoted by $|E(G)|$, is the number of edges of $G$. Let $e = uv$ be an edge joining the vertices $u$ and $v$ of a graph $G$. Then $u$ and $v$ are said to be adjacent, or $u$ and $v$ are incident with $e$, or $u$ and $v$ are the ends of $e$.

Distance in graphs

Distance
For any two vertices $u$ and $v$ in a graph $G$, the distance $d(u, v)$ from $u$ to $v$ is the number of edges of a shortest $u$ - $v$ path in $G$.

Eccentricity
For a vertex $v$ in a connected graph $G$, the eccentricity $e(v)$ is the distance from $v$ to a vertex farthest from $v$. That is, $e(v) = \max \{d(v, x)\}$.

Disjoint union
Let $X$ and $Y$ be sets. The disjoint union of $X$ and $Y$, denoted by $X \bigcup Y$, is found by combining the elements of $X$ and $Y$, treating all elements to be distinct. Thus, $|X \bigcup Y| = |X| + |Y|$.

Join and corona in graphs

Join
The join of two graphs $G$ and $H$, denoted by $G \bigcup H$, is the graph obtained by taking one copy of $G$ and $n$ copies of $H$, and then joining the $i$th vertex of $G$ to every vertex of the $i$th copy of $H$.

Corona
Let $G$ and $H$ be a graphs. The corona $G \circ H$ of two graphs $G$ and $H$ is the graph obtained by taking one copy of $G$ and $n$ copies of $H$, and then joining the $i$th vertex of $G$ to every vertex of the $i$th copy of $H$.

Ubat graph
A graph $G$ is an ubat graph if there exists $M \subseteq V(G)$ such that for any vertex $u \in V(G) \backslash M$, there exists a vertex $v \in M$ with $d(u, v) = e(u)$. The set $M$ is called an ubat-set of $G$. An ubat-set of $G$ of minimum cardinality is called an ubat-set of $G$. The cardinality of an ubat-set of $G$ is called the ubat number of $G$, denoted by $\alpha_u(G)$.

Example of ubat graphs
Consider graph $G'$ in Figure 2. Then $M = \{a, d\}$ is an ubat set, whence $G$ is an ubat graph. To see this, let $M = \{a, d\}$. Then $d(b; d) = 2 = e(b)$, $d(c; a) = 2 = e(c)$, $d(e, a) = 2 = e(e)$, and $d(f, d) = 2 = e(f)$. Hence, for every vertex $u \in V(G') \backslash M$, there exists a vertex $v \in M$ with $d(u, v) = e(u)$. Thus, $M$ is a ubat-set. Note that $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e\}$ and $\{f\}$ are not ubat-sets. Therefore, $M$ is a minimum ubat-set, i.e. $M$ is an ubat set. Accordingly, $G'$ is an ubat graph.

Preliminary results

The following present the results of this study:

1. Ubat Graphs

Theorem 3.4.1 Let $G$ and $H$ be graphs with isolated vertices $u \in V(G)$ and $v \in V(H)$. Then $\alpha_u(G + H) = 2$.

Proof: Let $G$ and $H$ be graphs with isolated vertices $u \in V(G)$ and $v \in V(H)$. Consider $M = \{u, v\}$ and let $w \in V(G + H) \backslash M$. Consider the following cases:

Case 1. $x \in V(G) \backslash M$
If $x \in V(G) \backslash M$, then $d(x, u) = e(x)$.

Case 2. $x \in V(H) \backslash M$ If $x \in V(H) \backslash M$, then $d(x, v) = e(x)$.

Hence, $M$ is an ubat-set.

Suppose $\alpha_u(G + H) = 1$. Let $M$ be an ubat-set with $|M| = \alpha_u(G + H) = 1$. Without loss of generality, let $M = \{v\}$ with say $v \in V(G)$ and let $w \in V(H)$. Then $d(v, w) = 1 \not= 2 = e(w)$. This is a contradiction since $M$ is an ubat-set.

Therefore, $\alpha_u(G + H) = 2$.

Corollary 3.4.2 Let $K_1, n$, and $K_m, n$ be a star of order $n + 1$ and a complete bipartite graph of order $n + m$, respectively. Then
1. $\alpha(K_1, n) = 2$,
2. $\alpha(K_m, n) = 2$.

Summary and Recommendations

This study encapsulates all the results of this study including particular cases. Moreover, since among results point to very interesting conjectures and very natural generalizations, we also express some recommendations.

Summary

1. Ubat-Graphs

1. Let $G$ and $H$ be graphs with isolated vertices $u \in V(G)$ and $v \in V(H)$. Then $\alpha_u(G + H) = 2$.

Theorem 3.4.1

2. Let $K_1, n$, and $K_m, n$ be a star of order $n + 1$ and a complete bipartite graph of order $n + m$, respectively. Then
1. $\alpha(K_1, n) = 2$;
2. $\alpha(K_m, n) = 2$.

Corollary 3.4.2

Fig 1: A Graph $G'$
Recommendations
It is recommended that the results in this study be used in endeavors that require the concepts ubat-sets. Moreover, since the concepts ubat-sets are a broad topics in graph theory, there are still a lot of open problems to research on. Some of these includes, finding some bounds or the exact value of the ubat-number of the following:
3. Composition of two arbitrary graphs.

References