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On *(Gr)-Continuous Functions and Contra*(Gr)-Continuous Functions in Topological Spaces

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Abstract
In this paper we introduced and study the notions of *(gr)-continuous and *(gr)-irresolute, contra *(gr)-continuous functions in topological spaces and also we discussed their properties.

Keywords: *(gr)-continuous, *(gr)-irresolute, contra *(gr)-continuous

1. Introduction

In 1970, Levine [11], introduced and investigated the notion of g-closed sets in topological spaces. In 1987, Bhattacharya, *et al.* [4], introduced and studied the notion of semi generalized closed sets in topological spaces. In 1990, Arya, *et al.* [2] introduced the notion of generalized semi closed sets.

In 1991, Balachandran, *et al.* [3], introduced and investigated the notion of generalized continuous function in topological spaces. In 1991, Sundaram, *et al.* [20], studied semi generalized continuous functions and in 1995, Devi, *et al.* [6], introduced and investigated the notion of generalized semi continuous function in topological spaces. In 1996, Dontchev [8], introduced Contra continuous function in topological spaces and in 1998, Noiri, *et al.* [16] formulated generalized pre- closed functions. Recently, Rajendran *et al.* [18] were introduced *(gr)-closed sets in topological spaces.

Let (X, τ) be a topological space with no separation axioms are assumed. If $A \subseteq X$, cl(A) and int(A) will respectively denote the closure and interior of A in (X, τ).

Definition 1.1 A subset A of a topological space (X, τ) is called
1) Pre- closed set [12], if $cl(int(A)) \subseteq A$.
2) Semi- closed set [10], if $int(cl(A)) \subseteq A$.
3) Semi- pre closed set [1], if $int(cl(int(A))) \subseteq A$.
4) Regular closed set [19], if $A = cl(int(A))$.
5) α- closed set [15], if $cl(int(cl(A))) \subseteq A$.

Definition 1.2 [17] For any subset A of (X, τ), $rcl(A) = \cap \{B: B \supseteq A$, B is a regular closed subsets of X$\}$.

Definition 1.3 A subset A of a topological space (X, τ) is called
1) g- closed set [11], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) gs- closed set [2], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
3) sg- closed set [4], if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X, τ).
4) gα - closed set [13], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α- open in (X, τ).
5) αg- closed set [13], if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
6) gp- closed set [14], if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
7) gsp- closed set [7], if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
8) rg- closed set [17], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
9) gpr- closed set [9], if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).
10) $\hat{g}$ - closed set [21], if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in (X, τ).
11) *(gr)-closed set [18], if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

Definition 1.4 A function f: (X, τ) ⟶ (Y, σ) is aid to be

1) g- continuous [3], if $f^{-1}(V)$ is g- closed in X for every closed subset V of Y.
2) sg- continuous [20], if $f^{-1}(V)$ is sg- closed in X for every closed subset V of Y.
3) gs- continuous [6], if $f^{-1}(V)$ is gs- closed in X for every closed subset V of Y.
4) gp- continuous [16], if $f^{-1}(V)$ is gp- closed in X for every closed subset V of Y.
5) gpr- continuous [9], if $f^{-1}(V)$ is gpr- closed in X for every closed subset V of Y.
6) gsp- continuous [7], if $f^{-1}(V)$ is gsp- closed in X for every closed subset V of Y.
7) rg- continuous [17], if $f^{-1}(V)$ is rg- closed in X for every closed subset V of Y.
8) regular continuous [17], if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
9) Contra continuous [8], if $f^{-1}(V)$ is closed in X for every open subset V of Y.

2. *(gr)-Continuous Functions

Definition 2.1A function f: (X, τ) → (Y, σ) is called *(gr)-continuous if $f^{-1}(V)$ is *(gr)-closed set in (X, τ)for every closed set V in (Y, σ).

Theorem 2.2 For a function f: (X, τ) → (Y, σ), every r-continuous function is *(gr)-continuous but not conversely.

Proof: Let f be a r-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is r-closed set in (X, τ). Since every r-closed set is *(gr)-closed, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is *(gr)-continuous.

Example 2.3 Let X = Y = {a, b, c, d} with τ = {∅, {c}, {a, b}, {a, b, c}, X}, σ = {∅, {a, c}, Y}. Let f: (X, τ) → (Y, σ) be the identity function, then f is *(gr)-continuous but not r-continuous. Since for the closed set {b, d} in Y, f^{-1}({b, d}) is *(gr)-closed, but not r-closed set in (X, τ).

Theorem 2.4 For the function f: (X, τ) → (Y, σ), the following hold.

(i) Every *(gr)-continuous function is g-continuous.
(ii) Every *(gr)-continuous function is gs-continuous.
(iii) Every *(gr)-continuous function is gsp-continuous.
(iv) Every *(gr)-continuous function is gp-continuous.
(v) Every *(gr)-continuous function is αg-continuous.
(vi) Every *(gr)-continuous function is rg-continuous.
(vii) Every *(gr)-continuous function is gpr-continuous.

Proof

(i) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is g-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is g-continuous.
(ii) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is gs-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is gs-continuous.
(iii) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is gsp-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is gsp-continuous.
(iv) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is gp-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is gp-continuous.
(v) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is αg-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is αg-continuous.
(vi) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is rg-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is rg-continuous.
(vii) Let F be a *(gr)-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is *(gr)-closed in (X, τ). Since Every *(gr)-closed set is gpr-closed set, $f^{-1}(V)$ is *(gr)-closed in (X, τ). Therefore f is gpr-continuous.

The converse of the above theorem need not be true as seen from the following examples.

Example 2.5

(i) Let X = Y = {a, b, c, d} with τ = {∅, {a}, {c}, {a, c}, {b, d}, {a, b, d}, {b, c, d}, X} and σ = { ∅, {d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function, then f is g-continuous but not *(gr)-continuous. Since for the closed set {a, b, c} in Y, f^{-1}({a, b, c}) = {a, b, c} is g–closed, but not *(gr) –closed set in (X, τ).
(ii) Let X = Y = {a, b, c, d} with τ = {∅, {d}, {a, b, c}, X} and σ = { ∅, {a}, {b}, {a, b}, Y }. Let the function f: (X, τ) → (Y, σ) be defined by f(a) = a, f(b) = c, f(c) = d and f(d) = b, then f is gs-continuous but not *(gr)-continuous. Since for the closed set {c, d} in Y, f^{-1}({c, d}) = {b, c} is gs–closed, but not *(gr) –closed set in (X, τ).
(iii) Let X = Y = {a, b, c, d} with τ = {∅, {c}, {a, b}, {a, b, c}, X} and σ = {∅, {c, d}, {a, c, d}, Y}. Let the function f: (X, τ) → (Y, σ) be defined by f(a) = c, f(b) = b, f(c) = a, and f(d) = d, then f is gsp-continuous but not *(gr)-continuous. Since for the closed set {a, b} in Y, f^{-1}({a, b}) = {b, c} is gsp–closed, but not *(gr) –closed set in (X, τ).
(iv) Let X = Y = {a, b, c, d} with τ = {∅, {d}, {a, b, c}, X} and σ = { ∅, {a}, {c}, {a, c, d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function, then f is gp-continuous but not *(gr)-continuous. Since for the closed set {c} in Y, f^{-1}({c}) = {c} is gp–closed, but not *(gr) –closed set in (X, τ).
(v) Let X = Y = {a, b, c, d} with τ = {∅, {d}, {b, c},{b, c, d}, X} and σ = { ∅, {a},{a, c}, Y }. Let the function f (X, τ) → (Y, σ) be defined by f(a) =a, f(b) = d, f(c) = c and f(d) = b. Then f is αg-continuous but not *(gr)-continuous. Since for the closed set {b, d} in Y, f^{-1}({b, d}) = {b, d} is αg–closed, but not *(gr) –closed set in (X, τ).
(vi) Let X = Y = {a, b, c, d} with τ = {∅, {c}, {d}, {c, d}, {a, c, d}, X} and σ = { ∅, {{b, c}, {b, c, d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function, then f is rg-continuous but not *(gr)-continuous. Since for the closed set {a, d} in Y, f^{-1}({a, d}) = {a, d} is rg–closed, but not *(gr) –closed set in (X, τ).
(vii) Let X = Y = {a, b, c, d} with τ = {∅, {a}, {c}, {a, c}, {a, c, d}, X} and σ = { ∅, {b, c}, {a, b, c}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function, then f is gpr-continuous but not *(gr)-continuous. Since for the closed set {d} in Y, f^{-1}({d}) = {d} is gpr–closed, but not *(gr) –closed set in (X, τ).

Theorem 2.6 Let f: (X, τ) → (Y, σ) be a function. Then the following statements are equivalent.

(i) f is *(gr) –continuous.

(ii) the inverse image of each open set in Y is *(gr) –open in X.

Proof: (i) ⇒ (ii) Assume that f: (X, τ) → (Y, σ) is *(gr) –continuous. Let G be an open sets in Y. Then G^C is closed in Y. Since f is *(gr) –continuous, $f^{-1}(G^C)$ is *(gr) –closed in X. But $f^{-1}(G^C) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is *(gr) –open in X.

(ii) ⇒ (i). Assume that the inverse image of each open set in Y is *(gr) –open in X. Let F be any closed set in Y. Then F^C is open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$ is *(gr) –open in X and so $f^{-1}(F)$ is *(gr) –closed in X. Therefore f is *(gr) –continuous.

Theorem 2.7 If a function f: (X, τ) → (Y, σ) is *(gr) –continuous, then f(*(gr)–cl(A)) ⊆ cl(f(A)) for every subset A of X.

Proof: Let f: (X, τ) → (Y, σ) be *(gr) –continuous. Let A ⊆ X. Then cl(f(A)) is closed in Y. Since f is *(gr) –continuous. f^{-1}(cl(f(A))) is *(gr) –closed in X and A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))), implies *(gr)-cl(A) ⊆ f^{-1}(cl(f(A))).Hence f(*(gr)–cl(A)) ⊆ cl(f(A)).

Theorem 2.8 Let f: (X, τ) → (Y, σ) is *(gr) –continuous function and g: (Y, σ) → (Z, η) is continuous function then g o f: (X, τ) → (Z, η) is *(gr) –continuous.

Proof: Let g be a continuous function and V be any open set in Z, then $g^{-1}(V)$ is open in Y. Since f is *(gr) –continuous, $f^{-1}(g^{-1}(V)) = (g \text{ o } f)^{-1}(V)$ is *(gr) –open in X. Hence g o f is *(gr) –continuous.

Definition 2.9A function f: (X, τ) → (Y, σ) is said to be *gr –irresolute, if $f^{-1}(V)$ is *(gr) –open set in (X, τ) for every *(gr)-open set V in (Y, σ).

Theorem 2.10 Every *(gr) –irresolute function is *(gr) –continuous but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be *gr –irresolute and V be a closed set in Y which is *(gr) –closed then $f^{-1}(V)$ is *(gr) –closed in X. Hence f is *(gr) –continuous.

Example 2.11 Let X = Y = {a, b, c, d} with τ = {∅, {c}, {d}, {c, d}, {a, c, d}, X} and σ = { ∅, {{a}, {c}, {a, c}, {a, c, d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function, then f is *(gr)-continuous but not *(gr)-irresolute.

Theorem 2.12 Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be any two functions, then the following hold.

(i) g o f is *(gr)-continuous if f is *(gr)-irresolute and g is *(gr)-continuous.

(ii) g o f is *(gr)-irresolute if f is *(gr)-irresolute and g is *(gr)-irresolute.

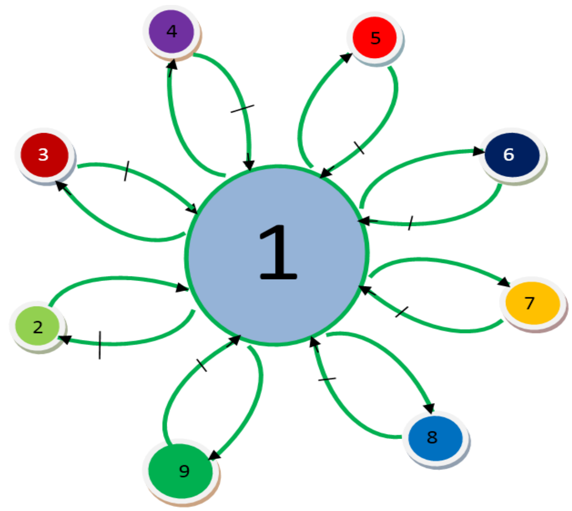
Proof

(i) Let V be a closed set in Z. Since g is *(gr)-continuous, $g^{-1}(V)$ is *(gr)-closed in Y. Since f is *(gr)-irresolute, $f^{-1}(g^{-1}(V))$ is *(gr)-closed in X. Hence g o f is *(gr)-continuous.

(ii) Let V be *(gr)-closed set in Z. Since g is *(gr)-irresolute, $g^{-1}(V)$ is *(gr)-closed in Y. Since f s is *(gr)-irresolute, $f^{-1}(g^{-1}(V))$ is *(gr)-closed in X. Hence g o f is *(gr)-irresolute.

Remark 4.1.13

For the functions defined above, we have the following implications.



1.*(gr) - continuous

2. r – continuous

3. g – continuous

4. gs – continuous

5. gp – continuous

6. gsp – continuous

7. gpr – continuous

8. rg – continuous

9. αg – continuous

3. Contra *(gr)-Continuous Functions

Definition 3.1 A function f: (X, τ) → (Y, σ) is called a contra *(gr)-continuous if $f^{-1}(V)$ is *(gr)-closed set in (X, τ) for every open set V in (Y, σ).

Definition 3.2 A function f: (X, τ) → (Y, σ) is called a contra *(gr)-irresolute if $f^{-1}(V)$ is *(gr)-closed set in (X, τ) for every *(gr)-open set V in (Y, σ).

Theorem 3.3 For the function f: (X, τ) → (Y, σ), the following hold.

(i) Every contra *(gr)- continuous function is contra g-continuous.

(ii) Every contra *(gr)- continuous function is contra gs-continuous.

(iii) Every contra *(gr)- continuous function is contra gsp-continuous.

(iv) Every contra *(gr)- continuous function is contra rg-continuous.

(v) Every contra *(gr)- continuous function is contra gpr-continuous.

(vi) Every contra *(gr)- continuous function is contra gp-continuous.

(vii)Every contra *(gr)- continuous function is contra αg-continuous.

Proof

(i) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is g-closed, $f^{-1}(V)$ is g-closed in X. Hence f is contra g-continuous.

(ii) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is gs-closed, $f^{-1}(V)$ is gs-closed in X. Hence f is contra gs-continuous.
(iii) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is gsp-closed, $f^{-1}(V)$ is gsp-closed in X. Hence f is contra gsp-continuous.
(iv) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is rg-closed, $f^{-1}(V)$ is rg-closed in X. Hence f is contra rg-continuous.
(v) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is gpr-closed, $f^{-1}(V)$ is gpr-closed in X. Hence f is contra gpr-continuous.
(vi) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is gp-closed, $f^{-1}(V)$ is gp-closed in X. Hence f is contra gp-continuous.
(vii) Let V be an open set in Y. Since f is contra *(gr)-continuous, then $f^{-1}(V)$ is *(gr)- closed in X. Since every *(gr)- closed set is αg--closed, $f^{-1}(V)$ is αg-closed in X. Hence f is contra αg-continuous.

The converse of the above theorem need not be true as seen from the following examples.

Example 3.4
(i) Let X = Y = {a, b, c, d} with τ = {∅, {d},{a, b, c}, X} and σ = { ∅, {a}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function. Then f is contra g-continuous, contra gs-continuous and contra gp-continuous but not in contra*(gr)-continuous. Since for the open set {a} in Y, f^{-1}({a}) = {a} is g–closed, gs-closed and gp-closed but not *(gr) –closed set in (X, τ).
(ii) Let X = Y = {a, b, c, d} with τ = {∅, {c},{a, b},{a, b, c}, X} and σ = { ∅, {a, c}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function. Then f is contra rg-continuous, contra gsp-continuous but not in contra*(gr)-continuous. Since for the open set {a, c} in Y, f^{-1}({a, c}) = {a, c} is rg–closed, gsp-closed but not *(gr) –closed set in (X, τ).
(iii) Let X = Y = {a, b, c, d} with τ = {∅, {a},{c}, {a, c}{a, c, d}, X} and σ = { ∅, {d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function. Then f is contra gpr-continuous, but not in contra*(gr)-continuous. Since for the open set {d} in Y, f^{-1}({d}) = {d} is gpr–closed but not *(gr) –closed set in (X, τ).
(iv) Let X = Y = {a, b, c, d} with τ = {∅, {b},{c}, {b, c}{b, c, d}, X} and σ = { ∅, {d}, Y }. Let the function f: (X, τ) → (Y, σ) be the identity function. Then f is contra αg-continuous, but not in contra*(gr)-continuous. Since for the open set {d} in Y, f^{-1}({d}) = {d} is αg–closed but not *(gr) –closed set in (X, τ).

Theorem 3.5 Let f: (X, τ) → (Y, σ) be contra *gr – continuous function and g: (Y, σ) →(Z, η) is continuous function then g o f: (X, τ) → (Z, η) is contra *(gr) – continuous.

Proof: Let V be any open set in Z. Since g: (Y, σ) → (Z, η) is continuous, $g^{-1}(V)$ is open in Y. Since f: (X, τ) → (Y, σ) is contra *(gr) –continuous, $f^{-1}(g^{-1}(V))$ is *(gr) –closed in X. Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is *(gr) – closed in X, which implies that g o f: (X, τ) → (Z, η) is contra *(gr) – continuous.

Theorem 3.6 Let f: (X, τ) → (Y, σ) be contra *(gr) – irresolute and g: (Y, σ) → (Z, η) is contra *(gr) –continuous function then g o f: (X, τ) → (Z, η) is contra *(gr) – continuous.

Proof: Let V be any open set in Z. Since g: (Y, σ) → (Z, η) is contra *(gr) – continuous, $g^{-1}(V)$ is *(gr) –closed in Y. Since f: (X, τ) → (Y, σ) is contra *(gr) –irresolute, $f^{-1}(g^{-1}(V))$ is *(gr) –open in X. Hence g o f: (X, τ) → (Z, η) is contra *(gr) –continuous.

Theorem 3.7 The following are equivalent for a function f: (X, τ) → (Y, σ)
(i) f is contra *(gr) –continuous.
(ii) the inverse image of every closed set of Y is *(gr) –open in X.

Proof: Let U be any closed set of Y. Since Y – U is open, then by (i), it follows that $f^{-1}(Y - U) = X - f^{-1}(U)$ is *(gr) – closed. This shows that $f^{-1}(U)$ is *(gr) – open in X.
Converse is similar.

Theorem 3.8 Suppose that X and Y are spaces and *(gr) – open in X is closed under arbitrary unions. If a function f: (X, τ) → (Y, σ) is contra *(gr) –continuous and Y is regular then f is *(gr) –continuous.

Proof: Let x be an arbitrary point of X and V be an open set of Y containing f(x). Since Y is regular, there exist an open set G in Y containing f(x) such that cl(G) ⊂ V. Since f is contra *(gr) –continuous, there exists U ∈ *(gr) –open in X containing x such that f(U) ⊂ cl(G). Then f(U) ⊂ cl(G) ⊂ V. Hence f is *(gr) –continuous.

Theorem 3.9 Let f: (X, τ) → (Y, σ) be surjective, *(gr) – irresolute and *(gr) –open and g: (Y, σ) → (Z, η) be any function then g o f: (X, τ) → (Z, η) is contra *(gr) – continuous iff g is *(gr) –continuous.

Proof: Suppose g o f is contra *(gr) –continuous. Let F be closed set in Z.Then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is *(gr) –open in X. Since f is *(gr) –open and surjective $f(f^{-1}(g^{-1}(F)))$ is *(gr) –open in Y. That is $g^{-1}(F)$ is *(gr) –open in Y. Hence g is contra *(gr) –continuous.
Conversely, Suppose that g is contra *gr –continuous. Let V be closed in Z. Then $g^{-1}(V)$ is *gr –open. Since f is *gr – irresolute, $f^{-1}(g^{-1}(V))$ is *gr –open. That is $(g \circ f)^{-1}((g \circ f)^{-1}(V)$ is *gr –open in X. Hence g o f: (X, τ) → (Z, η) is contra *gr –continuous.

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