International Journal of Applied Research 2015; 1(10): 1015-1020

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(10): 1015-1020
www.allresearchjournal.com
Received: 14-07-2015
Accepted: 16-08-2015

H Jude Immaculate
Department of Mathematics
Nirmala College for Women,
Coimbatore, Tamilnadu, India.

I Arockiarani
Department of Mathematics
Nirmala College for Women,
Coimbatore, Tamilnadu, India.

Correspondence
H Jude Immaculate
Department of Mathematics
Nirmala College for Women,
Coimbatore, Tamilnadu, India.

On Intuitionistic Fuzzy d-Quotient Mappings

H Jude Immaculate, I Arockiarani

Abstract
In this paper, we initiate the concept of IFd-quotient mapping and intuitionistic fuzzy strongly d-quotient mappings. The relation between the new mappings and intuitionistic fuzzy quotient mapping are investigated with counter examples.

Keywords: Intuitionistic, Fuzzy, Mappings

1. Introduction
The concept of fuzzy sets was proposed by Zadeh [22]. On the other hand Coker [7] introduced intuitionistic fuzzy topological spaces using the notion of fuzzy sets. After the introduction of fuzzy sets by Chang [6] in 1968, there have been several generalizations of notions of fuzzy set and fuzzy topology. By adding the degree of non-membership to fuzzy sets, Atanassov [3] proposed intuitionistic fuzzy sets (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precise model the problem based on the existing knowledge and observations. Fuzzy quotient mapping was introduced by Ramakrishnan [14] *et al.* in 2005. The same form of mapping was introduced in intuitionistic fuzzy topological space by Jun [12] *et al.* In this paper the concepts of intuitionistic fuzzy d-quotient mapping, strongly d-quotient mapping, ds-quotient mapping are introduced and some related properties are discussed throughout this paper. Examples are given to clarify the relationship between these concepts and existing mappings in intuitionistic fuzzy topological spaces

2. Reliminaries
Definition 2.1 [3]: Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS, for short) A is an object having the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ where the function $\mu_A : X\to I$ and $\nu_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\nu_A(x)\le 1$ for each $x\in X$.
Obviously, every fuzzy set A on a nonempty set X is an IFS having the form $A\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$.

Definition 2.2 [3]: Let X be a nonempty set and the IFS's A and B be in the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, $B=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$, and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. then we define

1. $A\subseteq B$ if and only if $\forall x\in X$, $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$;
2. $\overline{A}=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$;
3. $\cap A_j=\{\{\langle x,\wedge\mu_{A_j}(x),\vee\nu_{A_j}(x)\rangle : x\in X\}$;
4. $\cup A_j=\{\{\langle x,\vee\mu_{A_j}(x),\wedge\nu_{A_j}(x)\rangle : x\in X\}$;
5. $1_\sim=\{\langle x,1,0\rangle : x\in X\}$ and $0_\sim=\{\langle x,0,1\rangle : x\in X\}$;

Definition 2.3 [6]**:** Let X and Y be two nonempty sets and f: X$\rightarrow$Y be a function

(i) If $B=\{\langle y,\mu_B(x),\nu_B(x)\rangle : y\in Y\}$, is an IFS in Y, then the preimage of B under f is denoted and defined by $f^{-1}(B)=\{\langle x, f^{-1}(\mu_b)(x), f^{-1}(\nu_B)(x) : x\in X\}$;

(ii) If $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ is an IFS in X then the image of A under f is denoted and defined by $f(A)=\{\langle y, f(\mu_A)(y), f(\nu_A)(y)\rangle : y\in Y\}$ where

$$f(\mu_A)(y)=\begin{cases} sup_{x\in f^{-1}(y)}\mu_A(x) \ if\ f^{-1}(y)\neq 0 \\ 0\ otherwise, \end{cases}$$

and

$$f(\nu_A)(y)=\begin{cases} inf_{x\in f^{-1}(y)}\nu_A(x) \ if\ f^{-1}(y)\neq 0 \\ 1\ otherwise, \end{cases}$$

Definition 2.4 [6]**:** An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family τ of IFS's in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$.

(ii) $A_1\cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\cup A_j \in \tau$ for any $\{A_j : j\in J\}\subseteq \tau$.

Definition 2.5 [6]**:** The complement $\overline{A}$ of IFOS A in IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS, for short).

Definition 2.6 [6]**:** Let (X, τ) be an IFTS and A= $\langle x,\mu_A(x),\nu_A(x)\rangle\rangle$ be an IFS in X. Then the fuzzy interior and fuzzy closure of A is are denoted and defined by

cl(A)= $\cap$ {K:K is an IFCS in X and $A\subseteq K$}

int(A)= $\cup$ {G:G is an IFOS in X and $G\subseteq A$}

Note that, for any IFS A in (X,τ), we have cl($\bar{A}$)= $\overline{int(A)}$ and $int\overline{(A)}$=$\overline{cl(A)}$.

Definition 2.7 [8]**:** Let A be an IFS in an IFTS (X,τ), then A is

1. An *intuitionistic fuzzy semi open set* (IFSOS) if $A\subseteq cl(\text{int}(A))$.
2. An *intuitionistic fuzzy α-open set* (IFαOS) if $A\subseteq \text{int}(cl(\text{int}(A)))$.
3. An *intuitionistic fuzzy preopen set* (IFPOS) if $A\subseteq \text{int}(cl(A))$.

An IFS A is called an *intuitionistic fuzzy semiclosed set, intuitionistic fuzzy α-closed set, intuitionistic fuzzy preclosed set* (IFSCS, IFαCS, IFPCS),if the complement of A is an IFSOS, IFαOS, IFPOS, respectively.

Definition 2.8 [2]**:** Let A be an IFS in an IFTS (X,τ), then A is an *intuitionistic fuzzy d- open set* (IFdOS) if $A\subseteq scl(b\,\text{int}(A))\cup cl(\text{int}(A))$.

Definition 2.9: Let f be a mapping from an IFTS (X,τ) into IFTS (Y,κ). Then f is said to be

(i) intuitionistic fuzzy continuous [3] if $f^{-1}(B)\in IFO(X)$ for every $B\in\kappa$.

(ii) intuitionistic fuzzy semicontinuous [5] if $f^{-1}(B)\in IFSO(X)$ for every $B\in\kappa$.

(iii) intuitionistic fuzzy α-continuous [5] if $f^{-1}(B)\in IF\alpha O(X)$ for every $B\in\kappa$.

(iv) intuitionistic fuzzy precontinuous [5] if $f^{-1}(B)\in IFPO(X)$ for every $B\in\kappa$.

Definition 2.10 [2]**:** Let f be a mapping from an IFTS (X,τ) into IFTS (Y,κ). Then f is said to be intuitionistic fuzzy d-continuous mapping if $f^{-1}(B)\in IFDO(X)$ for every $B\in\kappa$.

Definition 2.11 [12]**:** A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ be an onto mapping from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,σ). Then f is said to be an intuitionistic fuzzy semi-quotient mapping if f is an intuitionistic fuzzy semicontinuous and (for all $G\in$IFS(Y)) $(f^{-1}(G)\in\tau\Rightarrow G\in IFSOS(Y))$.

Definition 2.12 [12]**:** A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ an onto mapping from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,σ). Then f is said to be an intuitionistic fuzzy α-quotient mapping if f is an intuitionistic fuzzy α-continuous and(for all $G\in$IFS(Y)) $(f^{-1}(G)\in\tau\Rightarrow G\in IF\alpha OS(Y))$.

Definition 2.13 [12]**:** A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ an onto mapping from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,σ). Then f is said to be an intuitionistic fuzzy strongly semi-quotient mapping if f it satisfies (for all $G\in$IFS(Y)) $(G\in IFOS(Y)\Leftrightarrow f^{-1}(G)\in IFSOS(X))$.

Definition 2.14 [12]**:** A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ an onto mapping from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,σ). Then f is said to be an intuitionistic fuzzy strongly α-quotient mapping if f it satisfies(for all $G\in$IFS(Y)) $(G\in IFOS(Y)\Leftrightarrow f^{-1}(G)\in IF\alpha OS(X))$.

3. Intuitionistic fuzzy d-quotient mapping

Definition 3.1: Let (X,τ) and (Y,σ) be intuitionistic fuzzy topological spaces. Let $f:X\rightarrow Y$ be an onto map. Then f is said to be an intuitionistic fuzzy d-quotient map if f is d-continuous and $f^{-1}(V)$ is intuitionistic fuzzy open in X implies V is an intuitionistic fuzzy d-open set in Y.

Definition 3.2: A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,σ) is said to be an intuitionistic fuzzy d-open mapping if f (A) is an IFdOS in Y for every IFOS A in X.

Example 3.3: Let $X=\{a,b,c\}$ and $Y=\{u,v\}$, $A=\langle x,(1,0.7,0,7),(0,0.2,0.2)\rangle$, $B=\langle x,(1,0.5,0,5),(0,0.4,0.4)\rangle$, $C=\langle x,(1,0.5),(0,0.4)\rangle$ where $\tau=\{0\sim,1\sim,A,B\}$ and $\sigma=\{0\sim,1\sim,C\}$ are intuitionistic fuzzy topology on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f (a) = u and f (b) = f(c) = v. Then $f^{-1}(0\sim)=0\sim$, $f^{-1}(1\sim)=1\sim$ and $f^{-1}(C)=B$ are intuitionistic fuzzy open set in X and hence intuitionistic fuzzy d-open set in X. Therefore f is an intuitionistic fuzzy d-continuous mapping. Clearly, $f^{-1}(0\sim)=0\sim$, $f^{-1}(1\sim)=1\sim$ are intuitionistic fuzzy open sets in X implies 0~ and 1~ are intuitionistic fuzzy d-open set in Y. Now $f^{-1}(C)=B$ is an intuitionistic fuzzy open set in X. Clearly dint(C) =C, therefore C is an intuitionistic fuzzy d-open set in Y. Therefore f is an intuitionistic fuzzy d-quotient mapping.

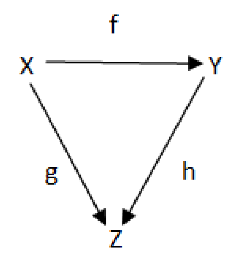
Theorem 3.4: Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy d-continuous and intuitionistic fuzzy d-open mapping from an intuitionistic fuzzy topological space (X,τ) onto an intuitionistic fuzzy topological space (Y,σ). Then f is an intuitionistic fuzzy d-quotient mapping.

Proof: Let $f^{-1}(B)$ be an intuitionistic fuzzy open set in X, for any intuitionistic fuzzy set B in Y. Then $f(f^{-1}(B))=B$ is an intuitionistic fuzzy d-open set in Y, as f is an intuitionistic fuzzy d-open and onto mapping. Hence f is an intuitionistic fuzzy d-quotient mapping.

Theorem 3.5: Let $f:(X,\tau)\to(Y,\sigma)$ be an onto intuitionistic fuzzy open and intuitionistic fuzzy d-irresolute mapping. If $g:(Y,\sigma)\to(Z,\delta)$ is an intuitionistic fuzzy d-quotient mapping then so is $g\circ f$.

Proof: Let B be an intuitionistic fuzzy open set in Z. Then $g^{-1}(B)$ is an intuitionistic fuzzy d-open set in Y as g is intuitionistic fuzzy d-quotient mapping. Since f is an intuitionistic fuzzy d-irresolute mapping. $f^{-1}(g^{-1}(B))=(g\circ f)^{-1}(B)$ is an intuitionistic fuzzy d-open set in X. So $g\circ f$ is an intuitionistic fuzzy d-continuous mapping. Suppose $(g\circ f)^{-1}(B)$ is an intuitionistic fuzzy open set in X. Then $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy open set in X. Since f is an intuitionistic fuzzy open and onto mapping $f(f^{-1}(g^{-1}(B)))=g^{-1}(B)$ is an intuitionistic fuzzy open set in Y. Since g is an intuitionistic fuzzy d-quotient mapping, B is an intuitionistic fuzzy d-open set in Z. Hence $g\circ f$ is an intuitionistic fuzzy d-quotient mapping.

Theorem 3.6: Let $(X,\tau),(Y,\sigma),(Z,\eta)$ be IFTS and let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy d-quotient mapping. IF $g:(X,\tau)\to(Z,\eta)$ is an intuitionistic fuzzy continuous mapping such that it is constant on each $f^{-1}(y)$ for $y\in Y$, then there exists an intuitionistic fuzzy d-continuous mapping $h:(Y,\sigma)\to(Z,\eta)$ such that the following diagram commutes:



Proof: Since g is constant on each $f^{-1}(y)$ for each $y\in Y$, the set $\{g(f^{-1}(y)\}$ is a singleton subset of Z. Let $h:(Y,\sigma)\to(Z,\eta)$ be a mapping defined by $h(y)=g(f^{-1}(y))$ for all $y\in Y$. Then clearly h is well defined and h (f(x)) = g(x) for all $x\in X$, that is $h\circ f=g$ .Let B be an IFOS in Z. Then $g^{-1}(B)$ is an IFOS in X, as g is an intuitionistic fuzzy continuous mapping. But $g^{-1}(B)=(hf)^{-1}(B)=(f^{-1}(h^{-1}(B)))$. Since f is an intuitionistic fuzzy d-quotient mapping it follows that $h^{-1}(B)$ is an IFdOS in Y. Hence, h is an intuitionistic fuzzy d-continuous mapping.

Theorem 3.7: If $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy semi-quotient mapping from an IFTS (X,τ) onto an IFTS (Y,σ), then f is an intuitionistic fuzzy d-quotient mapping.

Proof: Since f is an intuitionistic fuzzy semi-quotient mapping, f is an intuitionistic fuzzy semi continuous mapping. Since every semi-continuous is d-continuous, f is an intuitionistic fuzzy d-continuous mapping. Let $f^{-1}(B)$ is an IFOS in X. Then B is an IFSOS in Y as f is an intuitionistic fuzzy semi-quotient mapping. Since every IFSOS is an IFdOS, B is an intuitionistic fuzzy d-open set in Y. Hence, f is an intuitionistic fuzzy d-quotient mapping.

Theorem 3.8: If $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy α- quotient mapping from an IFTS (X,τ) onto an IFTS (Y,σ), then f is an intuitionistic fuzzy d-quotient mapping.

Proof: Similar to Theorem 3.7.

Definition 3.9: Let (X,τ) and (Y,σ) be intuitionistic fuzzy topological space. Let $f:(X,\tau)\to(Y,\sigma)$ be an onto map. Then f is said to be an intuitionistic fuzzy strongly d-quotient map provided a intuitionistic fuzzy subset A of Y is intuitionistic fuzzy open in Y if and only if $f^{-1}(\mathrm{A})$ is an intuitionistic fuzzy d-open set in X.

Example 3.10: Let $X=\{\mathrm{a,b,c}\}$, $Y=\{u,v\}$.
Let $A=\langle x,(0.6,0.6,0),(0.1,0.1,1)\rangle$ and $B=\langle y,(0.6,t),(0.1,s)\rangle$, where $t,s\in[0,1]$ with $t+s\le 1$. Let $\tau=\{0\sim,1\sim,A\}$ and $\sigma=\{0\sim,1\sim,B\}$ be IFT on X and Y respectively. Define a mapping $f:X\to Y$ by f(a) = f(b) =u and f(c)=v. Consider an IFS $G=\langle y,(0.7,0.4),(0.1,0.3)\rangle$ in Y. Then $f^{-1}(G)=\langle y,(0.7,0.7,0.4),(0.1,0.1,\ 0.3)\rangle$ is an intuitionistic fuzzy d-open set in X, but G is not an intuitionistic fuzzy open set in Y. Therefore f is not an intuitionistic fuzzy strongly d-quotient mapping.

Theorem 3.11: An intuitionistic fuzzy strongly semi-quotient map is an intuitionistic fuzzy strongly d-quotient map.
Proof: Let $f:X\to Y$ be an intuitionistic fuzzy strongly semi-quotient mapping. Clearly f is an intuitionistic fuzzy d-continuous mapping. Let B be an IFS in Y such that $f^{-1}(B)$ is an IFSOS in X. Since every intuitionistic fuzzy semi-open set is intuitionistic fuzzy d-open, $f^{-1}(B)$ is an IFdOS in X. Since f is an intuitionistic fuzzy strongly semi-quotient mapping, B is an IFOS in Y. Hence f is an intuitionistic fuzzy strongly d-quotient mapping.

Theorem 3.12: Every intuitionistic fuzzy strongly α-quotient mapping is an intuitionistic fuzzy strongly d-quotient mapping.

Proof; Similar to Theorem before.

Theorem 3.13: An intuitionistic fuzzy strongly d-quotient map is an intuitionistic fuzzy d-quotient map.
Proof: Let $f:X\to Y$ be an intuitionistic fuzzy strongly d-quotient mapping. Let B be an intuitionistic fuzzy open set in Y, by hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy d-open set in X. So f is an intuitionistic fuzzy d-continuous mapping. Let G be an intuitionistic fuzzy set in Y such that $f^{-1}(G)$ is an IFdOS in X. Since f is strongly d-quotient mapping G is an IFOS in Y which implies G is an IFdOS in Y. Hence f is an intuitionistic fuzzy d-quotient mapping.

Example 3.14: Let $X=\{\mathrm{a,b,c}\}$, $Y=\{u,v\}$.
Let $A=\langle x,(0.6,0.6,0),(0.1,0.1,1)\rangle$ and $B=\langle y,(0.6,t),(0.1,s)\rangle$, where $t,s\in[0,1]$ with $t+s\le 1$. Let $\tau=\{0\sim,1\sim,A\}$ and $\sigma=\{0\sim,1\sim,B\}$ be IFT on X and Y respectively. Define a mapping $f:X\to Y$ by f(a) = f(b) =u and f(c)=v. Consider an IFS $G=\langle y,(0.7,0.4),(0.1,0.3)\rangle$ in Y. Then $f^{-1}(G)=\langle y,(0.7,0.7,0.4),(0.1,0.1,\ 0.3)\rangle$ is an intuitionistic fuzzy d-open set in X, but G is not an intuitionistic fuzzy open set in Y. Therefore f is not an intuitionistic fuzzy strongly d-quotient mapping.

Definition 3.15: Let (X,τ) and (Y,σ) be intuitionistic fuzzy topological space. Let $f:(X,\tau)\to(Y,\sigma)$ be an onto map. Then f is said to be an intuitionistic fuzzy ds-quotient map if f is an intuitionistic fuzzy d-irresolute map and $f^{-1}(U)$ is an intuitionistic fuzzy d-open set in X implies U is an intuitionistic fuzzy open set in Y.

Definition 3.16: Let (X,τ) and (Y,σ) be intuitionistic fuzzy topological space. A function $f:(X,\tau)\to(Y,\sigma)$ is called an intuitionistic fuzzy strongly d-open map if the image of every fuzzy d-open set in X is an intuitionistic fuzzy d-open set in Y.

Theorem 3.17: Let (X,τ), (Y,σ) and (Z,η) be intuitionistic fuzzy topological spaces. Let $f:(X,\tau)\to(Y,\sigma)$ be an onto intuitionistic fuzzy strongly d-open map and an intuitionistic fuzzy d-irresolute map. Let $g:(Y,\sigma)\to(Z,\eta)$ be an intuitionistic fuzzy ds-quotient map. Then $g\circ f$ is an intuitionistic fuzzy ds-quotient map.

Proof: We claim $g\circ f$ is an intuitionistic fuzzy d-irresolute map. Let V be an intuitionistic fuzzy d-open set in Z. Then $g^{-1}(V)$ is an intuitionistic fuzzy d-open set in Y as g is an intuitionistic fuzzy ds-quotient map. Since f is an intuitionistic fuzzy d-irresolute map $f^{-1}(\mathrm{g}^{-1}(\mathrm{V}))=(\mathrm{g}\circ \mathrm{f})^{-1}(\mathrm{V})$ is an intuitionistic fuzzy d-open set in X. So, $g\circ f$ is an intuitionistic fuzzy d-irresolute map. Suppose $f^{-1}(\mathrm{g}^{-1}(\mathrm{V}))=(\mathrm{g}\circ \mathrm{f})^{-1}(\mathrm{V})$ is an intuitionistic fuzzy d-open set in X. Since f is fuzzy strongly d-open set, $f(f^{-1}(\mathrm{g}^{-1}(\mathrm{V})))$ is an intuitionistic fuzzy d-open set in Y. Since f is an onto map $f(f^{-1}(\mathrm{g}^{-1}(\mathrm{V})))=\mathrm{g}^{-1}(\mathrm{V})$. So $\mathrm{g}^{-1}(\mathrm{V})$ is an intuitionistic fuzzy d-open set Y. This implies that V is an intuitionistic fuzzy open set in Z as g is an intuitionistic fuzzy ds-quotient map. Hence $g\circ f$ is an intuitionistic fuzzy ds-quotient map.

Theorem 3.18: Every intuitionistic fuzzy ds-quotient mapping is an intuitionistic fuzzy strongly d-quotient mapping.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy ds-quotient mapping and let B be an intuitionistic fuzzy open set in Y. Then $f^{-1}(\mathrm{B})$ is an intuitionistic fuzzy d-open set in X, as f is an intuitionistic fuzzy d-irresolute mapping. Since f is an intuitionistic fuzzy ds-quotient mapping it follows that B is an intuitionistic fuzzy open set in Y. Hence f is an intuitionistic fuzzy strongly d-quotient mapping.

Theorem 3.19: Let $f:(X,\tau)\to(Y,\sigma)$ be an onto intuitionistic fuzzy ds-open map and an intuitionistic fuzzy d-irresolute map. Let $g:(Y,\sigma)\to(Z,\eta)$ be an intuitionistic fuzzy ds-quotient map. Then $g \circ f$ is an intuitionistic fuzzy strongly ds-quotient map.

Proof: Follows from Theorem 3.17 and Theorem 3.18.

Theorem 3.20: Every intuitionistic fuzzy quotient mapping is an intuitionistic fuzzy d-quotient mapping but not conversely.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy quotient mapping. Then f is an intuitionistic fuzzy continuous mapping and hence an intuitionistic fuzzy d-continuous mapping. Let B be an intuitionistic fuzzy set in Y such that, $f^{-1}(\mathrm{B})$ is an intuitionistic fuzzy open set in X. Since f is an intuitionistic fuzzy quotient mapping, B is an intuitionistic fuzzy open set in Y. Hence B is an intuitionistic fuzzy d-open set in Y and therefore f is an intuitionistic fuzzy d-quotient mapping.

Example 3.21: In Example 3.3 clearly f is an intuitionistic fuzzy d-quotient mapping. Let $G=\langle x,(0.1,0.7),(0,0.2)\rangle$ be any intuitionistic fuzzy set in Y, $f^{-1}(G)=\langle x,(0.1,0.7,0.7),(0,0.2,0.2\rangle$ is an intuitionistic fuzzy open set in X. But G is not an intuitionistic fuzzy open set in Y. Hence f is not an intuitionistic fuzzy quotient mapping.

Remark 3.22: Every intuitionistic fuzzy quotient mapping need not be an intuitionistic fuzzy strongly d-quotient mapping.

Example 3.23: Let $X=\{\mathrm{a,b,c}\}$, $Y=\{u,v\}$. $A=\langle x,(0.1,0.6,0.6),(0,0.4,0.4)\rangle$ and $B=\langle y,(1,0.6),(0,0.4)\rangle$. Let $\tau=\{0\sim,1\sim,A\}$ and $\sigma=\{0\sim,1\sim,B\}$ be IFTs on X and Y respectively. Define a mapping $f:X\to Y$ by f(a) = u and f(b) =f(c)=v. Clearly $f^{-1}(0\sim)=0\sim,\mathrm{f}^{-1}(1\sim)=1\sim$ and $f^{-1}(\mathrm{B})=\mathrm{A}$ are intuitionistic fuzzy open sets in X. Therefore f is an intuitionistic fuzzy continuous mapping. Also for an intuitionistic fuzzy set G in Y such that G is an IFOS in Y. Hence f is an intuitionistic fuzzy quotient mapping. Let $G=\langle x,(1,0.7),(0,0.2)\rangle$ be an IFS in Y. Since $d\,\mathrm{int}(f^{-1}(\mathrm{B}))=f^{-1}(\mathrm{B})$, $f^{-1}(\mathrm{B})$ is an intuitionistic fuzzy d-open set in X, but G is not an intuitionistic fuzzy open set in Y. Hence, f is not an intuitionistic fuzzy strongly d-quotient mapping.

Remark 3.24: An intuitionistic fuzzy quotient mapping need not be an intuitionistic fuzzy ds-quotient mapping.

Example 3.25: In Example 3.23 the mapping f is an intuitionistic fuzzy quotient mapping. Let $G=\langle x,(1,0.7),(0,0.2)\rangle$ be an IFS in Y. Since $d\,\mathrm{int}(f^{-1}(\mathrm{B}))=f^{-1}(\mathrm{B})$, $f^{-1}(\mathrm{B})$ is an intuitionistic fuzzy d-open set in X, but G is not an intuitionistic fuzzy open set in Y. Hence, f is not an intuitionistic fuzzy ds-quotient mapping.

Theorem 3.26: Let $(X,\tau),(Y,\sigma)$ be intuitionistic fuzzy topological spaces. I $f:(X,\tau^d)\to(Y,\sigma^d)$ is an intuitionistic fuzzy quotient map then $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy d-quotient map.

Proof: Let $V\in\sigma$. So $V\in\sigma^d$. Since f is an intuitionistic fuzzy quotient map $f^{-1}(V)\in\tau^d$. Hence it is proved that when V is an intuitionistic fuzzy open set in Y, then $f^{-1}(V)$ is an intuitionistic fuzzy d-open set in X. So f is an intuitionistic fuzzy d-continuous map. Suppose $f^{-1}(V)$ is an intuitionistic fuzzy open set in (X,τ) then $f^{-1}(V)\in\tau^d$. Since f is an intuitionistic fuzzy quotient map, $V\in\sigma^d$ and so V is an intuitionistic fuzzy d-open set in Y. Hence $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy d-quotient map.

Definition 3.27: Let $(X,\tau),(Y,\sigma)$ and (Z,η) be intuitionistic fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is called intuitionistic fuzzy quasi d-open if the image of every d-open set in X is an intuitionistic fuzzy open set in Y.

Theorem 3.28: Let $(X,\tau),(Y,\sigma)$ be intuitionistic fuzzy topological spaces. If $f:(X,\tau^d)\to(Y,\sigma^d)$ is an intuitionistic fuzzy quasi d-open map then $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic fuzzy strongly d-open map.

Proof: Let V be an intuitionistic fuzzy d-open set in τ. So $V\in\tau^d$. Since $f:(X,\tau^d)\to(Y,\sigma^d)$ is intuitionistic fuzzy quasi d-open, f(V) is intuitionistic fuzzy open in (Y,σ^{α}). So f(V) is an intuitionistic fuzzy d-open set in (Y,σ). Hence it follows that $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic fuzzy strongly d-open map.

References

1. Ahn BH, Cho YU, Jun YB, Intuitionistic fuzzy irresolute and quotient mapping, Far East Jour Math Sci. 2005; 17:189-200
2. Arockiarani I, Jude Immaculate H. A Note on intuitionistic fuzzy d-continuous mappings, Proceeding of National Seminar on Recent Development in Topology, 2015, 236-244.
3. Atanassov K. Intuitionistic fuzzy sets, Fuzzy Sets and System 1986; 20:87-96.
4. Caldas M. Weak and strong forms of irresolute maps, International Journal of Mathematics and Mathematical Sciences. 2000; 23(4):253-259.
5. Caldas M, Jafari S. Weak and Strong Forms of β-Irresoluteness, The Arabian Journal for Science and Engineering. 2006; 31(1A):31-39.

6. Chang CL. Fuzzy topological spaces, J Math Anal Appl. 1968; 24:182-190.
7. Coker D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems 1997; 88:81-89.
8. Ekici E, Kerre E. On fuzzy contra-continuities, Advances in Fuzzy Mathematics 2006; 1:35-44.
9. Gurcay H, Coker D. On fuzzy continuity in intuitionistic fuzzy topological spaces, J Fuzzy Math. 1997; 5:365-378.
10. Hanafy IM, El-Arish. Completely continuous functions in intuitionistic fuzzy topological spaces, Czechoslovak Mathematical Journal, 2003, 793-803.
11. Joung Kon Jeon, Young Bae Jun, Jin Han Park. Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, 2005, 3091-3101.
12. Jun YB, Kim JH, Roh EH. Intuitionistic fuzzy quotient mapping set, Far East Jour. Math. Sci. 2005; 17:175-188.
13. Krsteska B, Ekici E. Intuitionistic fuzzy contra strong precontinuity, Faculty of Sciences and Mathematics, University of Nis, Siberia, 2007, 273-284.
14. Ramakrishna PV, Regina Mary A. On fuzzy quotient mapping, Bull. Math. Sci. Soc 2005; 28:215-221.
15. Rajarajeswari P, Senthil Kumar L. Intuitionistic Fuzzy Completely Regular Weakly Generalized Continuous Mappings, International Journal of Applied Information Systems. 2012; 3:34-38.
16. Saraf RK, Seema Mishra, Caldas M. Weak and strong forms of Irresolute Fuzzy maps, Proc. Pakistan Acad. Sci 2008; 45(2):75-80.
17. Seok Jong Lee, Eun Pyo Lee. The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc, 2000, 63-76.
18. Shyla Isac Mary T, Thangavelu P. On Regular Pre-Semi closed Sets in Topological Spaces, KBM Journal of Mathematical Sciences & Computer Applications. 2010; 1:9-17.
19. Thakur SS, Rekha Chaturvedi. Regular Generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau Studii Si Cercertari Stiintifice, 2006; 6:257-272.
20. Thakur SS, Singh S. On fuzzy semi-preopen sets and fuzzy semi-precontinuity, Fuzzy sets and systems, 1998, 383-391.
21. Young Bae Jun, Jung Ok Kang, Seok Zun Kang. Intuitionistic fuzzy irresolute and continuous mappings, Far East J Math Sci [FJMS]. 2005; 17(2):201-216.
22. Zadeh LA. Fuzzy sets, Information and Control, 1965; 9:338-353.