Fractional Brownian motion and fractal analysis of brain MRI images: A review

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Abstract
Each and every particle in this world is moving in random and it is impossible to find the motion of such particles. In short, motion of such particles is uncertain. This uncertain movement can be seen in particles ranging from simple atoms to the complex biological cells and tissues in human brain and other human organs. This random motion has been analyzed by various researchers and this is being considered very important in the field of science and technology. This paper aims at providing the concept of Brownian motion and fractional Brownian motion. And also, fractal analysis of brain MRI images is also discussed since the brain MRI images are easily affected by this fractional Brownian motion.

Keywords: Brownian motion, fBm (fractional Brownian motion), FD (Fractal dimension), MRI (Magnetic Resonance Imaging)

1. Introduction
Human lives are full of uncertainties, same as many natural phenomena. No one can precisely foresee what will happen in the future. Rather than accepting the fact that the future is always uncertain, many models and algorithms have been continuously formulated for the prediction of matters involving uncertain elements. One of them is the Brownian model. This paper discusses the concept of Brownian motion and fractional Brownian motion in medical imaging. This paper is organized as follows. Chapter 2 deals with Brownian motion and fractional Brownian motion. Chapter 3 deals with fractal analysis of brain MRI images and conclusion is given in chapter 4.

2. Brownian motion and Fractional Brownian Motion
Before we go deep into the detailed applications of Brownian motion, the concept 'fractal' has to be introduced, as it plays a major part in many important applications of Brownian motion. The concept 'fractal' was introduced by IBM researcher Benoit B. Mandelbrot. Expressed in its simplest form, 'fractals' refer to images in the real world which tend to consist of many complex patterns that recur at various sizes. One example of fractal pattern is Von Koch curve and it is shown in the figure 1.

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Mandelbrot proposed the idea of a fractal (short for "fractional dimension") as a way to deal with problems of scale in the real world. According to his definition, a fractal is any curve or surface that is independent of scale. This property, referred to as self-similarity, means that any portion of the curve, if magnified and seen, would appear identical to the whole curve. The fractals are not differentiable at any cause and this makes the fractal curves differ from the normal ones. That is, although fractals are continuous (smooth), they are "linked" everywhere. Fractals can be characterized by the fractal dimension and it is the ratio of change in details of the image to the change in scale of the image.

The notion of "fractional dimension" provides a way to measure how rough fractal curves are. We normally consider lines to have a dimension of 1, surfaces a dimension of 2 and solids a dimension of 3. However, a rough curve (say) wanders around on a surface; in the Fractal dimension can be measured by many ways and it determines the roughness of the fractals. Another important aspect of the fractals is that, the roughness of the fractals is controlled by a parameter called "Hurst Parameter (H)". If Hurst parameter is larger, then the fractals will be smooth and if we keep on decreasing the Hurst parameter, the fractals will be very rough. Usually Hurst parameter takes the value from 0 to 2. An example of fractal surface is a tree's foliage or the internal surfaces of lungs. A rough curve has a dimension between 1 and 2, and a rough surface has a dimension somewhere between 2 and 3. The dimension of a fractal curve is a number that characterizes the way in which the measured length between given points increases as scale decreases. Whilst the topological dimension of a line is always 1 and that of a surface always 2, the fractal dimension may be any real number between 1 and 2.

### 2.1 Fractals and Brownian motion

Fractals are self-similar. Self-similarity means that if we shrink or enlarge a pattern, its appearance should remain unchanged. Fractal pattern remains unchanged on enlarging and hence they are said to be self similar in nature.

One method of generation of fractal pattern is that, fractal patterns usually arise when simple patterns are transformed repetitively on smaller and smaller scales. The process that produces fractal patterns is the Brownian motion, which produces images of fractal objects. The procedure is similar to using a pen to mark dots at random on a sheet of paper. However, instead of being completely random, the movement of the pen from one position to the next is selected, at random, from a set of rules, each having a fixed probability of being chosen and similarly, random movement of fractal objects is also subjected to some rules and regulations.

Brownian motion is an example of a process that has a fractal dimension of 2. For example, if the water molecules jostle in a medium, the path of such water molecules is said to be random. The number of uniformly distributed variables in such motion determines the dimension of such random motion.

One important result of combining the theory of fractals and Brownian motion is the 'fractional Brownian motion model'. This model regards naturally occurring rough surfaces (like mountains, clouds and trees) as the end result of random walks, and utilizes a random iteration algorithm to produce fractal patterns. The applications of this Brownian model is widespread and it is been widely used in all areas of research. Another point of Brownian motion is its effect on the formation of aggregates such as crystals. Such properties are used in animation to generate pictures of artificial plants and landscapes.

### 2.2 Applications of brownian motion in medical imaging

One of the more successful engineering applications of the fractal theory is that, it has been used as image models in medical image processing. These applications include tissue characterization studies and textural image segmentation. With our human vision only regular lines are visible and only by this Brownian model, nameless line can be identified easily. Mandelbrot's fractal theory leads to the concept of Brownian motion in the field of medical image processing.

Medical images, like other natural phenomena, have a degree of randomness associated with their structure and the random noise superimposed on the image. The fractional Brownian motion model regards natural occurring surfaces as the result of random walks. Thus, an intensity of medical image can be treated fractionally by the Brownian motion model. Especially Brownian motion model is used for analyzing the brain images. Brain is the complex part of our human body and it has high degree of randomness because of the presence of grey matter and white matter inside the human brain. Because of these complex patterns brain images can be easily modeled by “Fractional Brownian Motion”.

#### 2.2.1 Classification

Classification refers to the identification of normal and abnormal tissues in the images. Conventional statistical techniques have been used in the past to distinguish among these images. Image can be classified by computing the Fourier transform of the image and determining its power spectrum. And then by applying a linear regression technique on the log of the power spectrum to estimate the fractal dimension. Apart from the conventional methods, a normalized fractional Brownian motion feature vector is defined to represent the statistical features of the image surface from the Brownian motion estimation concept. This concept aims at pairing the intensity of the pixels in an image at different scales. The real surfaces in medical images are not perfect fractal surfaces and their statistical features cannot be represented by a single value for the fractal dimension and hence multiple values are needed for their representation.

#### 2.2.2 Edge Enhancement

This basic approach was suggested for image segmentation and edge detection is that instead of using the Fourier power spectrum analysis, a transformed image can be obtained by calculating the fractal dimension of each pixel over the whole medical image. The fractal dimension distribution appears to hold the fact that, as edge enhancement that does not increase noise as convolution in fast Fourier transform. The transformation can thus enhance the detection of edges over the original medical image. These two techniques are time consuming but could provide a useful alternative to many tissue distinguishing techniques. The main aim of all the researchers was to assess the actual effectiveness of fractal theory in the area of medical image analysis for texture description. Their specific goal was to utilize fractal dimension to discriminate between normal and abnormal human cells. The 'fractional Brownian motion
model was employed to compute the fractal dimension of the different kinds of cells in medical image processing. In the experiments with real images (of cells), the conclusion is that the range of over which the abnormal cells exhibit fractal property compared to that of normal, healthy cells differed quite significantly, and hence that property can be used as a discriminatory feature to identify abnormal cells. This method can be used for quick and accurate identification of other forms of malignant or abnormal cells, and this could prove invaluable to researchers and doctors in the profession.

3. Fractal analysis of brain MRI images

3.1 Fractal analysis of brain tumors

In [1], N. Jayarat and Dr. P. Mohanalah proposed that segmentation plays a vital role in determining tumors in MR images. Brain tumor segmentation depends upon two techniques-feature based and atlas based. In proposed method, a simple assumption is made i.e., tumor is associated with single continuous region. This analysis is done using multi fractional Brownian motion. The segmentation is carried out based on the multi fractional features. This uses modified Ada boost algorithm called Ada boost support vector machine. The complexity of brain tumors in MRI may be more flexible for multi fBm (fractional Brownian motion) analysis and in this work, a stochastic model for fractal dimensional analysis for brain tumors texture extraction. This proposed method helps the classifiers to concentrate on difficult-to-classify patterns during detection and training steps. This model effectively models spatially varying heterogeneous tumor texture.

3.2 Fractal analysis of tumors in brain MRI images

In [2], Khan M. Iftekharuddin, Wei Jia and Ronald Marsh proposed that magnetic resonance images typically have a degree of noise and randomness associated with natural random nature of structure. Three methods are proposed based on fractal analysis models. First method involves thresholding the pixel intensity values (also called PTBC (Piecewise Threshold Box counting) in which the differences in intensity histogram and fractal dimension between normal and tumor images is calculated). In the other two methods, intensity is treated as third dimension and is called PTPSA (Piecewise Triangular Prism Surface Area), which detects and locate tumors accurately. The important property of fractals is that, they are self similar and have non-integer fractal dimension and they are applied in measuring textural images and the roughness of the surfaces. Medical images have high degree of randomness associated with its structure and therefore fractal models are needed to analyze wide variety of medical images especially for brain MRI images since they are highly complex in nature.

3.3 Computing the probability of location brain connectivity using diffusion tensor imaging

In [3], Joshua S Shimony, Adrian A Epstein and G. Larey Brett Horst proposed that diffusion tensor model is used to analyze magnetic resonance diffusion data which is used in neuro-scientific and clinical applications. Here the diffusion is based on Brownian motion. When diffusion occurs in isotropic medium, it is uniform in all directions and can be described easily by a single parameter. In biological tissue, diffusion displacements are hindered unevenly in 3 dimensional spaces by cell membrane and other sub cellular constituents. This uneven diffusion is called diffusion anisotropy. From this work, using diffusion tensor imaging, brain tumor is located and its probability is also computed, which can be used for further analysis of brain tumors.

3.4 Brain connectivity mapping and analysis using diffusion MRI

In [4], Brain G Booth and Ghasan Hamarneh proposed that MRI (Magnetic resonance Imaging) allows the assessment of organization and integrity of fibrous tissue. Diffusion profiles obtained from dMRI (diffusion Magnetic Resonance Imaging) had significant impact on analysis of neural connectivity within white matter of brain. The biological basis for diffusion MRI is that the motion of water molecules is due to thermal agitation and this motion is known as Brownian motion. This diffusion process is random and the cell structures in our brain restrict or hinder the motion of water molecules. This show the existence of Brownian motion in brain structures and thus it is replicated in brain MRI images. Generally, in our human brain, the functional regions, called grey matter are connected by a collection of neural pathways, which are also called as white matter. In addition to this, the brain contains CSF (Cortico Spinal Fluid or Cerebro Spinal Fluid). These differences within the brain are potentially useful in analyzing brain structure and function.

3.5 Artifacts and pitfalls in diffusion MRI

In [5], Denis Le Bihan, Cyril Poupon, Alexis Amadon and Frank Lethimonnier proposed that diffusion MRI is subjected to many artifacts and pitfalls such as eddy currents and their sensitivity to motion. Molecular diffusion resulting from thermal energy carried by these molecules is called Brownian motion. In brain, water molecular displacements significantly differ from true “Brownian motion”, which is defined for free molecules. When the physiological state changes, the features also changes, this makes this method a more powerful one. There is slight attenuation in MRI signal due to phase shift in the molecules inside the brain. Thus MRI is prone to diffusion. The second artifact is due to the use of strong gradient pulses in patient’s motion. When MRI sequence is prone to motion artifacts, diffusion MRI is sensitive to motion. As a result the images will exhibit ghost structure due to the distribution of such phase shifts over time. These two artifacts will leave the image prone to Brownian motion, which degrades the useful features of the image which is needed for the diagnosis of the disease and also with this image it is difficult to track the progress of the disease.

4. Conclusion

Thus this paper gives the detailed explanation about the concepts of Brownian motion, fBm (fractional Brownian motion) and Brownian motion model. Brain MRI images are fractal in nature and thus the fluids within the brain such as white matter and grey matter experiences uncertain movement and there exists a Brownian motion inside the human brain. This motion can be modeled as “fractional Brownian motion model”. From this model the structure of brain can be effectively analyzed from its fractal nature and it can be used for further diagnosis of brain diseases if any.
5. References

Author’s Bibliography
Gokilavani Chinnasamy was born in Tami Nadu, India on 1991. She completed her B.E., degree in Electronics and Communication Engineering in Hindusthan College of Engineering and Technology, Coimbatore. And currently pursuing her M.E., degree Specialized in Electronics and Communication Engineering in SNS College of Technology, Coimbatore. Her area of interest includes Digital Image segmentation, Image enhancement and Image restoration. She has published papers in various international journals. She is a member of IETE and the IRED.