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B^3-Graphs and its application

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Abstract
In this paper, we construct a class of planar graphs with m, n and p vertices of degree 3, 4 and 2 respectively and then investigate various properties relating to domination number and block graph. An application of chemical substances has been put forwarded through the 3- regular and 3-coloring properties. Finally, a new class of graphs which is n-colorable has been constructed from the planar graph and some theoretical properties have been studied.
AMS SUB classification: 05C30, 05C45

Keywords: dominating set, block graph, coloring of a graph, chromatic number, n-critical graph

1. Introduction

The planar and non-planar graphs and their properties have been focused by various authors. The applications of such type's graphs have been found in VLSI(Very Large Scale Integrated circuit) design technology [1]. The planar and its dual have also played an important role in floor planning [2]. Recently, Kalita [3] has discussed the application of lower and upper bound of crossing of complete graph. He has also discussed various properties of graphs which have many applications in VLSI design.

Again, it is known that the domination number has an important role in Communication network. This concept of domination number was first introduced by Claude Berge in 1958 in his book on graph theory [4]. It was defined under the name "coefficient of external stability". Thereafter, in 1962 Oystein Ore used the terminology "domination set" and "domination number" for the first time in his book on graph theory [5]. Frank Harary *et al.*[6] pointed out an application in voting situations using the concept of domination number. Again, another interesting application was pointed out by Liu [7], where a dominating set represents a set of cities which are acting as transmitting stations, and can transmit message to every city in the network. Igor E. and Vadim E. [8] established some relations about domination parameters of cubic graph which had settled a problem posed by many workers of reference [9].

Recently, A. Nagoorgani and R. J. Hussain [10] introduced the concept of global domination number, total domination number and connected domination number in fuzzy graph with some important properties. Again, K.A. Bibi and R. Selvakumar [11] established some relations concerning about domination partition set, exact domination partition set and well-dominated partition in the same year.

Another important topic of graph theory is the coloring of graphs. It is true that there are different types of graphs which are four colorable, five colorable, six colorable and so on. But the known n-colorable graphs are complete graphs. It is not known till date, what type of graphs are n-critical or n colorable [12]. Hence this is also an important problem to be discussed.

In this paper, we introduce a new class of graphs known as B^3 - graph (defined later) and study some of their properties relating to coloring, domination number and block graphs, and the properties of 3-regular graphs. In addition to this, we construct another class of graph which is always n-colorable.

The paper is organized as follows

The section-1 deals with the information of previous works relating to our topics. Section-2 consists of preliminaries. The construction of B^3-graphs and some of their results are discussed in section-3.Section-4 contains application of B^3-graph.The discussion is drawn in section-5 and significance of the outcomes from this paper is reflects in section-6

2. Preliminaries

For a graph G, V (G) denotes the set of vertices in G, N(x) denotes the neighborhood of a vertex 'x'. Then the closed neighborhood of x is denoted by N[x] and defined as N[x] =N(x) U{x}, which is the set of vertices dominated by x. For a set X, which is a subset of V (G), N(X) =U N(x) and N[X] = N(X) U X. A set X is called a dominating set if N[X] =V(G). The domination number **γ** (G) of a graph G is the minimum cardinality of a minimal dominating set of G.

Again, the block of a graph is defined as a maximal non separable sub graph. A non-separable graph is connected, non-trivial and has no cut point. The chromatic number of a graph G is denoted by χ(G) and is defined as the minimum n for which G has an n-coloring. A coloring of a graph is an assignment of colors to its vertices so that no two adjacent vertices have the same color.

If the chromatic number of a graph is 'n', that is χ(G)=n and chromatic number of the graph (G-v), χ(G-v) ≤n-1, for each vertex v, then the graph is said to be n-critical. However, every n-chromatic graph, for n ≥ 2, contains an n-critical sub graph.

3. Construction of B^3-graphs and some of their results

3(a) Construction of B^3-graph: Let us consider a tree of four vertices having one vertex of degree 3 and the other three vertices of degree one (as shown in figure 1.1).Then producing one pair of vertices in each of the vertex of degree one and then adding the each pair of vertices by one edge, we have a graph whose four vertices of degree three each and six vertices of degree two each as shown in figure (1.2).

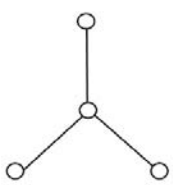

Fig 1.1

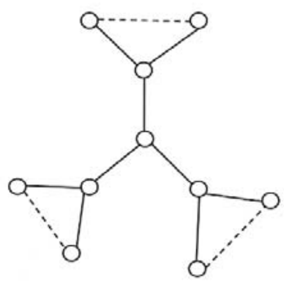

Fig 1.2

Thus, continuing the process of addition of a pair of vertices and adding the each pair of vertices by an edge (as discussed above), we have a graph of the following pattern:

G [4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], for r=0,1,2,3…, where 'm' indicates a vertex of degree 3, 'n' indicates a vertex of degree 4, 'p' indicates a vertex of degree 2. For r=1, the graph is G(4m+ 6n +12p, 30), which is shown in the following figure (1.3):

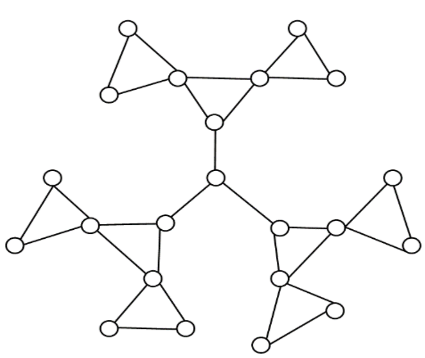

Fig 1.3: G (22, 30)

These types of graph are named as B^3 graph, in conformity with initial letters of the three authors Banamali, Bichitra and Bhaben.

3(b) Properties of B^3-graphs

The B^3-graphs have the following interesting properties:

Theorem (3.1): The graph G [4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], for r=0, 1, 2, 3…, is planar and 3- colorable; where m, n, and p indicate a vertex of degree 3, degree 4 and degree 2 respectively.

Proof: We know that the necessary condition for a simple planar graph G with at least three vertices is e(G)≤ 3.n(G)-6, where e(G) and n(G) denote the number of edges and number of vertices of G respectively. Hence, if we apply this condition in B^3-graph, we have, 3.n(G)-6=3.[4+6.(2^r-1)+6.2^r]-6 =2.[6(3. 2^r-1)]=2.e(G).Thus, we have e(G) ≤ 3.n(G)-6. Moreover, it is clear that from the construction process one can draw the B^3-graph on a plane without intersecting the edges, which is the simple definition of a planar graph. Hence, the B^3-graph is planar. Again, we know that a graph having at least one triangle is always 3-colorable. Therefore, our graph G is 3-colorable.

Theorem (3.2): The domination number of the graph G[4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], for r=0,1,2,3,……is

$$\gamma(G) = \frac{3[2^{r+3} - 1]}{7}, \ r= 0,3,6,9\ldots$$

$$= \frac{3[2^{r+3} - 2]}{7} + 1, \ r= 1, 4, 7\ldots$$

$$= \frac{3[2^{r+3} - 4]}{7} + 1, \ r= 2, 5, 8\ldots$$

Proof: Case-1 (for r=0, 3, 6, 9…..)

For r=0, the graph is G (4m + 6p, 12). Clearly D_o= {v_1, v_2, v_3} is a minimal dominating set (as shown in figure 1.4). Therefore the domination number is 3, that is 3[2^{o+3}-1] / 7. Thus the result is true for r=0.

Next, for r=3, the graph is G (4m + 42n + 48p, 138). Here the end vertices of degree 2 are not adjacent to any vertices of the set D_o and as they are adjacent to 24 vertices, which is minimum, therefore the domination number is (3+24) =27, that is 3[2^{3+3}-1] / 7. Thus the result is true for r=3.

Then, let us suppose that the result is true for r=k (where k is a multiple of 3), that is the domination number is 3[2^{k+3}-1] / 7, in which minimal dominating set D_k.

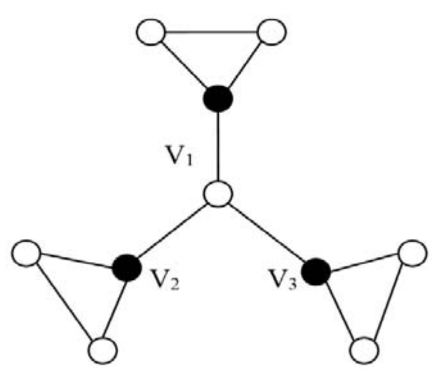

Fig 1.4

Now for r=k+3, the number of two degree vertices in the graph G is 6.2^{k+3} and these end vertices of degree 2 are not adjacent to any vertices of the set D_k (figure 1.4 gives six end vertices having degree 2). Since, they are adjacent with $\frac{1}{2}(6.2^{k+3})$ vertices, that is3.2^{k+3} vertices, therefore, the domination number is $[3(2^{k+3}-1) / 7] + 3. 2^{k+3}$or $3[2^{k+6}-1] / 7$. Thus the result is true for r= k + 3, which complete the proof for r=0, 3, 6, 9….

Case-2(for r= 1, 4, 7…)

For r=1, the graph is G (4m + 6n+ 12p, 30). Clearly D_1= {v_o, v_1, v_2, v_3, v_4, v_5, v_6} is a minimal dominating set (as shown in figure 1.5). Therefore, the domination number is 7, that is $\{3(2^{1+3}-2) / 7\} + 1$. Thus the result is true for r=1.

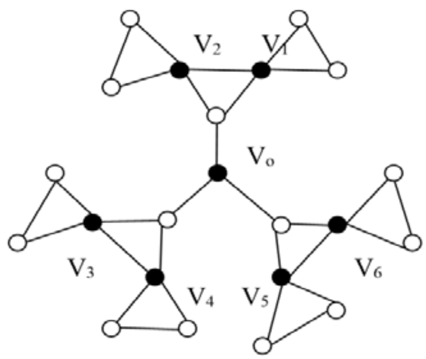

Fig 1.5: G (22, 30)

Next, suppose that the result is true for r=k, that is the domination number is $\{3[2^{k+3}-2] / 7\} +1$, in which minimal dominating set D_k (say).

Now for r=k+3, the number of two degree vertices in the graph G is 6.2^{k+3} and these end vertices of degree 2 are not adjacent to any vertices of the set D_k (figure 1.5 gives twelve end vertices having degree 2).Since they are adjacent with $\frac{1}{2}(6.2^{k+3})$ vertices, that is $3. 2^{k+3}$ vertices which are not in the set D_k, therefore, the domination number is $[\{3(2^{k+3}-2) / 7\}+1]+ 3. 2^{k+3}$ or $\{3[2^{k+6}-2] / 7\} +1$. Thus the result is true for r= k + 3, which complete the proof for r=1, 4, 7, 10….

Case-3 (for r= 2, 5, 8, 11…)

For r=2, the graph is G (4m + 18n+ 24p, 66). Here, D_2= {v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}} is a minimal dominating set (as shown in figure 1.6).Therefore the domination number is 13, that is $\{3(2^{2+3}-4)/7\}+ 1$. Thus the result is true for r=2.

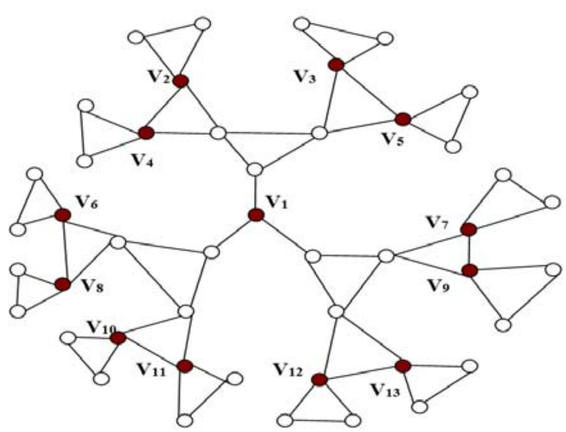

Fig 1.6: G (46, 66)

Next, suppose that the result is true for r=k, that is the domination number is $\{3[2^{k+3}-4] / 7\} +1$, in which minimal dominating set D_k (say).

Now for r=k+3, the number of two degree vertices is 6.2^{k+3} and these end vertices of degree 2 are not adjacent to any vertices of the set D_k (figure 1.6 gives twenty four end vertices having degree 2). Since they are adjacent with $\frac{1}{2}(6.2^{k+3})$ vertices, that is 3.2^{k+3} vertices which are not in the set D_k, therefore, the domination number is $[\{3(2^{k+3}-4) / 7\} +1] + 3.2^{k+3}$ or $\{3[2^{k+6}-4]/7\} +1$. Thus the result is true for r= k + 3, which complete the proof for r=2, 5, 8, 11….

Theorem(3.3): The Block graph H [$3. 2^r$ (m+s), $6. 2^r$] of G[4m + $6(2^r-1)$.n + $6. 2^r$.p, $6(3. 2^r -1)$] is 3-colorable, wherer=0,1,2,3…, 'm' indicates a vertex of degree 3, 'n' indicates a vertex of degree 4,'p' indicates a vertex of degree 2 and 's' indicates a vertex of degree 1.

Proof: For r=1, we have the graph H (in figure 1.8) is the block graph of G (in figure 1.7). It is observed that the block graph H contains one and only one triangle. On the other hand, it is known that a graph having one triangle is always 3-colorable. Therefore, the graph of figure 4.8 is always 3-colorable. From the construction process of B^3-graph, it is easy to find the block graph of B^3-graph for any value of r=0, 1, 2, 3, 4..., which contains only one triangle.

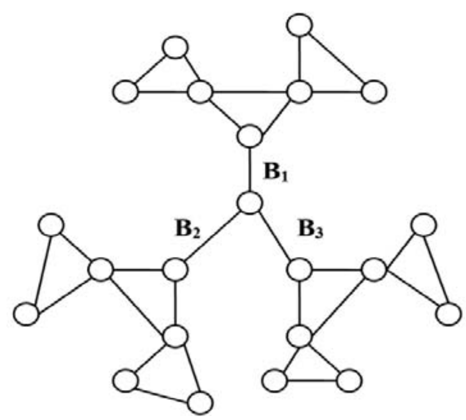

Fig 1.7: G (22, 30)

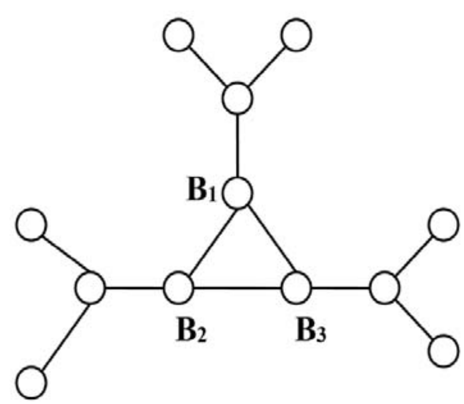

Fig 1.8: H (12, 12)

Therefore, the Block graph H of G is 3-colorable for r=0, 1, 2, 3…. Hence completes the proof.

Corollary: The number of vertices of the block graph H of G is equal to its number of edges.

Theorem(3.4): The Block graph H [3. 2^r (m+s), 6. 2^r] of G[4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)]will be 3-regular if we introduce (3. 2^r) edges with the vertices of degree one, and this graph is C[6. 2^r.m, 9. 2^r], for r = 0,1,2,3…,which is3-regular but planar.

Proof: For r=1, the block graph of G is H(6m+6s, 12). Then from the figure (1.9) we have the six vertices u_1, u_2, u_3, u_4, u_5 and u_6 having degree one each. So if we introducedsix edges with these six vertices by making degree three each, then the new graph C (12,18) will be 3-regular.

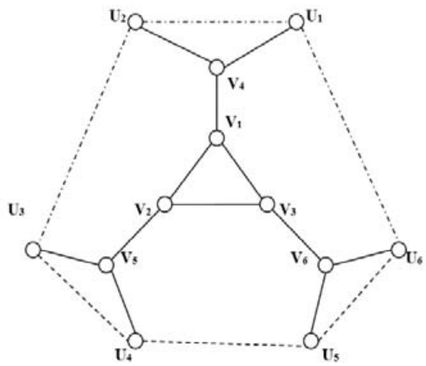

Fig 1.9: C (12, 18)

In the same way, let us consider the block graph H [3. 2^r (m+s), 6. 2^r] for any value of r. Then in this graph the number of one degree vertices is (3. 2^r). If we connect these vertices by one with another such that all of them lie in a cycle, then the number of new edges will be (3. 2^r)and the resulting graph will be 3-regular. Thus we have the 3-regular graph

C [6. 2^r.m,6. 2^r + 3. 2^r] i.e. C [6. 2^r.m, 9. 2^r], r=0,1,2,3...

3(b) Another construction of n-coloring Graph:

In our graph C [6. 2^r.m, 9. 2^r], where r=1, 2, 3…; and m is a vertex of degree 3, we consider n vertices such that there does not exist any triangle formed by them and they lie in a path of length (n-1) and they do not form a complete cycle. Then we construct a new graph by joining these n-vertices by ½[n^2-3n+2] edges in such a way that two vertices of degree (n+1) and the (n-2) vertices of degree n and all the remaining vertices of degree 3. Let us construct it for n=4 (figure 4.10). Then the new pattern of this graph will be C_r [(6. 2^r - n) m + (n-2) u + 2v, 9. 2^r+ ½(n^2-3n+2)] where r=1, 2, 3...; n=4, 5, 6…; m is a vertex of degree 3; u is a vertex of degree n; v is a vertex of degree (n+1).

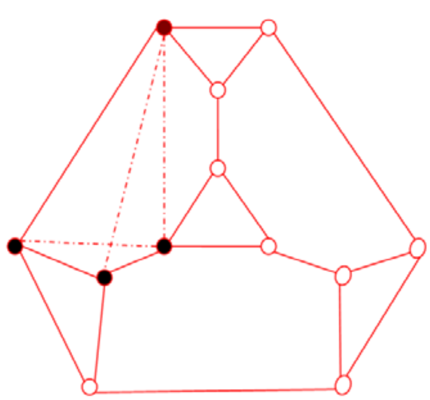

Fig 1.10: G (12, 21)

Theorem (3.5) (n-coloring theorem)

The graph C_r [(6. 2^r -n) m + (n-2) u + 2v, 9. 2^r + ½(n^2-3n+2)], where r=1, 2, 3..; n= r+3; m is a vertex of degree 3; u is a vertex of degree n; v is a vertex of degree (n+1), is always n-colorable or (r+3)-colorable.

Proof: Here, we have n = r + 3 for r=1, 2, 3, 4….Now for r=1, the graph is C_1 [12, 21]. From the figure (1.10) we have the number of (1+3), i.e. 4-degree vertices is 2 and number of (1+3+1),i.e. 5-degree vertices is 2 and the number of edges to be introduced in the original graphC [6. 2^r.m, 9. 2^r],for r=1, is 3 which can be express as ½(n^2-3n+2) = ½[$(r+3)^2$-3(r+3) +2], for r=1.

Again, from the figure (1.10), it is observed that the 4-degree and 5-degree vertices together form a complete sub graph having (2+2) =4 vertices and since these four vertices can be colored only by four distinct colors therefore the graph is 4-colorable i.e. (1+ 3) colorable. Thus the result is true for r=1.

Next, let the result is true for r=k, i.e. the graph is (k+ 3) colorable or n-colorable. Then the graph must contain a complete sub graph having (k+3) vertices (as our construction process). Then if we consider a neighbor vertex according to our construction process together with these (k+3) vertices and making it a complete graph having (k+ 3+1) vertices by adding (k+3)-1 i.e. (k+2) edges, then total number of edges to be introduced is ½[$(k+3)^2$-3(k+3) +2]+ (k+2)= ½[k^2+5k+6]= ½[$(k+4)^2$-3(k+4) +2].Thus the result is true for r=k+1, that is the graph is (k+4) colorable or n+1 colorable as n=r+3. Hence the result is true for n (≥ 4).

4. Application of B^3 graph

TheB^3graph has the following special application for chemical substances--

Suppose, there are r (≥10) products (or substances) to be produced by a chemical factory. The different r products may reacts with others. The company wants to keep the three kinds of products so that they do not react at all for non reactions of them with only three categories. Design a structure of graph with the above three properties such that no reactions is take place with others.

The company must reserve the right to produce the number of product so that they can pack only three packets with same quality of substances. Hence, when a company produces 10 substances and wants to keep them only three stores or packets then it can be represented by the B^3 graph as a graphical pattern in which one packet contains four products of same kind and the other two contain 3 products each of same kind. This result is nothing but our B^3 graph for r=0. Hence the company must produce the products as 22, 46, 94…; for r= 1, 2, 3…

Here, we impose a question: "how the company will produce the products so that they can be placed in to three boxes only"? The answer is given by our B^3 graph.

5. Discussion

In this chapter, we introduce a construction mode of a class of planar graphs G[4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], for r=0, 1, 2, 3…, where m, n, and p indicatevertex of degree 3, degree 4 and degree 2 respectively, which is 3- colorable. Besides, it is found that the domination number of the graph G [4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], for r=0, 1, 2, 3 … is

$$\gamma(G)\ \gamma(G) = \frac{3[2^{r+3} - 1]}{7}, \ r=0,3,6,9,\ldots\ldots\ldots$$

$$= \frac{3[2^{r+3} - 2]}{7} + 1 \text{, r=1, 4, 7……}$$

$$= \frac{3[2^{r+3} - 4]}{7} + 1 \text{, r=2, 5, 8…….}$$

Moreover, from the construction of the block graph H [3. 2^r (m+ s), 6. 2^r] of G [4m + 6(2^r-1).n + 6. 2^r.p, 6(3. 2^r -1)], it is proved that the graph H is 3-colorable but not 3-regular. Interestingly, introducing (3. 2^r) edges with this edges of block graph H, a new class of planar graph, namely C[6. 2^r.m,9. 2^r], for r = 0,1,2,3…; is found which is3-regular.

Finally, we formulate a new class of n-colorable graph C_r [(6. $2^{r.}$ -n) m + (n-2) u + 2v, 9. 2^r + ½(n^2-3n+2)], where r=1,2,3…, by means of joining the original vertices by ½(n^2-3n+2) edges subject to the condition that the two vertices are of degree (n+1), (n-2) vertices are of degree n and the rest are of degree 3.

Moreover, an application in company products is advanced forward. In this context, formation of three non-reactive chemical substances is proposed for business purposes.

6. Significance of the outcomes

In this chapter, we have established a fine property relating to domination number of a new class of graph, named B^3 – graph. Since domination number plays an important role in the field of communication network, therefore, this result may act as a powerful tool in solving communication network problem. Finally, we have forwarded a class of graph which is always n- colorable. It is an important problem to identify what types of graphs are n-critical or n-colorable (except the complete graph), which is not known till date. Therefore, we may expect, our findings will lead to solve this type of problem in near future.

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