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Birendra Prasad
Research Scholar, Department of Maths, V.K.S.U. ARA, Arrah, Bihar, India

Correspondence
Birendra Prasad
Research Scholar, Department of Maths, V.K.S.U. ARA, Arrah, Bihar, India

A note on the necessary and sufficient condition for transformation in a linear space

Birendra Prasad

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Abstract
The object of this work is to establish the necessary and sufficient conditions for transformation in a linear space.

Keywords: Linear space, transformation, Sharan and Simmon

Introduction
In this paper the efforts has been made to extend some of results with the concept the transformation for L-convex set studies by Sharan.

Definition: For definition we refer to Sharan and Simmon. However we give below some of the definition to serve as ready reference.

L-convex set : Let S be a non empty subset of a linear space E then S is called a L-convex set if for x, y Є s and $\alpha, \beta \geq O$
$\alpha x + \beta y$ is in S for $\alpha + \beta \leq 1$.

Theorem 1: Let E and E' be any two linear space over a field K, such that
T: E → E' be a linear transformation from E into E'.
A and B be any two L – convex sets in E then
$T(A) \subseteq T(B) \Rightarrow A \subseteq B$

Proof: By hypothesis A and B are two L-convex sets in E.
Also, T: E → E' is a linear transformation.
We first prove that T(A) and T(B) are L-convex sets.
Clearly $T(A) \subseteq E'$ and $T(B) \subseteq E'$.
Let Z1, Z2Є T(A), then ∃X1, X2 in A such that,
Z1 = T(X1),
Z2 = T(X2),
Also $\alpha x1 + \beta x2$ Є A for $\alpha, \beta \geq O$ and $\alpha + \beta \leq 1$ (as A is L-convex set)
Also, $\alpha z1 + \beta z2 = \alpha T(x1) + \beta T(x2) = T(\alpha x1 + \beta x2)$ Є T(A)
Thus T(A) is L-convex set.
Similarly T(B) is also L-convex set.
Now let T(A) Є T(B)
Thus $\alpha z1 + \beta z2$ Є T(A) ⇒ $\alpha z1 + \beta z2$ Є T(B)
and T(B) is a L-convex set ⇒Z1, Z2Є T(B)
⇒$T(\alpha x1 + \beta x2)$ Є T(B)
⇒$\alpha x1 + \beta x2$ Є B
Thus $\alpha x1 + \beta x2$ Є A ⇒$\alpha x1 + \beta x2$Є B, hence $A \subseteq B$.

Corollary 2: Let T: E → E' be a linear transformation from linear space E into linear space E' (over the same field K), Now if A and B be any two L-convex sets in E then

$A \subseteq B \Rightarrow T(A) \subseteq T(B)$

Proof: Since $A \subseteq B \Rightarrow T(A) \subseteq T(B)$ ..(1)

For proof we refer to Sharan [1].
Also by Theorem – 1

$T(A) \subseteq T(B) \Rightarrow A \subseteq B$ ..(2)

Thus by (1) and (2)

$A \subseteq B \Leftrightarrow T(A) \subseteq T(B)$

Corollary – 2: Under the set of condition given in corollary -1. It can also be seen easily that $A = B \Leftrightarrow T(A) = T(B)$.

Proof: Since it is also need that, $A \subseteq B \Leftrightarrow T(A) \subseteq T(B)$
That is $B \subseteq A \Leftrightarrow T(B) \subseteq T(A)$

Consequently $A = B \Leftrightarrow T(A) = T(B)$

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