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Influence of the rod shape on the critical flutter speed articulated at the ends

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Abstract
The task about influence of a rod form on critical flutter speed pivotally fixed on the ends in a gas flow taking into account nonlinear dependences is considered. Problem definition and a method of a solution on a flutter of a viscoelastic rod taking into account physical and aerodynamic not linearities is given. The mathematical model of a task is constructed. By means of the method of Bubnov-Galyorkina based on polynomial approximation of deflections, the task comes down to a solution of a system of the nonlinear integral-differential equations like Voltaire. The numerical method based on use of quadrature formulas is applied to a solution of the received system, at weak singular kernel of Koltunov-Rzhanitsyn.

Keywords: Viscoelasticity, viscoelastic rod, flutter, physical nonlinearity, aerodynamic nonlinearity, critical velocity, Bubnov-Galerkin method, relaxation kernel, numerical method, nonlinear integral-differential equation

Introduction
The hereditary theory of viscoelasticity has provided a wide opportunity for describing the dynamic processes of deformation of various materials. Since rods are used as structural elements in many branches of industry and technology, the study of their dynamic behavior in various shapes and studies of structures for vibrations and dynamic stability, taking into account the physical nonlinearity of the material, are relevant.
Despite existence of numerous works, the devoted physically nonlinear fluctuations and stability of rods, there are not enough researches on a flutter of a viscoelastic rod considering at the same time both physical and aerodynamic nonlinearity so far.
Bubnov-Galerkina's method for a solution of stability problems is for the first time offered in I.G. Bubnov's review of S.P. Tymoshenko's work [14]. Afterwards this method, it was developed by B.G. Galerkin [15] and it is successfully applied including for a solution problems of stability of rods [16].

Formulation of the problem
Consider the problem of flutter of a hereditarily deformable bar taking into account physical nonlinearity (1) [7, 13].

$$\sigma = (1 - R^*)\left(m_1\varepsilon + m_2\varepsilon^3\right), \quad \varepsilon = u_x,\ u = -zw_x \tag{1}$$

where

$$R^* f(t) = \int_0^t R(t-\tau) f(\tau)\,d\tau$$

where m_1 and m_2 are elastic constants that determine in the process of testing the material for tensile or torsion, moreover, $m_2 < 0$ for a material with soft, and $m_2 > 0$ for hard characteristics, R (t) is the kernel of heredity, which has weakly singular features of the Abel type.
The definition of aerodynamic forces acting on a deformable body is rather complicated [17]. In the case of sufficiently high supersonic speeds, the pressure on the body surface *q*, according to the piston theory [9], can be calculated as follows [4]:

$$q = k\left[V\frac{\partial w}{\partial x} + \chi_1 v^2\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial t}\right] \tag{2}$$

where $q = p - p_0, \quad k = \frac{\chi p_0}{c_0}, \quad \chi_1 = \frac{\chi + 1}{4c_0}.$

P_0 and c_0 - pressure and speed in a no perturbed flow of w-movement of a system on norm to its initial surface. χ – weed an indicator a gas track.

Let's solve a problem about a flutter in nonlinear viscoelastic setting, considering physical and aerodynamic nonlinearity. For this purpose we will construct a mathematical model for a research of a hereditary and deformable rod in a gas flow taking into account these not linearities.

Taking the hypothesis of flat sections for the bending moment, we write [7]:

$$M_x = (1 - R^*)\left[m_1 J_2 W_{xx} + m_2 J_4 W_{xx}^3\right] \tag{3}$$

where equal width $b(x)$ and height $h(x)$ for beams

$$J_2 = \frac{b(x)h^3(x)}{12}, \quad J_4 = \frac{b(x)h^5(x)}{80}$$

Substituting (3) into the equilibrium equation [18], we obtain the main, solving equation:

$$(1 - R^*)\frac{\partial}{\partial x^2}\left[d(x)W_{xx} + Q_1 d_2(x)W_{xx}^3\right] + F(x)W_{tt} + PW_x + Q_2 P^2 W_x^2 + \gamma W_t = 0 \tag{4}$$

where

$$W = h_0\overline{W}, \; x = a\overline{x}, \; t = t_1\overline{t}, \; m = m_0 F(x), \; h(x) = h_0\overline{h(x)}, \; b(x) = b_0\overline{b(x)},$$

$$J_2 = J_2^0 d(x), \quad J_4 = J_4^0 d_2(x), \quad d(x) = b(x)h^3(x), \quad d_2(x) = b(x)h^5(x),$$

$$J_2^0 = \frac{b_0 h_0^3}{12}, \quad Q_1 = \frac{m_2 J_4^{(0)}}{m_1 J_2^{(0)} h_0^2}\left(\frac{h_0}{a}\right)^4, \quad Q_2 = \frac{m_1 b_0(\chi + 1)}{48\, k c_0}\left(\frac{h_0}{a}\right)^4,$$

$$P = \frac{kVa^3}{m_1 J_2^{(0)}}, \quad t_1 = \sqrt{\left(m_0 a^4\right)/\left(m_1 J_2^{(0)}\right)}, \quad \gamma = \frac{kza^4}{m_1 J_2^{(0)} t_1}, \; F(x) = b(x)h(x)$$

Nonlinear IDE in partial derivatives (4), together with boundary ones [8]

w=0, w_x=0 at x=0; w=0, w_x=0 at x=1 (5)

and initial

$$W\big|_{t=0} = d_0(x), \quad W_t\big|_{t=0} = d_1(x) \tag{6}$$

Conditions represent a mathematical model of the problem of rod flutter, taking into account physical and aerodynamic nonlinearity. It is required to find the critical velocities v, leading to an increasing vibration amplitude.

Solution method: We construct an approximate solution using the Bubnov-Galerkin method. We represent the solution to IDE (8) as following

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \tag{7}$$

where $\varphi_\kappa(x)$ - known basis functions satisfying given boundary conditions, $u_\kappa(t)$ - unknown functions from time to be determined.

For finding of the unknown $u_\kappa(t)$ functions we substitute (7) in (4), multiplying by $\varphi_i(x)$ and we will integrate on x, we will receive the following nonlinear systems of ordinary IDE.

$$\sum_{k=1}^{N}\Big\{a_{ki}\ddot{u}_k(t)+\gamma b_{ki}\dot{u}_k(t)+\omega_{ki}(1-R^*)u_k(t)+pd_{ki}u_k(t)+Q_2P^2\sum_{j=1}^{N}m_{kji}u_k(t)u_j(t)+$$
$$+Q_1(1-R^*)\sum_{j=1}^{N}\sum_{q=1}^{N}n_{kjqi}u_k(t)u_j(t)u_q(t)\Big\}=0,\quad i=\overline{1,N} \tag{8}$$

where

$$a_{ki}=\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx,\qquad b_{ki}=\int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

$$\omega_{ki}=\int_0^1\Big[d(x)\varphi_k''(x)\Big]''\varphi_i(x)dx,\qquad d_{ki}=\int_0^1\varphi_k'(x)\varphi_i(x)dx,$$

$$m_{kji}=\int_0^1\varphi_k'(x)\varphi_j'(x)\varphi_i(x)dx,\quad n_{kjqi}=\int_0^1\Big[d_2(x)\varphi_k''(x)\varphi_j''(x)\varphi_q''(x)\Big]''\varphi_i(x)dx$$

Integration of a nonlinear system (8) at Rjanitsyn-Koltunov's kernel over a wide range of change of physic mechanical parameters of a rod, $R(t)=A\cdot e^{-\beta t}t^{\alpha-1},\ A>0,\ \beta>0,\ 0<\alpha<1$ was carried out by the numerical method, based on analytical conversions [6].

Numerical examples and analysis of results

The calculations were carried out for a hinged-terminated bar on both sides. In this case, the rod was assumed to be in the shape of a trapezoid. It was studied how changing the width and thickness of a trapezoidal rod affects the critical speed, which reflects the occurrence of the flutter state of the rod.

It is assumed that the rod has the following form (fig. 01):

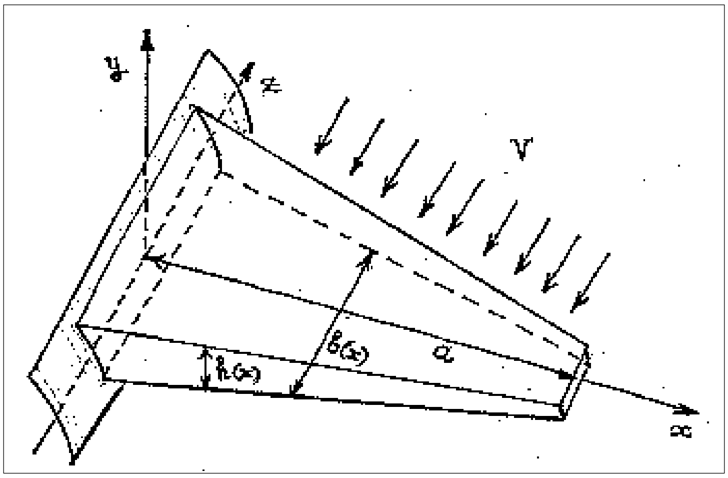


Fig 1.

The calculation was carried out taking into account the physical and aerodynamic nonlinearity with the selected rheological parameters and the shape of the bar in plan. The figure shows the width, meaning $b(x)=5-\alpha_1 x$, and the thickness – $h(x)=1-\alpha_2 x$. As the basis functions $\varphi_k(x)$, we take the beam functions

$$\varphi_k(x)=\sin\lambda_k x,\quad \lambda_k=k\pi$$

and for the initial conditions

$$u_k(0)=\int_0^1\alpha_0(x)\varphi_k(x)dx,\quad \dot{u}_k(0)=0 \text{ где } \alpha_0(x)=\big\{[x(1-x)]^4+\varphi_1(x)\big\}/10\ 0$$

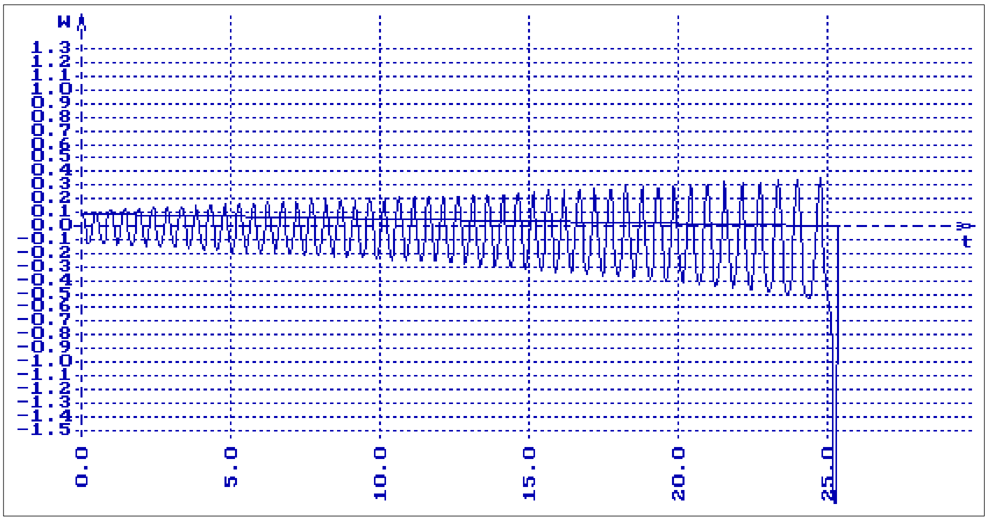
The shape of the bar in plan already largely redefines the value of the other design parameters of the wing.

Therefore, in order to investigate its influence on the critical speed, it is necessary to set a certain relationship between the shape of the bar in the plan and its stiffness, mass, that is, it is necessary to make a calculation for different α_1 and α_2. For this purpose, a series of rods with a trapezoid shape of variable thickness are considered. Then the shape of the bar depends on the parameter 2, this parameter characterizes the narrowing of the bar. At the same time, a series of rods with a trapezoid shape of variable width in plan is considered, then the shape of the rod depends on the parameter 1. This parameter characterizes the narrowing of the bar in width.

The following results were obtained:
with varying width
α_1=1.0 and α_2=0.2, *v*=486,35;
α_1=2.0 and α_2=0.2, *v*=436,67;
α_1=3.0 and α_2=0.2, *v*=388,15;

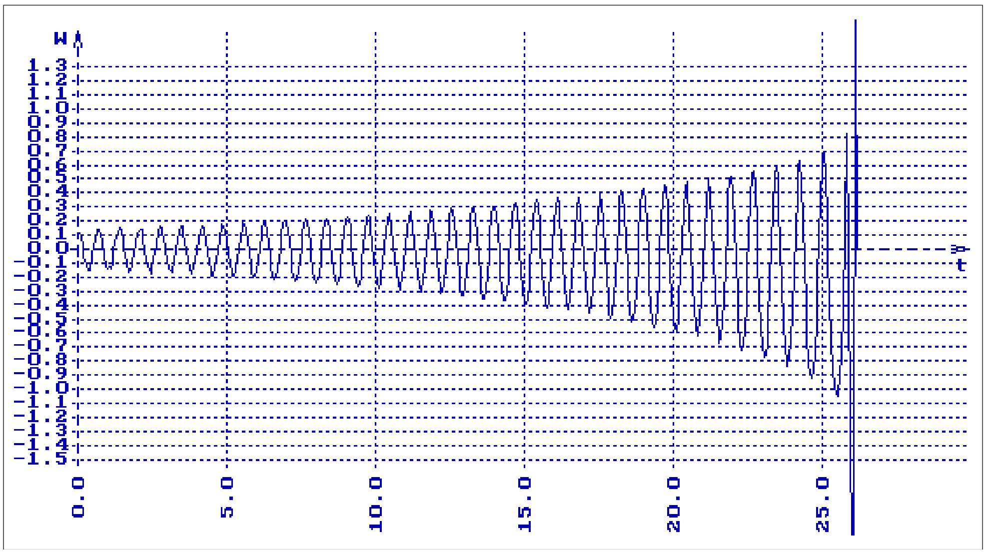
with varying thickness
α_1=4.0 and α_2=0.1, *v*=383,75;
α_1=4.0 and α_2=0.5, *v*=276,24;
α_1=4.0 and α_2=0.8, *v*=144,47.

In the process of obtaining the calculation results, it turned out that an increase in the amplitude of oscillations does not mean that flutter has occurred. The vibration amplitude gradually increases, and a moment comes when a sharp jump in vibration occurs. In this case, it can be assumed that a flutter state has occurred. This result will be the result of critical speed. The following pictures are proof of the above words:



N=2, Speed: *v*=486.34
Physically and aerodynamically nonlinear: *Q*1=1, *h/a*=1/50

Fig 2: A=0.05, α=0.25, β=0.05, α_1=1.0, α_2=0.2



N=2, speed: *v*=144.47
Physically and aerodynamically nonlinear: *Q*1=1, *h/a*=1/50

Fig 3: A=0.05, α=0.25, β=0.05, α_1=4.0, α_2=0.8

Conclusion

Основніе результаті работі сведені к следующему:

1. On the basis of the cubic theory of visco - elasticity, a boundary value problem is formulated for the dynamic calculation of a rod from a physically nonlinear - viscoelastic material, taking into account the shape of the rod.
2. A general computational algorithm for constructing the initial relations of the Bubnov - Galerkin method as applied to the boundary value problems of dynamic calculation of a rod made of a physically nonlinear viscoelastic material has been developed and implemented on a computer.

On the basis of the proposed method, a unified algorithmic system was built for solving problems of the dynamic theory of elasticity and viscoelasticity, which is implemented on a computer using standard programs written in the Turbo-Pascal language, which allow varying the accuracy as when choosing the number of terms of coordinate functions.

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