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On The negative Pythagorean equation $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$

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Abstract
Infinitely many non-zero distinct integer solutions for the negative Pythagorean equation $x^{-2}+y^{-2}=z^{-2}$ are presented. A few interesting properties among the solutions are also given.

Keywords: negative Pythagorean equation, integral solutions.

1. Introduction

The equation given by $x^2+y^2=z^2$ is known as Pythagorean equation. Here, x & y are called the sides or legs of the Pythagorean triangle and z, its hypotenuse. Note that Pythagorean triangle is nothing but the right angled triangle. The integer values of x, y, z satisfying the Pythagorean equation are given by $x=2mn, y=m^2-n^2, z=m^2+n^2$ where $m>n>0$ [1].

Now, the equation given by $x^{-2}+y^{-2}=z^{-2}$ is known as negative Pythagorean equation [2] and we illustrate below the method of obtaining integer values of x, y, z satisfying the negative Pythagorean equation.

2. Method of Analysis

Consider the negative Pythagorean equation

$$x^{-2}+y^{-2}=z^{-2} \tag{1}$$

(1) Is written as $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$

i.e, $\frac{(x^2+y^2)}{x^2y^2}=\frac{1}{z^2}$

$$\text{i.e } x^2+y^2=\frac{x^2y^2}{z^2}=\left(\frac{xy}{z}\right)^2 \tag{2}$$

$$\text{Let } t=\frac{xy}{z} \tag{3}$$

$$(2)\Rightarrow x^2+y^2=t^2 \tag{4}$$

Since x and y are integers, 't' should be an integer.

Let $d=\gcd(x,y,t)$

$$\text{Then } x=ad, y=bd, t=cd \text{ where } \gcd(a,b,c)=1 \tag{5}$$

Now, from (3) & (5), we get

$$cd = \frac{abd^2}{z}$$

$$ie, z = \frac{abd}{c}$$

Since $\gcd(a,c)=1, \gcd(b,c)=1$, we have c divides d as z is an integer.

Let $d = kc$

Therefore $z = kab$ (6)

Also, from (5), $x = kac, y = kbc, t = kc^2$ (7)

From (4) & (7), we have

$$k^2a^2c^2 + k^2b^2c^2 = k^2c^4$$

$$\Rightarrow a^2 + b^2 = c^2 \quad (8)$$

This is the well-known Pythagorean equation whose solutions are given by

$$a = 2mn, b = m^2 - n^2, c = m^2 + n^2, m > n > 0$$

Note that (8) is also solved as follows

Let p, q, r, s denote successive Fibonacci numbers

Then, observe that $ps = r^2 - q^2$

Squaring both sides, we have

$$p^2s^2 = (r^2 - q^2)^2 = (r^2 + q^2)^2 - 4q^2r^2$$

Thus, the solutions of (8) may also be taken as

$$a = ps, b = 2qr, c = q^2 + r^2$$

Substituting these values of a,b,c in (6) and (7), the non-zero distinct integer solutions to (1) are given by

$$x = \mathrm{kps}(\mathrm{q}^2 + \mathrm{r}^2)$$

$$\mathrm{y} = 2\mathrm{kqr}(\mathrm{q}^2 + \mathrm{r}^2) \quad (**)$$

$$\mathrm{z} = 2\mathrm{kpqrs}$$

Observed that the above solutions are different from the solutions presented in [2].

A few numerical examples are presented below

k	p	q	r	s	a	b	c	x	y	z	t
2	2	3	5	8	16	30	34	1088	2040	960	2312
2	3	5	8	13	39	80	89	6942	14240	6240	15842
1	5	8	13	21	105	208	233	24465	48464	21840	54289
1	8	13	21	34	272	546	610	165920	333060	148512	372100
2	2	4	6	10	20	48	52	2080	4992	1920	5408

The above solutions satisfy the following relations

[I] $k\left(\frac{xz}{y}\right)$ is a perfect square

[II] $k^2(x^2 - z^2)$ Is a Bi-Quadratic integer

[III] Each of the following triples forms an Arithmetic Progression

$$(i);\left(2(x^2 - x + y), t^2, 2(y^2 - y + x)\right)$$

$$(ii):\left(2(x \pm y)^2, t^2, \mp 4xy)\right)$$

$$(iii):\left(4x^2 - 2y^2, t^2, 4y^2 - 2x^2\right)$$

$$(iv);\left(2x^2, t^2, 2y^2\right)$$

[IV] Consider the solutions given by (**),

Case (1): Note that $6[\left(\frac{x+y+z}{k}\right) + q^2(4r^2 + q^2)]$ is a

Nasty Number and hence represents area of the Pythagorean triangle

Case(2): $\left(\frac{x+y+z}{k}\right) + 2q^2(3r^2 + 2qr + q^2)$ is a Bi-

Quadratic integer

3. References

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