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Application, properties and structure of soft semigroup

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Abstract
A soft semigroup is an algebraic structure that extends the concept of a traditional semigroup by incorporating soft sets, which are a generalization of classical sets that allow for the representation of uncertainty and vagueness. In a soft semigroup, the binary operation is defined over soft elements rather than traditional elements. The application of soft semigroups is diverse and spans various fields, including decision-making, expert systems, and data analysis, where uncertainties and imprecisions are present. Soft semigroups provide a powerful framework for modeling situations where the exact membership of elements in a set is not well-defined. The properties of soft semigroups encompass those of conventional semigroups, such as closure under the operation, associativity, and the existence of an identity element. However, they also introduce additional features due to the soft nature of elements, such as the manipulation of membership degrees and the propagation of uncertainty through operations. The structure of a soft semigroup consists of a non-empty set of soft elements equipped with a binary operation that respects the soft set operations, like union and intersection. Soft semigroups can be represented through matrices, mappings, or algebraic equations, showcasing their flexibility in modeling diverse scenarios with uncertain information. In soft semigroups provide a theoretical foundation and practical tools for handling uncertain and vague information within a semigroup framework. Their applications range from decision systems to data analysis, and their properties and structural components enable the manipulation and propagation of uncertainty in various contexts.

Keywords: Generalization, uncertainty and vagueness, traditional elements, matrices, mappings

Introduction
A soft semigroup is a mathematical structure that extends the concept of a traditional semigroup by incorporating the notion of "softness," which allows for degrees of membership or participation of elements in the operation. Soft semigroups find applications in various fields, including decision-making, uncertainty modeling, and fuzzy mathematics. Here's the definition and some properties of a soft semigroup:

Definition
A soft semigroup is defined as a set S equipped with a binary operation $\ast: S \times S \to S$ and a membership function $\mu: S \to [0, 1]$. The membership function assigns degrees of belonging to elements in S, indicating the extent to which each element participates in the semigroup operation.

In other words, a soft semigroup consists of two components
1. A binary operation $\ast$ that combines elements from S in the usual way.
2. A membership function μ that quantifies the degree of each element's involvement in the operation.

Properties
1. Soft Associativity: A soft semigroup satisfies soft associativity, which means that the operation is compatible with the membership degrees. Mathematically, for all a, b, c in S, the following condition holds:

$$\mu(a) \ast (\mu(b) \ast \mu(c)) \geq \mu((a \ast b) \ast c).$$

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This condition ensures that the degree of participation in the left-hand side is greater than or equal to the degree of participation in the right-hand side.

2. Monotonicity: The membership function μ is monotonic, meaning that if a is more strongly related to the operation than b (i.e., $\mu(a) \geq \mu(b)$), then a should have a greater degree of involvement in the operation than b.

3. Closure Property: Just like traditional semigroups, a soft semigroup maintains closure under the operation. This means that applying the operation to any pair of elements in S results in an element that is still in S.

4. Identity Element (Optional): A soft semigroup may or may not have an identity element. If an identity element exists, it is an element that, when combined with any other element using the operation, leaves the other element unchanged.

5. Invertibility (Optional): Similar to identity, invertibility might or might not be present in a soft semigroup. If an element has an inverse, combining it with another element using the operation should yield the identity element.

Applications

1. **Decision Analysis:** Soft semigroups are used to model situations where the level of participation or preference of decision criteria varies.
2. **Fuzzy Mathematics:** Soft semigroups are related to fuzzy mathematics, which deals with uncertainty and degrees of truth.
3. **Uncertainty Modeling:** Soft semigroups find applications in modeling uncertain data or scenarios where precise information is lacking.
4. **Fuzzy Control:** In control systems, soft semigroups can help model complex relationships involving degrees of control actions.

Soft semigroups provide a framework to incorporate softness and degrees of membership into the traditional concept of semigroups. The membership function allows for a more nuanced representation of elements' participation in the operation, making it suitable for scenarios involving varying levels of involvement or uncertainty.

1. Soft Natural Numbers Addition

Consider the set of natural numbers $\mathbb{N}$. Define the membership function $\mu: \mathbb{N} \to [0, 1]$ such that $\mu(n) = \frac{1}{n}$. The operation $\ast$ is standard addition. This soft semigroup reflects that larger natural numbers have smaller degrees of participation in the addition.

Example 1

Consider the set of natural numbers $\mathbb{N}$. Define the membership function $\mu: \mathbb{N} \to [0, 1]$ such that $\mu(n) = \frac{1}{n}$. Perform the soft addition operation $\ast$ for $a = 3$ and $b = 5$. Calculate the resulting value using the given membership function.

Answer:

Given: $a = 3$, $b = 5$, and $\mu(n) = \frac{1}{n}$.

To perform the soft addition $\ast$ for a and b, we need to calculate $\mu(a) \ast (a + b)$ using the given membership function.

1. Calculate $\mu(a) = \frac{1}{3}$.
2. Calculate $a + b = 3 + 5 = 8$.
3. Multiply $\mu(a)$ and $(a + b)$:

$$\frac{1}{3} \ast 8 = \frac{8}{3}.$$

So, the result of the soft addition operation $\ast$ for $a = 3$ and $b = 5$ is $\frac{8}{3}$.

In this example, the larger number $b = 5$ has a smaller degree of membership ($\mu(b) = \frac{1}{5}$), and therefore contributes less to the soft addition. The result $\frac{8}{3}$ indicates that the contribution of b to the addition is moderated by its smaller degree of membership.

2. Soft Matrix Multiplication

In the set of square matrices, the membership function could represent the condition number of each matrix. The operation $\ast$ could be matrix multiplication. Matrices with smaller condition numbers have a higher degree of participation in the multiplication.

Example 2

Consider two 2×2 matrices, A and B, defined as follows:

Matrix A =```| 2 1 || 3 4 |```

Matrix B =```| 5 6 || 7 8 |```

Additionally, the membership function μ assigns degrees of belonging based on the determinant of each matrix. For matrix A, $\mu(A) = \frac{1}{\det(A)}$, and for matrix B, $\mu(B) = \frac{1}{\det(B)}$. Calculate the result of the soft matrix multiplication $\ast$ for A and B using the given membership function.

Answer:

Given: Matrix A and $\mu(A) = \frac{1}{\det(A)}$, Matrix B and $\mu(B) = \frac{1}{\det(B)}$.

To perform the soft matrix multiplication $\ast$ for A and B, we need to calculate $\mu(A) \ast (A \times B)$ using the given membership function.

1. Calculate the determinant of matrix A:
   $$\det(A) = (2 \times 4) - (1 \times 3) = 8 - 3 = 5.$$
2. Calculate the determinant of matrix B:
   $$\det(B) = (5 \times 8) - (6 \times 7) = 40 - 42 = -2.$$
3. Calculate $\mu(A) = \frac{1}{\det(A)} = \frac{1}{5}$ and $\mu(B) = \frac{1}{\det(B)} = -\frac{1}{2}$.
4. Calculate the matrix product $A \times B$:
   ``` A × B =
   | 2 1 | × | 5 6 | = | (2 × 5) + (1 × 7) (2 × 6) + (1 × 8) | = | 19 20 |
   | 3 4 | | 7 8 | | (3 × 5) + (4 × 7) (3 × 6) + (4 × 8) | | 43 50 | ```
5. Calculate the soft matrix multiplication $\mu(A) \ast (A \times B)$:
   $$\mu(A) \ast (A \times B) = \frac{1}{5} \ast \begin{bmatrix} 19 & 20 \\ 43 & 50 \end{bmatrix} = \begin{bmatrix} \frac{19}{5} & \frac{20}{5} \\ \frac{43}{5} & \frac{50}{5} \end{bmatrix}.$$
   So, the result of the soft matrix multiplication $\ast$ for matrices $A$ and $B$ is:
   ```| 19/5 4 | | 43/5 10 |```

In this example, the membership degrees (\(\mu(A)\) and \(\mu(B)\)) affect the contribution of each matrix to the soft matrix multiplication, resulting in a matrix with elements adjusted according to their determinant-based degrees of belonging.

3. Soft Set Intersection

Consider the set of all subsets of a given universal set. Define the membership function such that \(\mu(A) = \frac{|A|}{|U|}\), where \(|A|\) is the size of set \(A\) and \(|U|\) is the size of the universal set. The operation \(\ast\) could be set intersection.

Example -3

Consider two soft sets \(A\) and \(B\) defined over the universal set \(U = \{1, 2, 3, 4, 5\}\). The membership functions for \(A\) and \(B\) are given as follows:

Membership function for set \(A\)
\[\begin{align*}
\mu_A(1) &= 0.8 \\
\mu_A(2) &= 0.6 \\
\mu_A(3) &= 0.4 \\
\mu_A(4) &= 0.7 \\
\mu_A(5) &= 0.5 \\
\end{align*}\]

Membership function for set \(B\)
\[\begin{align*}
\mu_B(1) &= 0.7 \\
\mu_B(2) &= 0.5 \\
\mu_B(3) &= 0.3 \\
\mu_B(4) &= 0.8 \\
\mu_B(5) &= 0.6 \\
\end{align*}\]

Calculate the soft intersection \(\ast\) of sets \(A\) and \(B\) using the given membership functions.

Answer

Given: Membership functions for sets \(A\) and \(B\).
To calculate the soft intersection \(\ast\) of sets \(A\) and \(B\), we need to determine the minimum membership value for each element in the intersection.
1. Calculate the soft intersection \(\ast\) for each element \(x\) in the universal set \(U\) using the formula:
\[\mu_{A \ast B}(x) = \min\{\mu_A(x), \mu_B(x)\}.\]
Calculating for each element:

- For \(x = 1\), \(\mu_{A \ast B}(1) = \min\{0.8, 0.7\} = 0.7\).
- For \(x = 2\), \(\mu_{A \ast B}(2) = \min\{0.6, 0.5\} = 0.5\).
- For \(x = 3\), \(\mu_{A \ast B}(3) = \min\{0.4, 0.3\} = 0.3\).
- For \(x = 4\), \(\mu_{A \ast B}(4) = \min\{0.7, 0.8\} = 0.7\).
- For \(x = 5\), \(\mu_{A \ast B}(5) = \min\{0.5, 0.6\} = 0.5\).

So, the membership function for the soft intersection \(A \ast B\) is:
\[\begin{align*}
\mu_{A \ast B}(1) &= 0.7 \\
\mu_{A \ast B}(2) &= 0.5 \\
\mu_{A \ast B}(3) &= 0.3 \\
\mu_{A \ast B}(4) &= 0.7 \\
\mu_{A \ast B}(5) &= 0.5 \\
\end{align*}\]

The soft intersection \(A \ast B\) consists of elements with membership values determined by taking the minimum value from the corresponding membership functions of sets \(A\) and \(B\).

4. Soft Graph Intersection

For the set of all subgraphs of a given graph, the membership function could represent the density of each subgraph. The operation \(\ast\) could be graph intersection.

Example 4

Consider two directed graphs \(G\) and \(H\) with vertex sets \(V_G = \{A, B, C, D\}\) and \(V_H = \{X, Y, Z\}\), respectively. The edges in \(G\) and \(H\) are defined as follows:

Graph \(G\) edges

- \(A \to B\)
- \(B \to C\)
- \(C \to D\)
- \(D \to A\)

Graph \(H\) edges

- \(X \to Y\)
- \(Y \to Z\)
- \(Z \to X\)

Additionally, the membership functions for vertices in \(G\) and \(H\) are given as follows
Membership function for graph \(G\)
\[\begin{align*}
\mu_G(A) &= 0.8 \\
\mu_G(B) &= 0.6 \\
\mu_G(C) &= 0.4 \\
\mu_G(D) &= 0.7 \\
\end{align*}\]

Membership function for graph \(H\)
\[\begin{align*}
\mu_H(X) &= 0.7 \\
\mu_H(Y) &= 0.5 \\
\mu_H(Z) &= 0.3 \\
\end{align*}\]
Calculate the soft graph intersection \(\ast\) of graphs \(G\) and \(H\) using the given membership functions.
Answer:
Given: Edges and membership functions for graphs \(G\) and \(H\).
To calculate the soft graph intersection \(\ast\) of graphs \(G\) and \(H\), we need to determine the minimum membership value for each common vertex between the two graphs.
Common vertices: \(V_{\text{common}} = \{A, B, C\}\).
1. Calculate the soft graph intersection \(\ast\) for each common vertex \(v\) using the formula:
\[\mu_{G \ast H}(v) = \min\{\mu_G(v), \mu_H(v)\}.\]

Calculating for each common vertex

- For \(v = A\), \(\mu_{G \ast H}(A) = \min\{0.8, 0.7\} = 0.7\).

- For \(v = B\), \(\mu_{G \ast H}(B) = \min\{0.6, 0.5\} = 0.5\).
- For \(v = C\), \(\mu_{G \ast H}(C) = \min\{0.4, 0.3\} = 0.3\).

So, the membership function for the soft graph intersection \(G \ast H\) is
\[\begin{align*}
\mu_{G \ast H}(A) &= 0.7 \\
\mu_{G \ast H}(B) &= 0.5 \\
\mu_{G \ast H}(C) &= 0.3 \\
\end{align*}\]

The soft graph intersection \(G \ast H\) consists of common vertices from graphs \(G\) and \(H\), with membership values determined by taking the minimum value from the corresponding membership functions of the vertices.

5. Soft String Concatenation
In the set of strings, the membership function could be based on the length of each string. Longer strings have a lower degree of membership. The operation \(\ast\) could be string concatenation.

Example 5
Consider two strings \(s\) and \(t\) defined as follows:
String \(s\) = "hello"
String \(t\) = "world"
Additionally, the membership functions for characters in strings \(s\) and \(t\) are given as follows
Membership function for string \(s\)
\[\begin{align*}
\mu_s('h') &= 0.9 \\
\mu_s('e') &= 0.7 \\
\mu_s('l') &= 0.5 \\
\mu_s('o') &= 0.8 \\
\end{align*}\]

Membership function for string \(t\):
\[\begin{align*}
\mu_t('w') &= 0.8 \\
\mu_t('o') &= 0.6 \\
\mu_t('r') &= 0.4 \\
\mu_t('l') &= 0.7 \\
\mu_t('d') &= 0.9 \\
\end{align*}\]

Calculate the soft string concatenation \(\ast\) of strings \(s\) and \(t\) using the given membership functions.
Answer:
Given: Strings \(s\) and \(t\) and their respective membership functions.
To calculate the soft string concatenation \(\ast\) of strings \(s\) and \(t\), we need to determine the membership value for each character in the concatenated string.
Concatenated string: \(u = "helloworld"\).
1. Calculate the soft string concatenation \(\ast\) for each character \(c\) in the concatenated string \(u\) using the formula:
\[\mu_{s \ast t}(c) = \begin{cases}
\mu_s(c) & \text{if } c \in s \\
\mu_t(c) & \text{if } c \in t \\
\end{cases}\]

Calculating for each character
- For \(c = 'h'\), \(\mu_{s \ast t}('h') = \mu_s('h') = 0.9\).
- For \(c = 'e'\), \(\mu_{s \ast t}('e') = \mu_s('e') = 0.7\).
- For \(c = 'l'\), \(\mu_{s \ast t}('l') = \mu_s('l') = 0.5\).
- For \(c = 'l'\), \(\mu_{s \ast t}('l') = \mu_s('l') = 0.5\).
- For \(c = 'o'\), \(\mu_{s \ast t}('o') = \mu_s('o') = 0.8\).
- For \(c = 'w'\), \(\mu_{s \ast t}('w') = \mu_t('w') = 0.8\).
- For \(c = 'o'\), \(\mu_{s \ast t}('o') = \mu_t('o') = 0.6\).
- For \(c = 'r'\), \(\mu_{s \ast t}('r') = \mu_t('r') = 0.4\).
- For \(c = 'l'\), \(\mu_{s \ast t}('l') = \mu_t('l') = 0.7\).
- For \(c = 'd'\), \(\mu_{s \ast t}('d') = \mu_t('d') = 0.9\).

So, the membership function for the soft string concatenation \(s \ast t\) is
\[\begin{align*}
\mu_{s \ast t}('h') &= 0.9 \\
\mu_{s \ast t}('e') &= 0.7 \\
\mu_{s \ast t}('l') &= 0.5 \\
\mu_{s \ast t}('l') &= 0.5 \\
\mu_{s \ast t}('o') &= 0.8 \\
\mu_{s \ast t}('w') &= 0.8 \\
\mu_{s \ast t}('o') &= 0.6 \\
\mu_{s \ast t}('r') &= 0.4 \\
\mu_{s \ast t}('l') &= 0.7 \\
\mu_{s \ast t}('d') &= 0.9 \\
\end{align*}\]

The soft string concatenation \(s \ast t\) consists of characters from both strings \(s\) and \(t\), with membership values determined by the corresponding membership functions of the characters.

6. Soft Polynomial Composition
Consider the set of all polynomials with real coefficients. The membership function could be based on the degree of each polynomial. The operation \(\ast\) could be polynomial composition.

Example 6
Consider two polynomials \(p(x)\) and \(q(x)\) defined as follows:
Polynomial \(p(x)\) = \(3x^2 + 2x + 1\)
Polynomial \(q(x)\) = \(2x^3 + x^2 - 4x + 5\)
Additionally, the membership functions for coefficients in polynomials \(p(x)\) and \(q(x)\) are given as follows
Membership function for polynomial \(p(x)\)
\[\begin{align*}
\mu_p(3) &= 0.9 \\
\mu_p(2) &= 0.7 \\
\mu_p(1) &= 0.5 \\
\end{align*}\]

Membership function for polynomial \(q(x)\)
\[\begin{align*}
\mu_q(2) &= 0.8 \\
\mu_q(1) &= 0.6 \\
\mu_q(-4) &= 0.4 \\
\mu_q(5) &= 0.7 \\
\end{align*}\]
Calculate the soft polynomial composition \(\ast\) of polynomials \(p(x)\) and \(q(x)\) using the given membership functions.

Answer

Given: Polynomials $p(x)$ and $q(x)$, and their respective membership functions.

To calculate the soft polynomial composition $\ast$ of polynomials $p(x)$ and $q(x)$, we need to determine the membership value for the coefficient of each term in the composed polynomial.

Composed polynomial: $r(x) = p(q(x))$.

1. Calculate the soft polynomial composition $\ast$ for each coefficient c in the composed polynomial $r(x)$ using the formula:

$$\mu_{p \ast q}(c) = \begin{cases}
\mu_p(c) & \text{if } c \text{ is a coefficient in } p(x) \\
\mu_q(c) & \text{if } c \text{ is a coefficient in } q(x) \\
\end{cases}$$

Calculating for each coefficient:

- For $c = 3$, $\mu_{p \ast q}(3) = \mu_p(3) = 0.9$.
- For $c = 2$, $\mu_{p \ast q}(2) = \mu_q(2) = 0.8$.
- For $c = 1$, $\mu_{p \ast q}(1) = \mu_q(1) = 0.6$.
- For $c = 1$, $\mu_{p \ast q}(1) = \mu_q(1) = 0.6$.
- For $c = -4$, $\mu_{p \ast q}(-4) = \mu_q(-4) = 0.4$.
- For $c = 5$, $\mu_{p \ast q}(5) = \mu_q(5) = 0.7$.

So, the membership function for the soft polynomial composition $p \ast q$ is:

$$\begin{align*}
\mu_{p \ast q}(3) &= 0.9 \\
\mu_{p \ast q}(2) &= 0.8 \\
\mu_{p \ast q}(1) &= 0.6 \\
\mu_{p \ast q}(1) &= 0.6 \\
\mu_{p \ast q}(-4) &= 0.4 \\
\mu_{p \ast q}(5) &= 0.7 \\
\end{align*}$$

The soft polynomial composition $p \ast q$ consists of coefficients from both polynomials $p(x)$ and $q(x)$, with membership values determined by the corresponding membership functions of the coefficients.

7. Soft Interval Arithmetic

In the set of real intervals, the membership function could represent the length of each interval. Longer intervals have a lower degree of membership. The operation $\ast$ could be interval intersection.

Example 7

Consider two intervals A and B defined as follows:

Interval A = $[2, 5]$

Interval B = $[3, 7]$

Additionally, the membership functions for the endpoints of intervals A and B are given as follows:

Membership function for interval A

$$\begin{align*}
\mu_A(2) &= 0.8 \\
\mu_A(5) &= 0.6 \\
\end{align*}$$

Membership function for interval B

$$\begin{align*}
\mu_B(3) &= 0.7 \\
\mu_B(7) &= 0.5 \\
\end{align*}$$

Calculate the soft interval arithmetic $\ast$ for intervals A and B using the given membership functions.

Answer:

Given: Intervals A and B, and their respective membership functions.

To calculate the soft interval arithmetic $\ast$ for intervals A and B, we need to determine the membership value for each endpoint of the resulting interval.

Resulting interval: $C = A \ast B$.

1. Calculate the soft interval arithmetic $\ast$ for each endpoint x of the resulting interval C using the formula:

$$\mu_{A \ast B}(x) = \max\{\mu_A(x), \mu_B(x)\}.$$

Calculating for each endpoint:

- For $x = 2$, $\mu_{A \ast B}(2) = \max\{0.8, 0\} = 0.8$.
- For $x = 3$, $\mu_{A \ast B}(3) = \max\{0, 0.7\} = 0.7$.
- For $x = 5$, $\mu_{A \ast B}(5) = \max\{0.6, 0.6\} = 0.6$.
- For $x = 7$, $\mu_{A \ast B}(7) = \max\{0, 0.5\} = 0.5$.

So, the membership function for the soft interval arithmetic $A \ast B$ is:

$$\begin{align*}
\mu_{A \ast B}(2) &= 0.8 \\
\mu_{A \ast B}(3) &= 0.7 \\
\mu_{A \ast B}(5) &= 0.6 \\
\mu_{A \ast B}(7) &= 0.5 \\
\end{align*}$$

The soft interval arithmetic $A \ast B$ consists of endpoints from both intervals A and B, with membership values determined by taking the maximum value from the corresponding membership functions of the endpoints.

8. Soft Vector Dot Product

In the set of real vectors, the membership function could be based on the norm of each vector. Vectors with larger norms have lower degrees of membership. The operation $\ast$ could be the dot product.

Example 8

Consider two vectors v and w defined as follows:

Vector v = $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

Vector w = $\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$

Additionally, the membership functions for the components of vectors v and w are given as follows:

Membership function for vector v

$$\begin{align*}
\mu_v(2) &= 0.8 \\
\mu_v(3) &= 0.6 \\
\mu_v(4) &= 0.4 \\
\end{align*}$$

Membership function for vector w

$$\begin{align*}
\mu_w(1) &= 0.7 \\
\mu_w(5) &= 0.5 \\
\mu_w(6) &= 0.3 \\
\end{align*}$$

Calculate the soft vector dot product $\ast$ for vectors v and w using the given membership functions.

Answer

Given: Vectors v and w, and their respective membership functions.

To calculate the soft vector dot product $\ast$ for vectors v and w, we need to determine the membership value for each component of the resulting vector.

Resulting vector: $u = v \ast w$.

1. Calculate the soft vector dot product $\ast$ for each component c of the resulting vector u using the formula:

$$\mu_{v \ast w}(c) = \mu_v(c) \cdot \mu_w(c).$$

Calculating for each component:

- For $c = 2$, $\mu_{v \ast w}(2) = \mu_v(2) \cdot \mu_w(2) = 0.8 \cdot 0.7 = 0.56$.
- For $c = 3$, $\mu_{v \ast w}(3) = \mu_v(3) \cdot \mu_w(3) = 0.6 \cdot 0.5 = 0.3$.
- For $c = 4$, $\mu_{v \ast w}(4) = \mu_v(4) \cdot \mu_w(4) = 0.4 \cdot 0.3 = 0.12$.

So, the membership function for the soft vector dot product $v \ast w$ is:

$$\begin{align*}
\mu_{v \ast w}(2) &= 0.56 \\
\mu_{v \ast w}(3) &= 0.3 \\
\mu_{v \ast w}(4) &= 0.12 \\
\end{align*}$$

The soft vector dot product $v \ast w$ consists of components from both vectors v and w, with membership values determined by multiplying the corresponding membership functions of the components.

9. Soft Probability Distributions

Consider the set of all probability distributions over a discrete set of outcomes. The membership function could represent the entropy of each distribution. The operation $\ast$ could be distribution convolution.

Example 9

Consider a discrete probability distribution X defined over the set of outcomes $\{1, 2, 3, 4, 5\}$. The probabilities of each outcome are given by:

$P(X = 1) = 0.2$

$P(X = 2) = 0.3$

$P(X = 3) = 0.1$

$P(X = 4) = 0.15$

$P(X = 5) = 0.25$

Additionally, the membership functions for each probability value are given as follows

Membership function for $P(X = 1)$: $\mu_{P(X=1)}(0.2) = 0.8$

Membership function for $P(X = 2)$: $\mu_{P(X=2)}(0.3) = 0.6$

Membership function for $P(X = 3)$: $\mu_{P(X=3)}(0.1) = 0.4$

Membership function for $P(X = 4)$: $\mu_{P(X=4)}(0.15) = 0.7$

Membership function for $P(X = 5)$: $\mu_{P(X=5)}(0.25) = 0.5$

Calculate the soft probability distribution $\ast$ for the given probability distribution X using the given membership functions.

Answer

Given: Discrete probability distribution X and its respective membership functions.

To calculate the soft probability distribution $\ast$ for the given probability distribution X, we need to determine the membership value for each probability value in the distribution.

Resulting probability distribution: $Y = X \ast \mu_{P(X)}$.

1. Calculate the soft probability distribution $\ast$ for each probability p in the resulting distribution Y using the formula:

$$\mu_{Y}(p) = \mu_{P(X)}(p).$$

Calculating for each probability

- For $p = 0.2$, $\mu_{Y}(0.2) = \mu_{P(X)}(0.2) = 0.8$.
- For $p = 0.3$, $\mu_{Y}(0.3) = \mu_{P(X)}(0.3) = 0.6$.
- For $p = 0.1$, $\mu_{Y}(0.1) = \mu_{P(X)}(0.1) = 0.4$.
- For $p = 0.15$, $\mu_{Y}(0.15) = \mu_{P(X)}(0.15) = 0.7$.
- For $p = 0.25$, $\mu_{Y}(0.25) = \mu_{P(X)}(0.25) = 0.5$.

So, the membership function for the soft probability distribution Y is:

$$\begin{align*}
\mu_{Y}(0.2) &= 0.8 \\
\mu_{Y}(0.3) &= 0.6 \\
\mu_{Y}(0.1) &= 0.4 \\
\mu_{Y}(0.15) &= 0.7 \\
\mu_{Y}(0.25) &= 0.5 \\
\end{align*}$$

The soft probability distribution Y consists of probabilities from the original distribution X, with membership values determined by the corresponding membership functions of the probabilities.

10. Soft Automata Composition

In the set of finite automata, the membership function could be based on the number of states of each automaton. Automata with more states have lower degrees of membership. The operation $\ast$ could be automata composition.

Example 10

Consider two finite automata A and B with states $Q_A = \{q_0, q_1\}$ and $Q_B = \{p_0, p_1\}$, respectively. The transition functions for A and B are defined as follows:

Transition function δ_A for automaton A:

- $\delta_A(q_0, 0) = q_0$
- $\delta_A(q_0, 1) = q_1$
- $\delta_A(q_1, 0) = q_1$
- $\delta_A(q_1, 1) = q_0$
- Transition function δ_B for automaton B:
- $\delta_B(p_0, 0) = p_1$
- $\delta_B(p_0, 1) = p_0$
- $\delta_B(p_1, 0) = p_1$
- $\delta_B(p_1, 1) = p_0$

Additionally, the membership functions for the states of automata A and B are given as follows:

Membership function for automaton A:

\[\begin{align*}
\mu_A(q_0) &= 0.9 \\
\mu_A(q_1) &= 0.7 \\
\end{align*}\]
Membership function for automaton \(B\):
\[\begin{align*}
\mu_B(p_0) &= 0.8 \\
\mu_B(p_1) &= 0.6 \\
\end{align*}\]
Calculate the soft automata composition \(\ast\) for automata \(A\) and \(B\) using the given membership functions.

Answer
Given: Finite automata \(A\) and \(B\), and their respective transition functions and membership functions.
To calculate the soft automata composition \(\ast\) for automata \(A\) and \(B\), we need to determine the membership value for each state in the resulting composed automaton.

Resulting composed automaton: \(C = A \ast B\).
1. Calculate the soft automata composition \(\ast\) for each state \(s\) in the resulting composed automaton \(C\) using the formula:

\[\mu_{A \ast B}(s) = \mu_A(s) \cdot \mu_B(s).\]

Calculating for each state
- For \(s = q_0\), \(\mu_{A \ast B}(q_0) = \mu_A(q_0) \cdot \mu_B(p_0) = 0.9 \cdot 0.8 = 0.72\).
- For \(s = q_1\), \(\mu_{A \ast B}(q_1) = \mu_A(q_1) \cdot \mu_B(p_1) = 0.7 \cdot 0.6 = 0.42\).

So, the membership function for the soft automata composition \(A \ast B\) is
\[\begin{align*}
\mu_{A \ast B}(q_0) &= 0.72 \\
\mu_{A \ast B}(q_1) &= 0.42 \\
\end{align*}\]

The soft automata composition \(A \ast B\) consists of states from both automata \(A\) and \(B\), with membership values determined by multiplying the corresponding membership functions of the states.
These examples demonstrate how the concept of soft semi groups can be applied to various mathematical structures, allowing for the incorporation of degrees of membership and softness into algebraic operations.

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