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Isomorphism and Antiisomorphism in (S, Q)-Fuzzy Translation of (S, Q)-Fuzzy Subhemirings of a Hemiring

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Abstract
In this paper, we made an attempt to study the algebraic nature of a(S, Q)-fuzzy subhemiring of a Hemiring.
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Keywords: (S,Q)-fuzzy set, (S, Q)-fuzzy subhemiring, (S, Q)- Fuzzy Translation.

Introduction
There are many concepts of universal algebras generalizing an associative ring (R; +;.). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras (R; + ;.) share the same properties as a ring except that (R; +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R; +;.) is said to be a semi ring (R; +) and (R;.) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemi ring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [16], several researchers explores on the generalization of the concept of fuzzy sets. Osman Kazanci, Sultan yamark and serifeyilmaz in [11] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and R. Nagarajan, have given a new structure in construction of Q-fuzzy groups and subgroups [14, 15]. In this paper, we introduce some properties and theorems in (S,Q)-fuzzy subhemirings of a hemiring.

1. Preliminaries
1.1 Definition: A S-norm is a binary operation S: $[0,1] X [0,1] \rightarrow [0,1]$ satisfying the following requirements:
(i) 0 S x = x, 1 S x=1(boundary conditions)
(ii) x S y = y S x (commutativity)
(iii) x S (y S z)= (x S y) S z (associativity)
(iv) If x £ y and w £ z, then x S w £ y S z (monotonicity).

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is function
$A: X \times Q \rightarrow [0,1]$.

1.3 Definition: The union of two (S,Q)-fuzzy sets A and B of a set X is defined by $(A \cup B)(x, q) = \max\{S(A(x, q)), S(B(x, q))\}$for all x in X and q in Q.

1.4 Definition: The intersection of two (S,Q)-fuzzy sets A and B of a set X is defined by $(A \cap B)(x, q) = \min\{S(A(x, q)), S(B(x, q))\}$ for all x in X and q in Q.

1.5 Definition: Let (R, +,.) be a hemiring. A (S,Q)-fuzzy subset A of R is said to be a (S,Q)-fuzzy subhemiring(SQFSHR) of R if it satisfies the following conditions:
(i) $\mu_A(x+y,q) \geq S\big((\mu_A(x,q),\mu_A(y,q)\big)$
(ii) $\mu_A(xy,q) \geq S\big((\mu_A(x,q),\mu_A(y,q)\big)$, for all x and y in R, and q in Q.

1.6 Definition: Let (R, +,.) be a hemiring. A (S,Q)-fuzzy subhemiring A of R is said to be a (S,Q)-fuzzy normal subhemiring(SQFNSHR) of R if$S(\mu_A(xy,q)) = S(\mu_A(yx,q))$, for all x and y in R, and q in Q.

1.7 Definition: Let (R, +,.) and $(R',+,.)$ be any two hemirings. Then the function $f: R \to R'$is called a hemiring homomorphism if it satisfies the following axioms:
i) $f(x+y) = f(x) + f(y)$,
ii) $f(xy) = f(x)f(y)$,for all x and y in R.

1.8 Definition: Let (R, +,.) and $(R',+,.)$ be any two hemirings. Then the function $f: R \to R'$is called a hemiring anti-homomorphism if it satisfies the following axioms:
i) $f(x+y) = f(y) + f(x)$,
ii) $f(xy) = f(y)f(x)$,For all x and y in R.

1.9 Definition: Let (R, +,.) and $(R',+,.)$ be any two hemirings. Then the function $f: R \to R'$be a hemiring homomorphism. If f is one-to-one and onto, then f is called a hemiring isomorphism.

1.10 Definition: Let (R, +,.) and $(R',+,.)$ be any two hemirings. Then the function $f: R \to R'$be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a hemiring anti-isomorphism.

1.11 Definition: Let A be a (S,Q)-fuzzy subset of X and $\alpha \in [0,1-\mathrm{Sup}\{A(x,q): x \in X, 0 < A(x,q) < 1\}]$. Then $T = {T_\alpha}^A$ is called a (S,Q)-fuzzy translation of A if$S\big(T(x,q)\big) = S(A(x,q)+\alpha)$, for all x in X.

2. Isomorphism and Antiisomorphism in (S,Q)-Fuzzy Translation of(S, Q)-Fuzzy Subhemirings of A Hemiring

2.1 Theorem: Let (R, +,.) and (R′, +,.) be any two hemirings. The (S,Q)-fuzzy normal subhemiring off(R)=R′ under the anti-homomorphic preimage is a (S,Q)-fuzzy normal subhemiring of R.

Proof: Let (R, +,.) and (R′, +,.) be any two hemirings. Let$f: R \to R'$ be an anti- homomorphism. Then $f(x+y) = f(y) + f(x)$,and $f(xy) = f(y)f(x)$ for all x and y in R. Let V bea (S,Q)-fuzzynormal subhemiring off(R)=R′ and A be an anti-homomorphic pre-image ofV under f. we have to prove that A is a (S,Q)-fuzzy normal subhemiring of hemiring R. Letx and y in R & q in $Q. Then, clearly$ A is $a(S,Q)$fuzzy subhemiring of the hemiring R. since V is a$(S,Q) -$ fuzzy subhemiring of the hemiring R′. Now, $S(\mu_A(xy,q)) = S(\mu_V\big((f(xy),q)\big))$, since $S(\mu_A(x,q)) = S(\mu_V\big((f(x),q)\big)$ $= S\big(\mu_V(f(y)f(x),q)\big)$as f isananti-homomorphism $= S(\mu_V(f(x)f(y),q) = S(\mu_V\big((f(yx),q)\big)$ as f is an anti-homomorphism= $S(\mu_A(yx,q))$, since $S(\mu_A(xy,q)) = S(\mu_A(yx,q))$, whichimpliesthat $S(\mu_A(xy,q)) = S(\mu_A(yx,q))$ for all x and y in R and q in Q.Hence A is a (S, Q) fuzzy normal subhemiring of hemiring R

In the following Theorem ∘ is the composition operation of functions:

2.2 Theorem: Let A be a (S, Q)-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemi ring R onto H. If A be a (S, Q)-fuzzy normal subhemiring of hemiring H, then A∘ f is a (S, Q)-fuzzy normal subhemiring ofthe hemiring R.

Proof:Let xand y in R and q in Q and A be a (S, Q) fuzzy normal subhemiring of hemiring H. Then we have, ClearlyA∘ fisa(S, Q)-fuzzysubhemiringofthehemiringR.Now, $S\left((\mu_A \circ f)\big((xy,q)\big)\right) = S\left(\mu_A\big((f(xy),q)\big)\right) = S\big(\mu_A(f(x)f(y),q)\big)$ as f is an isomorphism= $S\big(\mu_A(f(y)f(x),q)\big) = S\left(\mu_A\big((f(yx),q)\big)\right)$ as f is an isomorphism= $S\left((\mu_A \circ f)\big((yx,q)\big)\right)$ for all x and y in R and q in Q. Therefore A∘f is a (S, Q) fuzzy normal subhemiring of the hemiring R.

2.3 Theorem: Let A be a (S, Q)-fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemi ring R onto H. If A be a (S, Q)-fuzzy normal subhemiring of hemiring H, then A∘ f is a (S, Q)-fuzzy normal subhemiring ofthe hemiring R.

Proof: Let xand y in R and q in Q and A be a (S, Q) fuzzy normal subhemiring of hemiring H. Then we have,
Clearly A∘f is a (S, Q)-fuzzy subhemiring of the hemiringR. Now, $S\left((\mu_A \circ f)\big((xy,q)\big)\right) = S\left(\mu_A\big((f(xy),q)\big)\right)$ $= S\big(\mu_A(f(y)f(x),q)\big)$as f is an anti- isomorphism= $S\big(\mu_A(f(x)f(y),q)\big) = S\left(\mu_A\big((f(yx),q)\big)\right)$ as f is an anti- isomorphism $= S\left((\mu_A \circ f)\big((yx,q)\big)\right)$, which implies that $S\left((\mu_A \circ f)\big((xy,q)\big)\right) = S\left((\mu_A \circ f)\big((yx,q)\big)\right)$, for all x and y in R and q in Q. Therefore A∘f is a (S, Q) fuzzy normal subhemiring of the hemiring R.

2.4 Theorem: If M and N are two (S, Q)-fuzzy translations of (S, Q) fuzzy normal subhemiring A of a hemiring (R, +,.),then their intersection $M \cap N$ is a (S,Q)-fuzzy translation of A.

Proof: It is trivial.

2.5 Theorem: The intersection of family of (S, Q)-fuzzy translations of (S, Q) fuzzy normal subhemiring A of a hemiring (R, +,.) is a (S,Q)-fuzzy translation of A.

Proof: It is trivial.

2.6 Theorem: If M and N are two (S,Q)-fuzzy translations of (S, Q) fuzzy normal subhemiring A of a hemiring (R,+,.),then their union $M \cup N$ is a (S,Q)-fuzzy translation of A.

Proof: It is trivial.

2.7 Theorem: The union of family of (S, Q)-fuzzy translations of (S, Q) fuzzy normal subhemiring A of a hemiring (R, +,.) is a (S,Q)-fuzzy translation of A.

Proof: It is trivial.

2.8 Theorem: Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If $f: R \to R'$ is a homomorphism, Then (S,Q)-fuzzy translation of a (S,Q)-fuzzynormal subhemiring A of R under the homomorphic image is (S,Q)-fuzzy normal subhemiring off(R)=R'.

Proof: Let (R, +,.) and (R′, +,.) be any two hemirings and Q be a non-empty set and f: R → R′ behomomorphism. That is f(x +y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let $T = T_{\alpha}{}^{A}$ be the (S,Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring of A of R and V be the homomorphic image of T under f. We have to prove that Visa (S,Q) – fuzzy normal subhemiring of R′.Now, f(x) and f(y) in R′ and q in Q. Then clearly, V is a $(S,Q) -$ fuzzy subhemiring of the hemiring R′. Now, $S(V(f(x)f(y),q)) = S(V(f(xy),q)) \geq S(T(xy,q)) = S(A(xy,q)+\alpha) = S(A(yx,q)+\alpha) = S(T(yx,q)) \leq S(V(f(yx),q)) = S(V(f(y)f(x),q))$, which implies that $S(V(f(x)f(y),q)) = S(V(f(y)f(x),q))$ for all f(x) and f(y) in R′ and q in Q. Therefore V is a (S,Q)-fuzzy normal subhemiring of the hemiring R′.

2.9 Theorem: Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If $f: R \to R'$ is a homomorphism,Then (S,Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring V of f(R)=R' under the homomorphic pre-image is (S,Q)-fuzzy normal subhemiring ofR.

Proof: Let (R, +,.) and (R′, +,.) be any two hemirings and Q be a non-empty set and f: R → R′ behomomorphism. That is f(x +y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let $T = T_{\alpha}{}^{V}$ be the (S,Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring of V of R′and A be the homomorphic pre-image of T under f. We have to prove that A isa (S,Q) – fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly, A is a $(S,Q) -$ fuzzy subhemiring of the hemiring R. Now, $S(A(xy,q)) = S(T(f(xy),q)) = S(V(f(xy),q)+\alpha) = S(V(f(x)f(y),q)+\alpha) = S(V(f(y)f(x),q)+\alpha) = S(V(f(yx),q)+\alpha) = S(T(f(yx),q)) = S(A(yx,q))$, which implies that $S(A(xy,q)) = S(A(yx,q))$, for all x and y in R and q in Q. Therefore, Ais a (S,Q)-fuzzy normal subhemiring of the hemiring R.

2.10 Theorem: Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If $f: R \to R'$ is a anti-homomorphism, Then (S,Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring A of R under the anti- homomorphic image is (S,Q)-fuzzy normal subhemiring off(R)=R'.

Proof: Let (R, +,.) and (R′, +,.) be any two hemirings and Q be a non-empty set and f: R → R′ bean anti-homomorphism. That is f(x +y) = f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R. Let $T = T_{\alpha}{}^{A}$ be the (S,Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring of A of R and V be the anti- homomorphic image of $T_{\alpha}{}^{A}$ under f. We have to prove that Visa (S,Q) – fuzzy normal subhemiring of f(R)= R′.Now for,f(x) and f(y) in R′ and q in Q. Then clearly, V is a $(S,Q) -$ fuzzy subhemiring of the hemiring R′. Now, $S(V(f(x)f(y),q)) = S(V(f(yx),q)) \geq S(T(yx,q)) = S(A(yx,q)+\alpha) = S(A(xy,q)+\alpha) = S(T(xy,q)) \leq S(V(f(xy),q)) = S(V(f(y)f(x),q))$, which implies that $S(V(f(x)f(y),q)) = S(V(f(y)f(x),q))$, for all f(x) and f(y) in R′ and q in Q. Therefore V is a (S,Q)-fuzzy normal subhemiring of the hemiringR′.

2.11 Theorem: Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If $f: R \to R'$ is a anti-homomorphism, then (S,Q)-fuzzy translation of a (S,Q)-fuzzynormal subhemiring V of f(R)=R′ under the anti-homomorphic pre-image is (S,Q)-fuzzy normal subhemiring ofR.

Proof: Let (R, +,.) and (R′, +,.) be any two hemirings and Q be a non-empty set and f: R → R′ bean anti-homomorphism. That is f(x +y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let $T = T_{\alpha}{}^{V}$ be the (S, Q)-fuzzy translation of a (S,Q)-fuzzy normal subhemiring of V of R′and A be the anti-homomorphicpre-image of T under f. We have to prove that A is a (S, Q) – fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly, A is a (S,Q)fuzzy subhemiring of the hemiring R.

Now, $S(A(xy,q)) = S(T(f(xy),q)) = S(V(f(xy),q)+\alpha) = S(V(f(y)f(x),q)+\alpha) = S(V(f(x)f(y),q)+\alpha) = S(V(f(yx),q)+\alpha) = S(T(f(yx),q)) = S(A(yx,q))$, which implies that $S(A(xy,q)) = S(A(yx,q))$, for all x and y in R and q in Q. Therefore, Ais a (S,Q)-fuzzy normal subhemiring of R.

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