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Unsteady MHD flow of stratified fluid through porous medium past an infinite flat plate with variable suction in slip flow

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Abstract
In this analysis is carried out for velocity field and skin-friction under slip flow conditions at the plate. The effects of magnetic field parameter, permeability parameter, suction velocity amplitude and stratification factor have been presented graphically on velocity distribution and skin-friction. It is being observed that external velocity is attend early for increasing Hartmann number and decreasing stratification factor and permeability parameter. For skin-friction it is concluded that it decreases with increasing stratification factor and increasing with Hartmann Number and suction velocity amplitude.

Keywords: MHD flows, stratified fluid, slip conditions, variable suction

Introduction
The physical properties like density and viscosity of a fluid varies with temperature, if such variations are taken into consideration, the results are very useful in Scientific and engineering problems. As the flow behaviour of fluids in a petroleum reservoir rock depends to a large extent on the viscous stratification and also on porous properties of the rock the study of such problems provides better understanding of reservoir performance. Moreover, due to geophysical applications in oceans and atmosphere the study of such flows has greatly increased in recent years,

Beavers and Joseph [1], Beavers *et al.* [2] and Rajasekhara *et al.* [3] studied the flow past a porous medium without stratification. Channabasappa and Ranganna [4] studied the flow of a viscous stratified fluid on variable viscosity fluid past a porous bed with the idea that the stratification may provide a technique for studying the pore size in a porous medium. Flow problems between and across the permeable boundaries generate a coupled flow. In such coupled flow problems, the flow field, is divided into two regions. The study in two regions are to be made separately together with some suitable matching conditions at the interface of the two regions, Gupta and Sharma [5] solved such a problem with BJ condition at the interface. Unsteady stratified Couette flow using Laplace transform technique has been considered by Singh [6]. He concluded that stratification parameter is to increase the velocity field and to decrease the skin friction.

Fluctuating flows are important in paper industry involving porous flow equations. Light hill [7] and Stuart [8] initiated the study of fluctuating flows by considering the effects of fluctuations in the magnitude of the velocity of free stream flowing over fixed boundaries. Messiah [9] assumed the case when suction velocity also oscillates about a mean in the same phase as free stream velocity. The corresponding MHD case has been considered by Sondalgekar [10]. In another paper [11] he studied the free convection flow past a plane surface. Flow over a naturally permeable bed has been investigated by Om Prakash and Rajbanshi [12], Gupta and Gupta [13] and Kumar *et al.* [14]. It is being observed that not much work has been done to discuss stratified flows with slip boundary conditions at the plate. As it is known such boundary conditions occur when we deal with the flows at high altitudes. In this analysis investigations have been made on velocity as well as on temperature distributions for a stratified boundary layer flow.

In this research paper a theoretical analysis for velocity and skin-friction, of the flow of a viscous, incompressible, electrically conducting fluid in porous medium past an infinite flat plate with variable suction in slip flow regime under transverse magnetic field of uniform strength, has been carried out. The effects of magnetic field parameter (M) permeability of porous medium (K), stratification factor (α) and suction velocity amplitude (A) have been represented graphically on the velocity distribution. It is being observed that skin-friction increases with the increase of M and A, and decrease with the increase of K and α.

Formulation of the problem and basic equations

We consider a two dimensional electrically conducting, stratified, viscous fluid through an infinite porous medium of absolute permeability bounded by a parallel flat plate in presence of a transverse magnetic field. The plate is porous in nature and the suction velocity normal to the plate is directed towards it and varies periodically with time about a non-zero Constant meanv'_o. The external velocity is taken $U'_o(1+\varepsilon e^{i\omega' t'})$.

The x′-axis is taken along the plate, y'-axis normal to it, Dashes denote dimensional quantities. For this geometry the equations of motion and continuity are:

$$\rho\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p'}{\partial x'}+\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right)-\frac{\mu}{K'}u'-\sigma B^2u' \tag{1}$$

$$\rho\frac{\partial v'}{\partial t'}=-\frac{\partial p'}{\partial x'} \tag{2}$$

$$\frac{\partial v'}{\partial y'}=0 \tag{3}$$

In the writing the above equations, following assumptions are made

a. Electrical conductivity of the fluid is sufficiently large so that the displacement current is neglected.
b. No external electric field is applied.
c. The secondary effects of magnetic induction are neglected.

From the third equation it is clear that v′ is a function of time only. Hence, we consider v′ of the form

$$v'=v'_o(1+A\varepsilon e^{i\omega' t'}) \tag{4}$$

Where A is a real positive constant and ε is small such that $\varepsilon A \le 1$.
If $U'(t')$ is the stream velocity parallel to the wall just outside the boundary layer then

$$-\frac{\partial p'}{\partial x'}=\rho\frac{\partial U'}{\partial t'}+\frac{\mu}{K'}U'+\sigma B^2U' \tag{5}$$

From Equation (1), (4) and (5) we get

$$\rho\left[\frac{\partial u'}{\partial t'}-v'_o\left(1+A\varepsilon e^{i\omega' t'}\right)\frac{\partial u'}{\partial y'}\right]$$
$$=\rho\frac{\partial U'}{\partial t'}-\frac{\mu}{K'}(u'-U')-\sigma B^2(u'-U')+\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) \tag{6}$$

Let us take

$$P=\rho_o e^{-a'y'},\ \mu=\mu_o e^{-a'y'} \text{ and } B=B_o e^{-a'y'/2} \tag{7}$$

Where ρ_o, μ_o and B_o are density, viscosity of the fluid and magnetic induction respectively at $y'=0$ and $a'>0$ is stratification factor.

In the light of equation (7) equation (6) reduce

$$\rho_o\left[\frac{\partial u'}{\partial t'}-v'_o\left(1+A\varepsilon e^{i\omega' t'}\right)\frac{\partial u'}{\partial y'}\right]$$

$$=\rho_o\frac{\partial U'}{\partial t'}+\mu_o\frac{\partial^2u'}{\partial y'^2}-\mu_o a'\frac{\partial u'}{\partial y'}-\frac{\mu_o}{K'}(u'-U')-\sigma B^2(u'-U') \tag{8}$$

We take boundary conditions as:

(i) Slip flow B.C. that permits a slip velocity us the plate $y'=0$, i.e.

$$u' = u'_s = L_1\left(\frac{\partial u'}{\partial y'}\right) \text{ at } y' = 0 \quad (9)$$

Where L_1 is slip coefficient given by

$L_1 = \left(\frac{2-m}{m}\right) L$ and $L = u'(\frac{\pi}{2p})^{\frac{1}{2}}$ is mean free path and m is Maxwell's reflection coefficient.

(ii) Free stream boundary condition as

$$U' \to U'(t') = U'_0\left(1 + \varepsilon e^{i\omega' t'}\right) \text{ as } y \to \infty. \quad (10)$$

We introduce the non-dimensional quantities defined by

$$Y = \frac{y'(v_o)}{\nu},\ t = \frac{v'_o{}^2 t'}{\nu},\ \omega = \frac{\nu\omega'}{v'_o{}^2},\ \alpha = \frac{\nu\alpha'}{|v'_o|}$$

$$U = \frac{u'}{U'_o}, U = \frac{U'}{U'_o}, K = \frac{v'_o{}^2 K'}{\nu^2}, M^2 = \frac{\sigma B_o{}^2 \nu}{\rho_o v_o{}^2} \quad (11)$$

Where U'_o is a reference velocity and ω' is the frequency.

Equation (8) is non-dimensional form reduces to

$$\frac{\partial^2 u}{\partial y^2} + (1 - \alpha + A\varepsilon e^{i\omega t})\frac{\partial u}{\partial y} - \left(\frac{1}{K} + M^2\right)(u - U) - \frac{\partial u}{\partial t} = -\frac{\partial U}{\partial t} \quad (12)$$

Subject to the conditions

$$u = h_1\left(\frac{\partial u}{\partial y}\right) \text{ at } y = 0$$

And

$$U \to U(t) \text{ as } y \to \infty \quad (13)$$

Where $h_1 = L_1\frac{|v'_o|}{\nu}$ (Rarefaction parameter) and free stream velocity is given by

$$U = 1 + \varepsilon e^{i\omega t} \quad (14)$$

Solution of the problem
Assuming a periodic solution of the form [messiah (9)]

$$u = 1 + \varepsilon e^{i\omega t} - f_1(y) - \varepsilon e^{i\omega t} f_2(y) \quad (15)$$

Substituting (15) and (14) in (12) and comparing harmonic terms, neglecting coefficients of ε^2 and higher order, we get

$$\frac{d^2 f_1}{dy^2} + (1-\alpha)\frac{df_1}{dy} - (\frac{1}{K} + M^2)f_1 = 0 \quad (16)$$

$$\frac{d^2 f_2}{dy^2} + (1-\alpha)\frac{df_2}{dy} - (\frac{1}{K} + M^2 + i\omega)f_2 = -A\frac{df_1}{dy} \quad (17)$$

Subject to the conditions

$$1 + h_1 f'_1 = f_1 \text{ and } 1 + h_1 f'_2 = f_2 \text{ at } y = 0 \quad (18)$$

$$F_1 = f_2 \to 0 \text{ as } y \to \infty$$

Solution of (16) and (17) subject to (18) are

$$f_1 = \frac{1}{(1 + h_1 m_1)} e^{-m_1 y} \quad (19)$$

And

$$f_2 = \frac{1}{1+h_1 h}\left[1 - \frac{m_1 A}{(1-i\omega)}\right]e^{-hy} + \frac{m_1 A}{(1+h_1 m_1)(n-i\omega)}e^{-m_1 y} \tag{20}$$

Hence the velocity field is given by

$$u = 1- \frac{e^{-m_1 y}}{(1+h_1 m_1)} + \varepsilon(M_r \cos\omega t - M_i \sin\omega t) \tag{21}$$

Where

$$M_r = 1 - \frac{e^{-h_r y}}{c^2+d^2}\left[(c\cosh_i y - d\sinh_i y) - \frac{1}{n^2+\omega^2}\{m_1 AN(c\cosh_i y - d\sinh_i y) + m_1\omega A(d\cosh_i y - c\sinh_i y)\}\right] - \frac{m_1 AN}{(1+h_1 m_1)(N^2+\omega^2)}e^{-m_1 y}$$

$$M_i = \frac{e^{-h_r y}}{c^2+d^2}\left[(d\cosh_i y - c\sinh_i y) - \frac{1}{(N^2+\omega^2)}\{m_1 AN(d\cosh_i y + c\sinh_i y) + m_1 A\omega(d\sinh_i y - c\cosh_i y)\}\right] - \frac{m_1 A\omega}{(1+h_1 m_1)(N^2+\omega^2)}e^{-m_1 y}$$

$$P = (1-\alpha)^2 + 4\left(\frac{1}{K} + M^2\right)$$

$$H_r = \frac{1}{2}(1-\alpha) + \frac{1}{2}\left[\frac{1}{2}(P^2+16\omega^2)^{\frac{1}{2}} + P\right]^{1/2}$$

$$H_i = \frac{1}{2}\left[\frac{1}{2}(P^2+16\omega^2)^{\frac{1}{2}} - P\right]^{\frac{1}{2}}$$

$C = 1+h_1 h_r,\ d = h_1 h_i$

$$M_1 = \frac{1}{2}(1-\alpha) + \frac{1}{2}\left[(1-\alpha)^2 + 4\left(\frac{1}{K} + M^2\right)\right]^{1/2}$$

$$N = m_1^2 - (1-\alpha)m_1 - \left(\frac{1}{K} + M^2\right)$$

Skin-Friction

The non-dimensional skin-friction $\tau_\circ$ is given by

$$\tau_\circ = \frac{\tau'_\circ}{\rho_\circ \acute{U}_\circ |v'_\circ|} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{22}$$

With help of the (21), we get

$$\tau_\circ = \frac{m_1}{1+h_1 m_1} + \varepsilon|B|\cos(\omega t + \beta) \tag{23}$$

Where

$B = B_r + iB_i$

$$\beta = \tan^{-1}\frac{B_i}{B_r}$$

$$B_r = \frac{ch_r + h_i d}{(c^2+d^2)} - \frac{m_1 A}{(N^2+\omega^2)(c^2+d^2)}N(ch_r + h_i d) - \omega(ch_i - h_r d) + \frac{m_1^2 AN}{(N^2+\omega^2)(1+h_1 m_1)}$$

$$\frac{ch_i - h_r d}{(c^2+d^2)} - \frac{m_1 A}{(N^2+\omega^2)(c^2+d^2)}\{(ch_i - h_r d) + \omega(ch_r + h_i d)\} + \frac{{m_1}^2 A\omega}{(N^2+\omega^2)(1+h_1 m_1)}$$

Numerical Discussion

Numerical calculation have been carried out for velocity distribution and skin-friction for different values of parameters M (Hartman Number), K (Permeability parameter), α (stratification Factor) and A (Suction velocity amplitude).

from figure (1) and (2) in which velocity distribution is plotted for A = 0.0 $\omega t = \pi/3$ and A = 1.0, $\omega t = \pi/2$ it is being observed that external velocity is attend early for decrising α and K and for increasing M. it is clear from figure-3 that the velocity in the flow increases with the increasing suction velocity functions.

Figure-4 shows that skin-friction at the plate decreases with increasing stratification factor. Moreover τ_o increases with increasing M and A, and decreasing K.

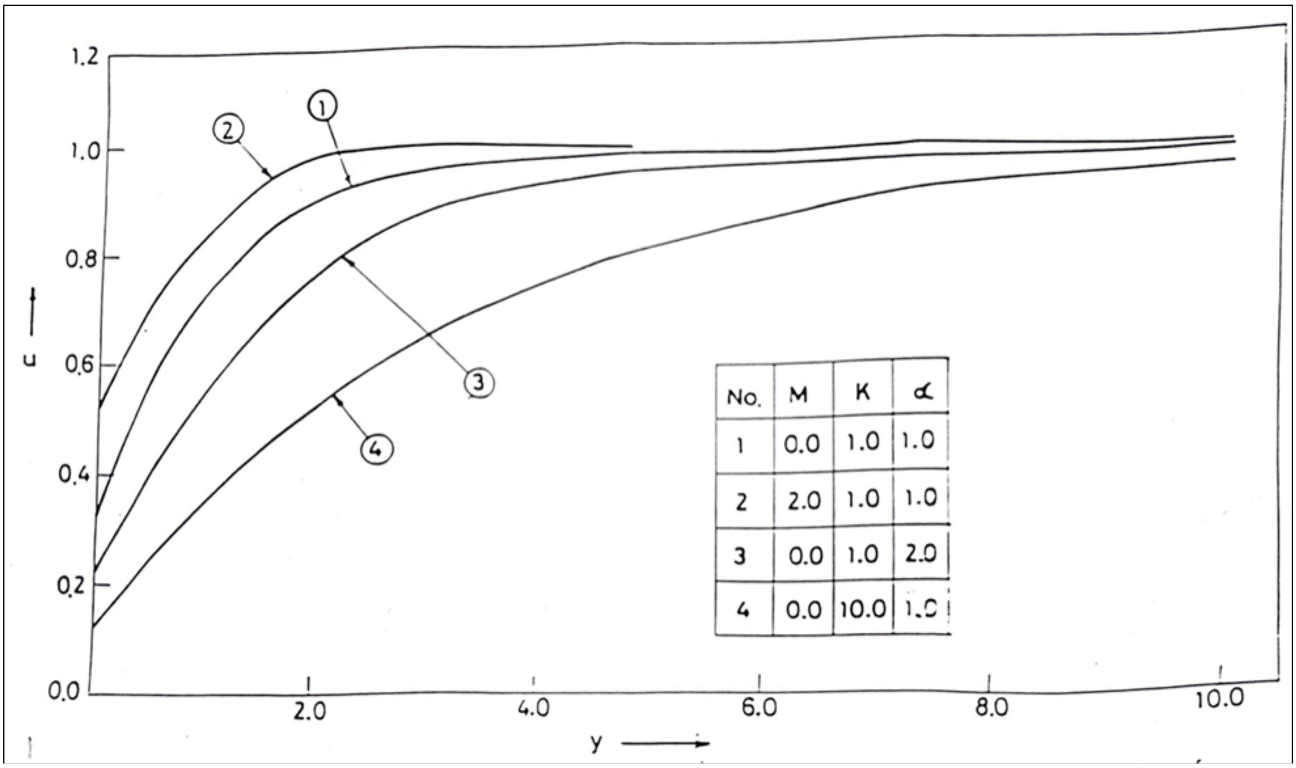

Fig 1: Velocity u plotted against perpendicular distance y for different values of M K and ac (ε = 1.0, A = 0.0, ω = 1.0, w t = π / 3, h_1 = 0.5)

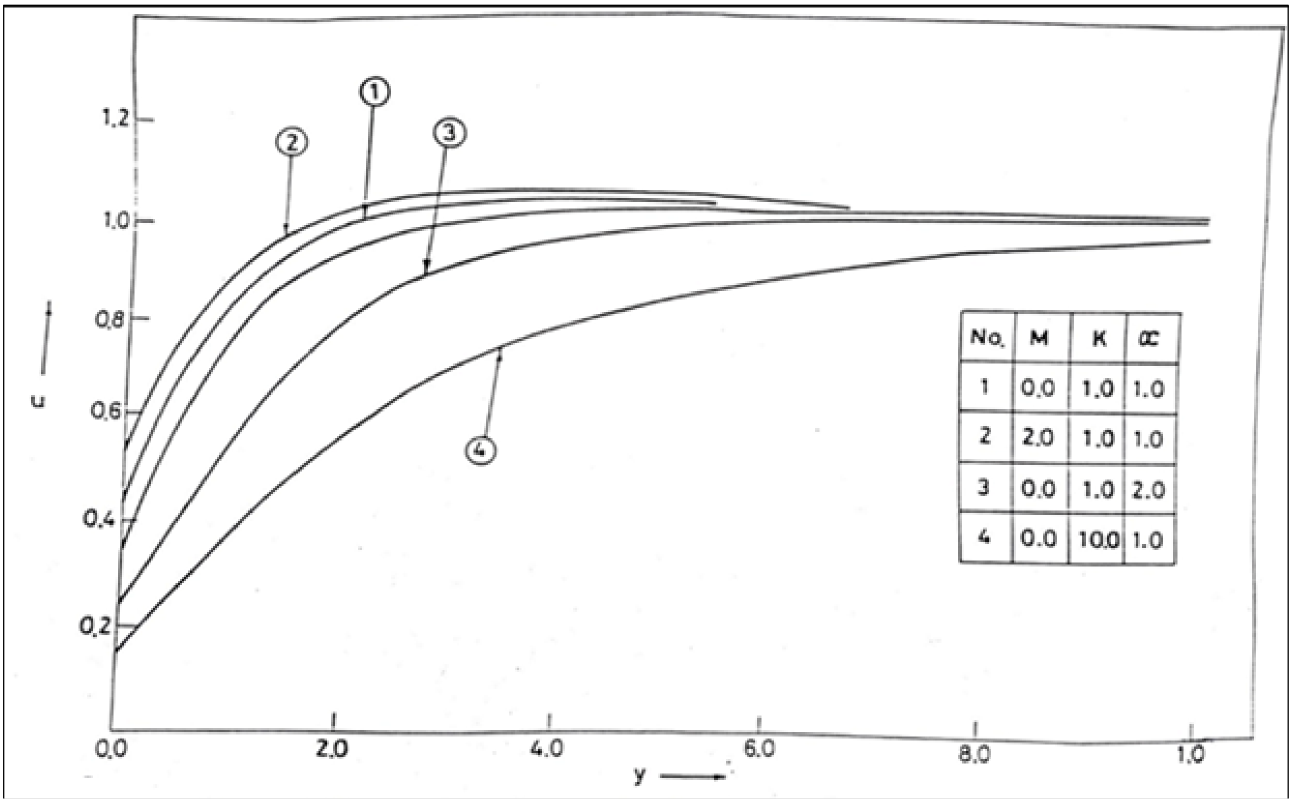

Fig 2: Velocity u plotted against the perpendicular distance y for different values of M, K and ac (ε = 0.1, A = 0.0, ω = 1.0, w t = π /2, h_1 = 0.5)

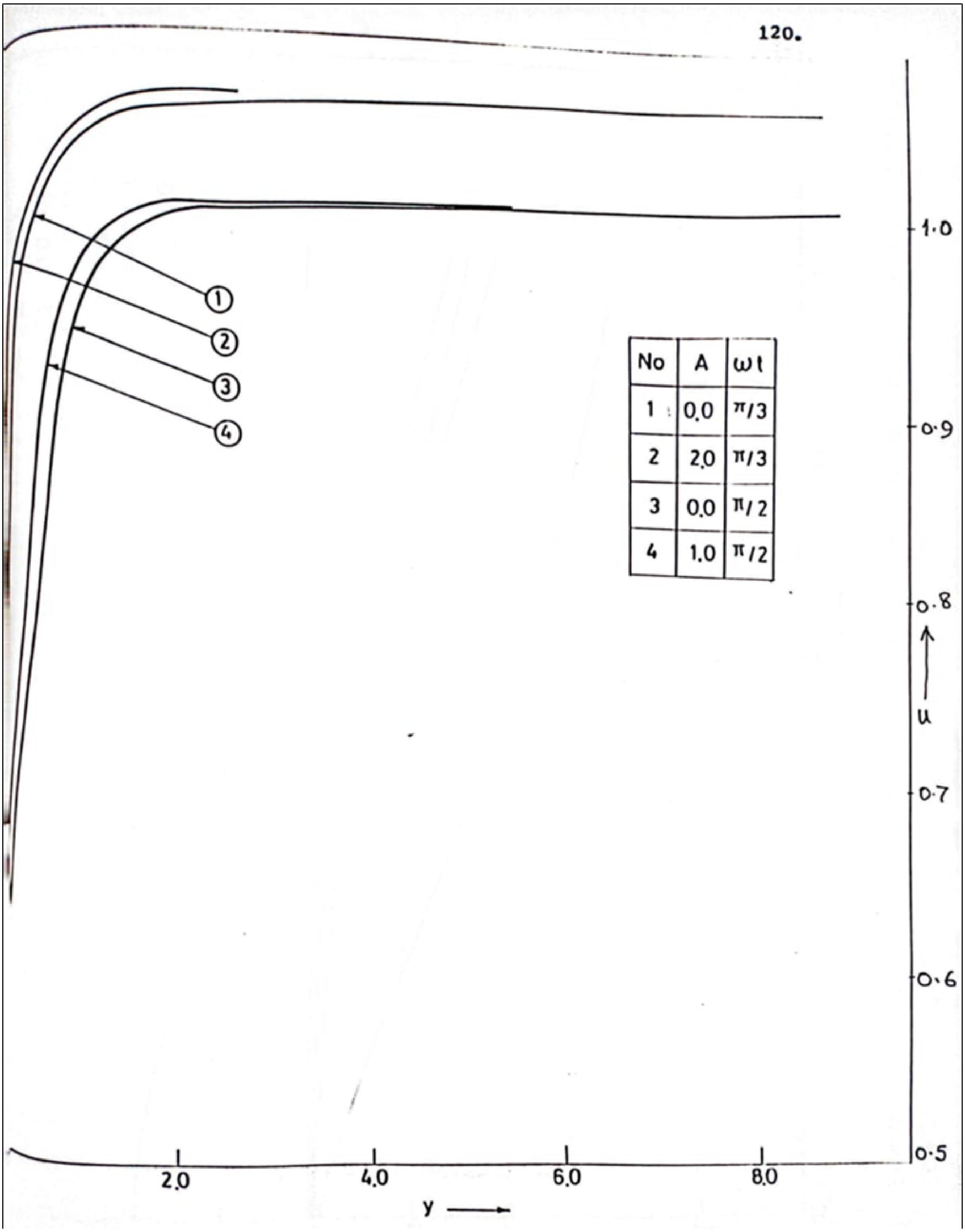

Fig 3: Velocity u plotted against the perpendicular distance y for different values and ω t ($\varepsilon = 0.1$, $\omega = 1.0$, $\alpha c = 1.0$, $K = 1.0$ $M = 2.0$, $h_1 = 0.5$)

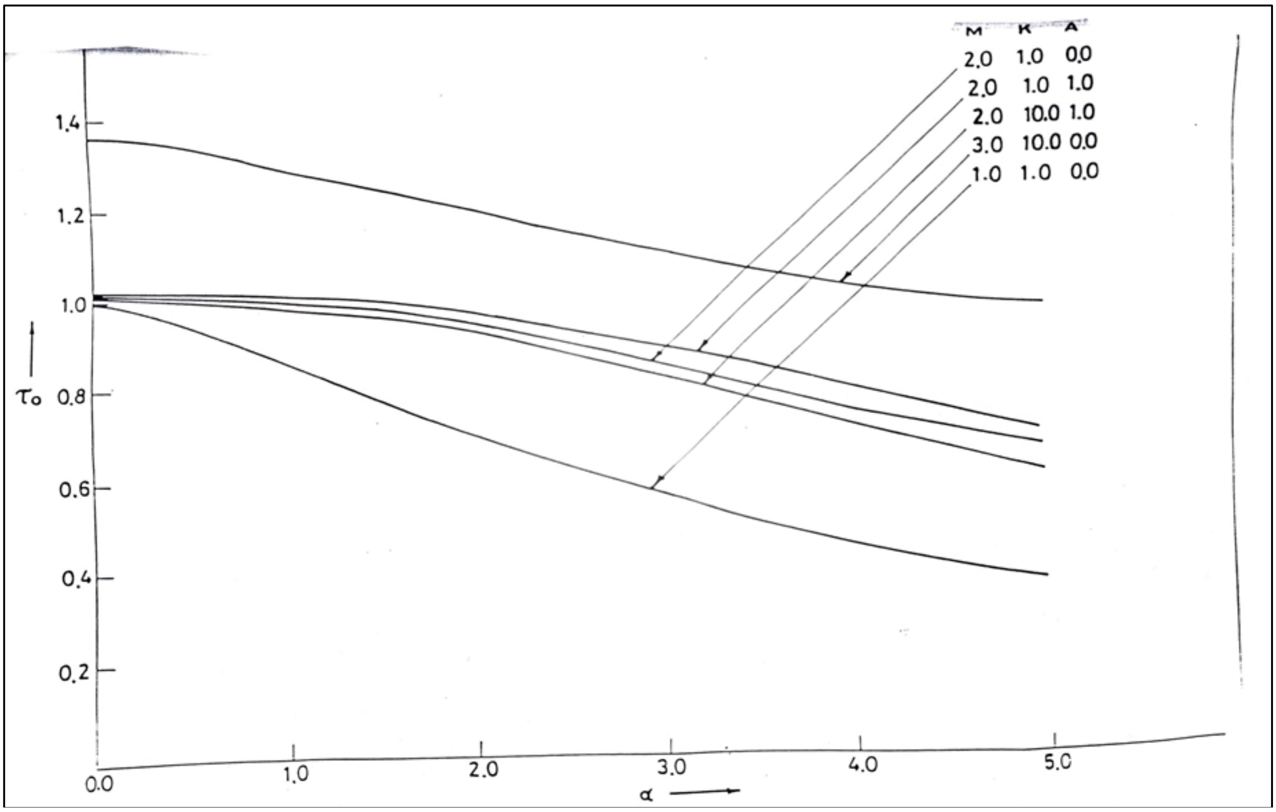

Fig 4: Skin friction T_0 plotted against the stratification factor ac for different values of M, K and A ($\mathcal{E} = 0.1$, $\omega = 1.0$, w t = $\pi/3$, $h_1 = 0.5$)

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