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Mathematical model that incorporate inter product category and technological substitution effects simultaneously

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Abstract
Technology development, which leads to product improvement, encourages adoption and extends the market. The growing market opportunities serve as a major incentive for industries to invest in improving the product. This study offers a new product diffusion model that simultaneously captures the effects of competition between different brands and those of technological substitution between successive generations in a product. Results show that the market potential of one category or generation is significantly affected by others and by the overall structure of a geographic market. The model studied may be used where there are three generations of technological substitution as the pattern of analysis for two generations technology, our models will be a good tool for the analysis of inter category and generations dynamics of any fast growing market.

Keywords: Bass model, Norton and bass model, multi generation diffusion, inter category product

1. Introduction
Firms have many ways of exploiting their technological assets for profitability and growth. While internal exploitation of technological assets, through designing, developing, manufacturing, and selling products and processes continues to be important, interest in their external exploitation through technology transfer has intensified in recent years. This may be attributed mainly to the globalization of business, liberalization of many developing economies, and greater emphasis on the protection of intellectual property after the formation of the World Trade Organization. Indeed, today, the transfer of manufacturing technology has become an important part of the international business strategy of firms. The theory of innovation diffusion is concerned with the demand for an innovation by relevant groups of potential users, Rogers and Shoemaker, define the innovation in the following words: in the context of the developing countries where most of the technological innovation have been transferred from the developed countries, they can be perceived as new and be treated as an innovation for the purpose of diffusion in the as and innovation for the purpose of diffusion recipient society. The extent to a new idea or product spread is a measure of diffusion. The diffusion itself is referred, more conventionally to the process by which an innovation is communicated to possible jobs. These technological substitution may be categorized in three groups depending upon the characteristics of substitutes. Mathematical models for innovation diffusion have been extensively used by Bayus [2], Bucklin and Sengupta [4], Mahajan and Muller [14], Mahajan and Bass [13] and analysts in the field of mathematical modeling. In these models carrying capacity of the system is taken as static variables and product growth have been studied with substitution effects of new technology. Due to rapid growth of market size and explosion of technology, market potential is treated as variable rather than a constant parameter. Based on the broader concept of competitive market, a dynamic market growth model required which is able to incorporate interproduct category and technological substitution effects simultaneously. The intercategory generational model for information technology industry have been studied by Yoffie [19] and Hayes [8]. To explain intercategory we take example of fast growing IT industry.

In the competitive market IT product from different category providing similar functionality can sometimes pose competitive threats that rival those from direct competitors. Even so rather than posing threat some IT product categories actually complement each other in the sense that they can be used in combination to provide greater consumer utility (i.e. computer hardware must be used with software). In addition because of their partial substitution the market growth of one category may increase the potential sales of another category even through they are not used complementarily (i.e. paper may eventually increase the potential market for cellular phones as some user want up graded service). To understand the inter category relationships and product demand a model is desirable. Besides these inter category effects we can also observe the technological substitution effects among the successive generation within one product category.
In this paper we have studied the model for three generations of three category product represented by graph (1) with help of mathematical model.

2. Mathematical Formulation
Bass [1] studied a basic diffusion model for new consumer durables to forecast the sales of the new product with the first purchase which divided the adopters into two groups innovators and imitators. Mahajan and Mullar [14] studied the model which deals with the technological substitution in successive generations within a single category with no competitive effects. They assumed the technological dominance by each successive generation. Two generation technological substitution models have been studied by Fisher and Pry [6] and Blackman [3]. Norton and Bass [16] describe the diffusion of technological innovation among a number of successive generations in which the diffusion and substitution are considered simultaneously. They extended the Single category model of diffusion model and proposed a new product growth model that explain adoption or sales two generations. Here we study the dynamic model for three generations. This model is

$$S_1(t) = m_1 F_1[1 - F_2(t - t_2)]$$

$$S_2(t) = m_2 F_2(t - t_2) + m_1 F_1(t) F_2(t - t_2)[1 - F_3(t - t_3)]$$

$$S_3(t) = m_3 F_3(t - t_3) + m_2 F_2(t - t_2) F_3(t - t_3) + m_1 F_1(t) F_2(t - t_2) F_3(t - t_3) \quad (1)$$

where $S_1(t), S_2(t), S_3(t)$ are cumulative sales of three generations in time period t and m_1, m_2, m_3 are the estimated market potentials assumed constant for the generations, 1, 2, and 3 respectively, t_2, t_3 are the time period in which generations 2, and 3 are introduce, $F_1(t)$, $F_2(t)$, $F_3(t)$ are cumulative fractions of adoptions for each generation.

$$F_2(t - t_2) = 0 \text{ for } t < t_2, F_3(t - t_3) = 0 \text{ For } t < t_3$$

The term m_1F_1 (t) $F_2(t - t_2)$ represents the portion of estimate sales for the first-generation G_1 product after the newer generation is introduced. $F_1(t)$, $F_2(t)$, $F_3(t)$ is calculated as follows:

$$F_1(t) = \frac{1 - e^{-(p_1 + q_1)t}}{1 + \frac{q_1}{p_1} e^{-(p_1 + q_1)t}}$$

$$F_2(t) = \frac{1 - e^{-(p_2 + q_2)t}}{1 + \frac{q_2}{p_2} e^{-(p_2 + q_2)t}}$$

$$F_3(t) = \frac{1 - e^{-(p_3 + q_3)t}}{1 + \frac{q_3}{p_3} e^{-(p_3 + q_3)t}}$$

Where p_1, p_2 and p_3 refers to the generations 1,2 and 3 respectively coefficient of innovation and q_1,q_2 and q_3 is the imitation coefficient for the generations 1,2 and 3 respectively.This model is made in such a way that newer generation can only reduce sales of older generation and older one cannot detract from the potential of a newer one. This defines the nature of intergeneration relationship. We take always this relation otherwise the model will be complicated for analysis.
Intercategory sales impact was studies by Peterson and Mahajan [17], Bayus [2], Bucklin and Sengupta [4], Hill and Brynjolfsson [9], Kim [12]. But intercategory generation model is desirable to discover which product category will have favourable, unfavourable or neutral impact upon furnish new existing products.

3. The Suggested Model
Now we study a more general modal for various types of interproduct category and intergeneration dynamics. Let us suppose that there are three categories A, B, C of a industry that interact with one another in a competitive market. Each category has

its generations, A_1, A_2, A_3 for A, B_1, B_2, B_3 for B and a single generation for newer category C. We can always say that category B is technologically more advance than A. This dynamics model is represented as follows:

$$S_{A_1}(t) = m_{A_1}(t)F_{A_1}(t - t_{A_1})[1 - F_{A_2}(t - t_{A_2})] \quad (2)$$

$$S_{A_2}(t) = m_{A_2}F_{A_2}(t - t_{A_2}) + m_{A_1}F_{A_1}(t - t_{A_1})F_{A_2}(t - t_{A_2})[1 - F_{A_3}(t - t_{A_3})] \quad (3)$$

$$S_{A_3}(t) = m_{A_3}F_{A_3}(t - t_{A_3}) + m_{A_2}F_{A_2}(t - t_{A_2})F_{A_3}(t - t_{A_3}) + m_{A_1}F_{A_1}$$

$$(t - t_{A_1})F_{A_2}(t - t_{A_2})F_{A_3}(t - t_{A_3}) \quad (4)$$

Similarly model for category B is

$$S_{B_1}(t) = m_{B_1}(t)F_{B_1}(t - t_{B_1})[1 - F_{B_2}(t - tB_2)] \quad (5)$$

$$S_{B_2}(t) = m_{B_2}F_{B_2}(t - t_{B_2}) + m_{B_1}F_{B_1}(t - t_{B_1})F_{B_2}(t - t_{B_2})[1 - F_{B_3}(t - t_{B_3})] \quad (6)$$

$$S_{B_3}(t) = m_{B_3}F_{B_3}(t - t_{B_3}) + m_{B_2}F_{B_2}(t - t_{B_2})F_{B_3}(t - t_{B_3}) + m_{B_1}F_{B_1}$$

$$(t - t_{B_1})F_{B_2}(t - t_{B_2})F_{B_3}(t - t_{B_3}) \quad (7)$$

$$Sc(t) = m_c(t)F_c(t - t_c) \quad (8)$$

Where S_{k_n} = number of units in use of generation n of category k at time t

$m_{k_n}(t)$ = Estimated market potential for generation n of category k at time t

$F_{k_n}(t - t_{k_n})$ = Market penetration rate by generation n of category k

t_{k_n} = The time period that generation of category k was launched

$$k = [A, B, C], n = [1,2,3]$$

In equation (2) – (8) estimated market potential m_{kn} (t) is represented as a function of time. This represented the real life phenomenon of the market because potential of the market is very much effected due to heavy inflow technology. In the model Norton and Bass it taken as constant. To include the effects of interactions among the product categories, we specify the market potentials m_{kn}(t) in the equation (2) – (8) as:

$$m_{A_1}(t) = m_{A_{10}}[(S_{B_1}(t))]^{r_{B_1-A_1}}\left(S_{B_2}(t)\right)^{r_{B_2-A_2}}\left(S_{B_3}(t)\right)^{r_{B_3-A_1}} \times \left(S_c(t)\right)^{r_{c-A_1}}] \quad (9)$$

$$m_{A_2}(t) = m_{A_{20}}[\left(S_{B_1}(t)\right)^{r_{B_1-A_2}}\left(S_{B_2}(t)\right)^{r_{B_2-A_2}}\left(S_{B_3}(t)\right)^{r_{B_3-A_2}}\left(S_c(t)\right)^{r_{c-A_2}} \quad (10)$$

$$m_{A_3}(t) = m_{A_{30}}[\left(S_{B_1}(t)\right)^{r_{B_1-A_3}}\left(S_{B_2}(t)\right)^{r_{B_2-A_3}}\left(S_{B_3}(t)\right)^{r_{B_3}-A_3} \times \left(S_c(t)\right)^{r_c-A_3} \quad (11)$$

$$m_{B_1}(t) = m_{B_{10}}[(S_{A_1}(t))]^{r_{A_1-B_1}}\left(S_{A_2}(t)\right)^{r_{A_2-B_2}}\left(S_{B_3}(t)\right)^{r_{A_3-B_1}}.\left(S_{A_c}(t)\right)^{r_{A_c-B_1}}] \quad (12)$$

$$m_{B_2}(t) = m_{B_{20}}[\left(S_{A_1}(t)\right)^{r_{A_1-B_2}}\left(S_{A_2}(t)\right)^{r_{A_2-B_2}}\left(S_{A_3}(t)\right)^{r_{A_3-B_2}}\left(S_c(t)\right)^{r_{c-B_2}} \quad (13)$$

$$m_{B_3}(t) = m_{B_{30}}[\left(S_{A_1}(t)\right)^{r_{A_1-B_3}}\left(S_{A_2}(t)\right)^{r_{A_2-B_3}}\left(S_{A_3}(t)\right)^{r_{A_3}-B_3}\left(S_c(t)\right)^{r_c-B_3} \quad (14)$$

$$m_c(t) = m_{c_0}(t)[\left(S_{A_1}(t_1)\right)^{r_{A_1-C}}\left(S_{A_2}(t)\right)^{r_{A_2-C}}\left(S_{A_3}(t)\right)^{r_{A_3-C}}$$

$$\left[\left(S_{B_1}(t_1)\right)\right]^{r_{B_1-C}}\left(S_{B_2}(t)\right)^{r_{B_2-C}}$$

$$\left(S_{B_3}(t)\right)^{r_{B_3-C}}\ ] \quad (15)$$

where $m_{k_{no}}$ = a base factor for the market potential m_{k_n} (t). Jain and Rao [10] has similar specification of the base factor impact of generation n' of category k' on the market potential of generation of category k.

In equation (9)–(15) the estimated market potential of one generation of a given product category is affected by the sales of the other categories. The influence of other generation in the same product category can be studied by incorporating equations (2) – (8) in the system of equation (9) – (15). The market potential of specific product generation have been studied by Kalish [11] and Mahajan and Peterson [15].

4. Application of the Model

A new product diffusion model incorporating brand competition could also have been suggested as a benchmark; however, without giving further consideration to customer dis-adoption behaviours, such a model would not have effectively explained why the cumulative number of users started to decline after certain time points. Information technology industry is one of the fast growing sectors. If only wireless telecommunication service industry is taken, it shows a dynamic intercategory character. The categories may be named as cellular phone (Analog), cellular phone digital, Namwoom Kim [12] studied the growing information technology industry using dynamic model. They used this model for wireless telecommunication services market in Hong-Kong and Taiwan.Hayes [8],Fike [5], Guy [7] also studied the information technology industry for generation changes from analog to digital for continuously changing market potential. Namwoom Kim took the Hong –Kong wireless telecommunication market because it is well establish in terms of market potential 32.8%of the total population use some kind of wireless telephone service and the region has high growth rate for various generation of communication services, ASIA week 1995.In Hong-Kong there are three product categories currently in use the pager,cellular telephone and cordless telephone $2CT_2$.The graph (2) represent the number of subscribers since 1984 to 1995.The first category (pager service) began sales in the early 1970,and after digital technology was introduced in 1980,market growth accelerated,form mid 1980 the pager market has been dominated by the digital type and no analog service has been available. In graph (2) pager sales represent only digital generation because analog data was not obtained.The pager digital market continues to grow and it is believed that it has not yet reached the saturation level.

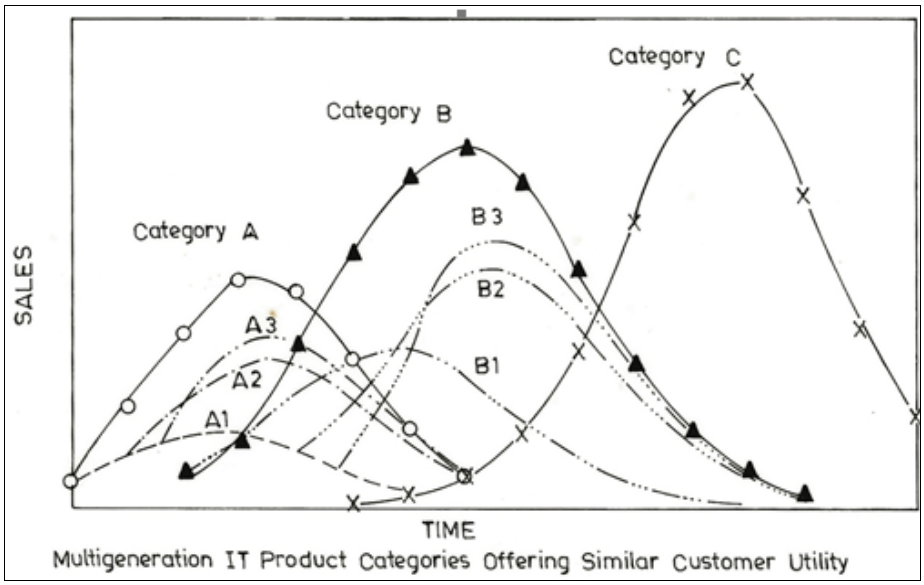
In the second category cellular phone there two generations of technology analog and digital.Analog was introduce in 1987 and digital in 1993.Williamson [18] studied that digital technology provides more than three; fold increase in available voice channels.As a result a cellular digital phone provides higher capacity in a specified radio spectrum, increased voice quality and enhanced call security. In graph (2) we observe that digital generation is adopted by larger number of subscribers in 1995.There is a technological substation between generations.The third category Cordless telephone originally developed in the United Kingdom,was introduced into Hong-Kong in 1992. It has targeted demand for low- cost communications that do not require the full functionality of cellular phones. Its monthly subscription fee and call charge per minute for cordless telephone 2 are much lower than those of cellular.Indeed cordless telephone has been called the poor man's cellular phone; for Eastern Economic review 1994. This model was used for wireless telecommunication for South-Korea.

The model studied may be used where there are three generations of technological substitution. As the pattern of analysis for two generations technology, our model will be a good tool for the analysis of intercategory and generations dynamics of any fast growing industry.

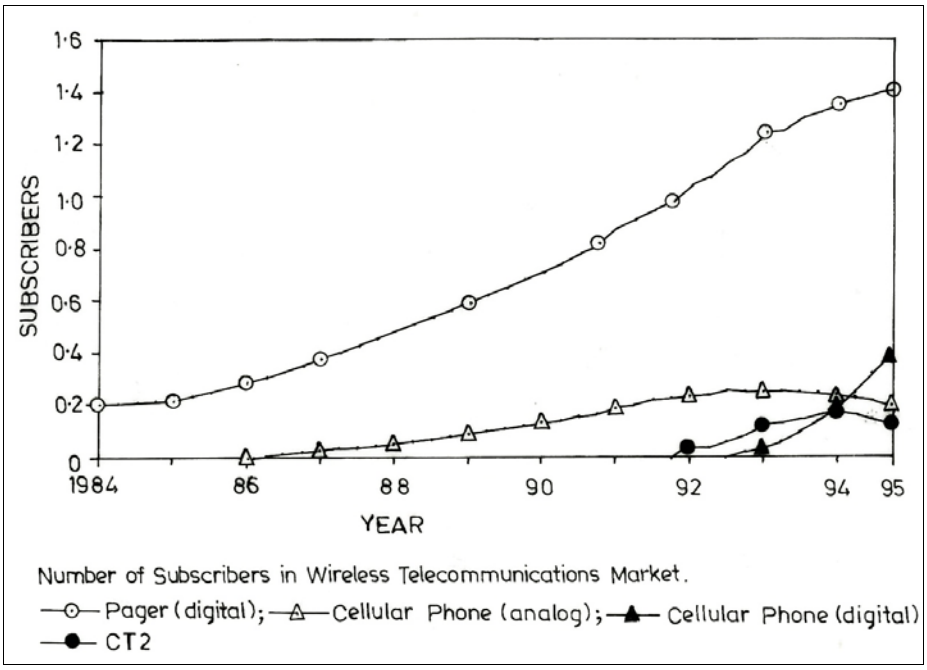
5. Conclusion

This study offers dynamic market growth model where intercategory effects and technological substitution simultaneously capture the market potential. We may be able to affirm that the sales of one related product category has an impact on other market potential and consequently affects the market growth. This model also helps in managerial decision making regarding

which category competitors or compliments to monitor, one of the most demanding tasks for diffusion models is to estimate appropriate market potentials that can provide reasonable long-run sales forecasts. This model has extensive application possibilities to many different information technology and high technology industries that involves technological interactions among related categories and generations. Examples of such industries include the computing industries where micro, mini, workstation and mainframe computers interact with each other for better performance. In display monitor industry and audio industry which has developed different kind music-playing technologies such as records, cassettes and CD. In home entertainment industry where the video-on-demand satellite television, cable television complete each other.



Graph 1



Graph 2

References

1. Bass FM. A new product growth model for consumer durables. Management Science. 1969;15(5):215-227.
2. Bayous BL. Forecasting sales of new products, J Prod. Innovation Management. 1987, 4.
3. Blackman AW. The market dynamics of technological substitution, Tech. Forecasting and Social Change. 1974;6:41-63.
4. Bucklin LP, Sengupta S. The co-diffusion of complementary innovations J Prod. Innovation Management. 1993;10:148-160.
5. Fike RL. Analog or digital. Telephony, 1994 Oct 17, 35-38.
6. Fisher CJ, Pray RH. A simple substitution model of technological change, Tech. Forecasting and Social Change. 1971;3:75-88.
7. Guy S. Wireless data show signs of untethering its potential" Telephony, 1996 Apr 22, 54-55.
8. Hayes JR. The bundler. Forbes, 1996 Apr 22, 82-85.

9. Hill L, Brynjolfsson E. The three faces of IT, Value Proc. 15th Internat. Conf. Inform. Systems, 1984, 263-277.
10. Jain DC, Rao RC. Effect of prince on the demand for durables, J Bus. Econom. Statist. 1990;8:163-170.
11. Kalish S. A new product adoption model with pricing, advertising and uncertainty". Management Sci. 1985;31:1569-1585.
12. Kim N. Management Science INFORMS. 2000 Apr;46(4):496-512.
13. Mahajan V, Bass FM. New product diffusion models in marketing, J Marketing. 1990;54:1-26.
14. Mahajan V, Muller E. Timing diffusion and substitution of successive generations of technological innovations, Tech. Forecasting and Social Change. 1966;51:109-132.
15. Mahajan V, Peterson RA. Innovation diffusion in a dynamic potential adopter population, Management Sci. 1978;24:1589-1597.
16. Norton JA, Bass FM. A diffusion theory model of adoption and substitution for successive generations of high technology products Management Sci. 1987;33:1069-1086.
17. Peterson RA, Mahajan V. Multi-product growth models, Jai Press, Greenwich, CT, 1978, 201-231.
18. Williamson J. The wireless industry erupts worldwide Telephony. 1992 Aug;3:26-32.
19. Yoffie DB. Strategic Management in Information Technology, Prentice-Hallo, Englewood Cliffs, N.J, 1994.