International Journal of Applied Research 2014; 1(1): 261-262

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2014; 1(1): 261-262
www.allresearchjournal.com
Received: 26-11-2013
Accepted: 29-12-2013

Birendra Prasad
Research Scholar, Department of Maths, V.K.S.U., Arrah, Bihar, India

A note on normed linear linear space with the concept of semi convex set

Birendra Prasad

DOI: https://doi.org/10.22271/allresearch.2014.v1.i1d.7397

Abstract
The object of this work is to establish a few results on Normad Linear Space with the concept of semi convex set.

Keywords: Normed linear, semi convex

Introduction
In the present paper we have paid the total attention to introduce the notion of semi convex set. Efforts has also been made to establish some of the results on Normad Linear Space with the notion of semi convex set.

Definition: For definition we refer to Jha [1], Rudin [2] and Simmons however we have given below some of the definition to serve as a ready reference.

Semi convex set: A non-empty subset C of linear space E is said to be semi convex set if for $x,y \in C$, $\alpha,\beta \geq 0$, $\alpha x+\beta y$ is in C for $\alpha+\beta \leq 1$
Clearly every semi convex set is a convex set. The converse may or may not be.

Zero Space: Alinear space may consist solely of the vector O with scalar multiplication defined by
$\alpha.0 = 0$for everyαwe call this linear space at zero space and we always denoted it by $\{0\}$.
Clearly every zero space is semi convex set.

Normed linear space: let E be a linear space over a field k then a norm on E we understand a map
$f: E \rightarrow R^+$ fromE into the set R^+, of non-negative real numbers if and only if the following condition are satisfied.

1. $f(x)=0$ iff $x= 0$ for all x in E
2. $f(\alpha x) = |\alpha|\, f(x)$ for all $\alpha \in K$, $x \in E$
3. $f(x + y) \leq f(x) + f(y)$ for all $x, y \in E$.

If there is no chance of confusion in writing $||x||$ in place of the above conditions takes the form

1. $||x|| = 0$ iff $x = 0$ for all $x \in E$
2. $|| \alpha x || = | \alpha ||| x ||$ for all $\alpha \in K$ and $x \in R$
3. $|| x + y || \leq ||x|| + ||y||$ for all $x, y \in E$

Whenever $|| \ ||$ is a norm on E the pair(E, ||| ||) is called linear space normed.
Also when K=R (the set of all real numbers) (E, ||| ||) is called a real normed linear space when K = C (the set of all complex numbers) the pair (E, ||| ||) is called normad linear space.

Corresponding Author:
Birendra Prasad
Research Scholar, Department of Maths, V.K.S.U., Arrah, Bihar, India

Metric Space: let M be non empty set then the map d: M × M →R from M × M into the set R (the set of all real numbers) is called a metric (or a distance function) if and only if the following condition are satisfied
1. $d(x, y) \geq 0$ for all x, y Є M
2. $d(x, y) = 0$ if and only if $x = y$ for all x, y Є M
3. $d(x, y) = d(y, x)$ for all x, y Є M
4. $d(x, z) \leq d(x, y) + d(y, z)$ for all x, y Є M

Here is d(x, y) called the distance between x and y. Whenever d is a metric on M then the pair (M,d) is called a Metric space.

We now given results: Every normad linear space E is a metric space with respect to the metric d defined by
$D(x, y) = \|x - y\|$ for all x, y Є E
[For the proof we prefer Jha [1]]

Closed Sphere: let a be a point in a metric space (M, d) andρ is a non negative real number then the
closed sphere is defined as
$S_\rho(a) = \{x \in M\ d(x, a) \leq \rho\}$
A closed sphere is also called a closed ball.
The closed sphere $S_\rho(a)$ contains its centre and if r = 0 it contain only its centre.
In this section we establish a few result using the definition given in the section

Theorem – 1: The intersection of any familyof semi convex set is semi convex.
Observation: let A_i be a family of semi convex set we shall show that $\cap \mathbf{i} A_i$is also semiconvex.

Let x, y Є$\cap A_i$

This implies that for every I, x ЄA_i, and y ЄA_i
Now let α, β≥ 0 and α+β≤ 1.
Now as A_i is semiconvex set for every i.
Hence αx + βy is in Ai for Every i
Thusαx + βy is in $\cap A_i$.
Thus$\cap A_i$ is semiconvex set.

Theorem - 2: The union of family {A} of semi convex sets is again semi convex set provided i ≤ j implies that $A_i \subseteq A_i$.
Proof: let x, y Є U A_i then there exists I, j such that x Є A_i and y Є A_j.

Now let $i \leq j \Rightarrow A_i \subseteq A_j \Rightarrow$ x Є A_i then x ЄA_j.
i.e., x, y Є$A_j$then taking α, β≥ 0 and α + β≤ 1, αx + βy is in A_i
(since A, is given to be semi convex set)
Hence, αx +βy is in $\cap A_i$.

Therefore, The family{ A_i} is a semi convex set thus the theorem.

Theorem- 3: A closed ball (i.e. sphere) in a normed linear space E is always a semi convex set.
Verification let us consider the closed ball B with centre a and radiusρ as

$$\begin{aligned} B_\rho(a) &= \{x \in E: d(x, a) \leq \rho \} \\ &= \{x \in E: \|x\| \leq \rho \}. \end{aligned}$$

Let us take any two points y, z Є$B\rho(a)$ and α, β≥ 0 such thatα + β≤ 1
In order to serve the purpose it is sufficient to show that
$\alpha y + \beta z \in B\rho(a)$ for α, β≥ 0 and α + β≤ 1.

$$\begin{aligned} \text{Now}\quad \|\alpha y + \beta z - a\| &\leq \|\alpha y + \beta z - (\alpha + \beta)a\| \\ &= \|\alpha (y - a) + \beta (z - a)\| \\ &\leq \|\alpha (y - a)\| + \|\beta (z - a)\| \\ &= |\alpha|\, \|y - a\| + |\beta|\, \|z - a\| \\ &= \alpha \|y - a\| + \beta \|z - a\| \qquad \text{(since } \alpha, \beta \geq 0) \\ &= \alpha\, d(y, a) + \beta\, d(z, a) \\ &\leq \alpha\rho + \beta\rho = (\alpha + \beta)\rho \leq \rho \qquad \text{(since } \alpha + \beta \leq 0) \end{aligned}$$

i.e. $\|\alpha y + \beta y - a\| \leq \rho$
So. $\alpha y + \beta z \in B_\rho(a)$
Therefore, $B_\rho(a)$ is a semi convex set.

Reference

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3. Simmons GF. Introduction to Topology and Modern Analysis International Student Edition, 1963.