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Dr. A.Nellai Murugan
Department of Mathematics
V.O.Chidambaram College
Tuticorin 628 008.

S.Heerajohn
Department of Mathematics
V.O.Chidambaram College
Tuticorin 628 008.

Correspondence
Dr. A.Nellai Murugan
Department of Mathematics
V.O.Chidambaram College
Tuticorin 628 008.

Special Class of Mean Square Cordial Graphs

A.Nellai Murugan, S.Heerajohn

Abstract
Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈x⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that the graphs Tree Tr(n), Umbrella U(n,3), Twig Tg_n are Mean Square Cordial Graphs.

Keywords: Mean Square Cordial Graph, Mean Square Cordial Labeling. 2000 Mathematics Subject classification05C78.

1. Introduction
A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Tree Tr(n), Umbrella U(n,3), Twig $Tg_n$are Mean Square Cordial Graphs. For graph theory terminology, we follow [2].

2. Preliminaries
Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.
The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that the graphs Tree Tr(n), Umbrella U(n,3), Twig Tg_n are Mean Square Cordial Graphs.

Definition: 2.1
Let Tr be any tree. Denote the tree obtained from Tr by considering two copies of Tr by adding an edge between them by Tr(Z) and in general the graph obtained from Tr(n-1) and Tr by adding an edge between them is denoted by Tr(n).

Definition: 2.2
Umbrella is a graph obtained from a Fan by joining a path of length m, P_m to a middle vertex of a path P_n is Fan F_n. It is denoted by U(m,n).

Definition: 2.3
A graph obtained from a path by attaching exactly two pendent edges to each internal vertex of a path is called a twig and is denoted by Tg_n, $n \geq 1$

3. Main Results
Theorem: 3.1
Tree Tr(n) is Mean Square Cordial Graph.

Proof

Let $V(Tr(n)) = \{ u_{ij}, v_{ij} : 1\leq i \leq n, 1\leq j \leq 2, w_i : 1\leq i \leq n \}$

Let $E(Tr(n)) = \{ [(u_{i1}u_{i2}) \cup (v_{i1}v_{i2}) \cup (u_{i1}v_{i1}) \cup (v_{i1}w_i) : 1\leq i \leq n] \cup [(w_iw_{i+1}) : 1\leq i \leq n-1] \}$

Define $f : V(Tr(n)) \rightarrow \{0,1\}$

Case: 1

When $n \equiv 1 \pmod 2$

The vertex labeling are,

$f(u_{i2}) = 1, 1\leq i \leq n$

$f(v_{i1}) = 0, 1\leq i \leq n$

$f(v_{i2}) = 1, 1\leq i \leq n$

$f(w_i) = 0, 1\leq i \leq n$

$$f(u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

The induced edge labeling are,

$f^*(u_{i1}u_{i2}) = 1, 1\leq i \leq n$

$f^*(v_{i1}v_{i2}) = 1, 1\leq i \leq n$

$f^*(w_iv_{i1}) = 0, 1\leq i \leq n$

$f^*(w_iw_{i+1}) = 0, 1\leq i \leq n-1$

$$f^*(v_{i1}u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph Tr(n) satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, Tree Tr(n) (n-odd) is Mean Square Cordial Graph.

Case: 2

When $n \equiv 0 \pmod 2$

The vertex labeling are,

$f(u_{i2}) = 1, 1\leq i \leq n$

$f(v_{i1}) = 0, 1\leq i \leq n$

$f(v_{i2}) = 1, 1\leq i \leq n$

$f(w_i) = 0, 1\leq i \leq n$

$$f(u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n \end{cases}$$

The induced edge labeling are,

$f^*(u_{i1}u_{i2}) = 1, 1\leq i \leq n$

$f^*(v_{i1}v_{i2}) = 1, 1\leq i \leq n$

$f^*(w_iv_{i1}) = 0, 1\leq i \leq n$

$f^*(w_iw_{i+1}) = 0, 1\leq i \leq n-1$

$$f^*(v_{i1}u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n \end{cases}$$

Here, $v_f(0) = v_f(1)$for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph Tr(n) satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, Tree Tr(n)(n-even) is Mean Square Cordial Graph.

For example, The Mean Square Cordial Labeling of Tr(2), Tr(3) are shown in figures 3.2 and 3.3.

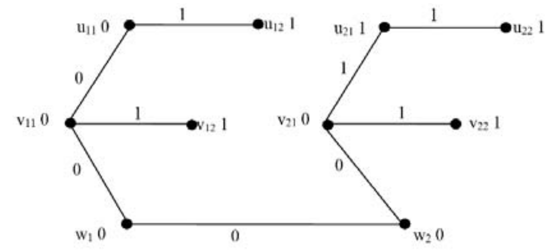

Fig 3.2

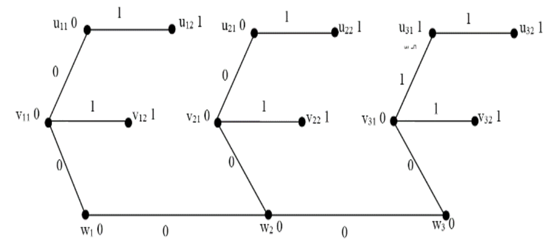

Fig 3.3

Theorem: 3.4

Umbrella U(n,3) (n-odd) isMean Square Cordial Graph.

Proof:

Let G be U(n,3)

Let $V(G) = \{ u, v, w, u_i : 1\leq i \leq n \}$

Let $E(G) = \{ [(vw)]\cup [(vu_{\frac{n+1}{2}})] \cup [(uu_i) : 1\leq i \leq n] \cup [(u_iu_{i+1}) : 1\leq i \leq n-1] \}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u) = 0$

$f(v) = 1$

$f(w) = 1$

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

The induced edge labeling are,

$f^*(vw) = 1$

$f^*(vu_{\frac{n+1}{2}}) = 1$

$$f^*(uu_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f^*(u_iu_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

Here, $v_f(1) = v_f(0)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, Umbrella U(n,3) (n-odd) isMean Square Cordial Graph.

For example, U(5,3) isMean Square Cordial Graph as shown in figure 3.5.

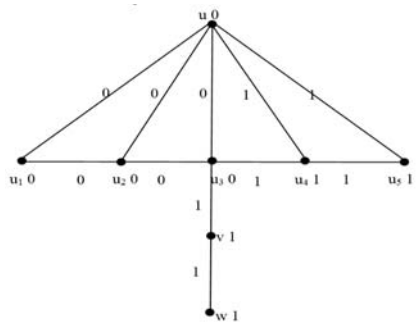

Fig 3.5

Theorem:3.6
Umbrella U(n,3) (n-even) isMean Square Cordial Graph

Proof
Let G be U(n,3)
Let V(G) = { u, v, w, u_i : $1\leq i \leq n$ }
Let E(G) = { [(vw)]∪ [($vu_{\frac{n}{2}}$)] ∪ [(uu_i) : $1\leq i \leq n$] ∪ [(u_iu_{i+1}) : $1\leq i \leq n-1$] }
Define f : V(G) → {0,1}

The vertex labeling are,
f(u) = 0
f(v) = 1
f(w) = 1

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+2}{2} \\ 1, \frac{n+4}{2} \leq i \leq n \end{cases}$$

The induced edge labeling are,
$f^*(vw) = 1$
$f^*(vu_{\frac{n+1}{2}}) = 1$

$$f^*(uu_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+2}{2} \\ 1, \frac{n+4}{2} \leq i \leq n \end{cases}$$

$$f^*(u_iu_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n-1 \end{cases}$$

Here, $v_f(1) + 1 = v_f(0)$for all n and
$e_f(1) + 1 = e_f(0)$for all n
Therefore, The Graph G satisfies the conditions
$| v_f(1) – v_f(0) |\leq 1$
$| e_f(1) – e_f(0) |\leq 1$
Hence, Umbrella U(n,3) (n-even) isMean Square Cordial Graph.
For example, U(4,3)isMean Square Cordial Graph as shown in figure 3.7.

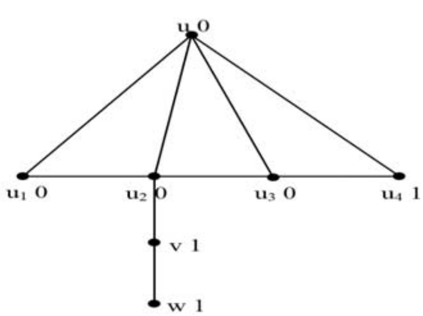

Fig 3.7

Theorem: 3.8
Twig Tg_n is Mean Square Cordial Graph.

Proof
Let V(Tg_n) = { u_i : $1\leq i \leq n$, v_i, w_i : $1\leq i \leq n-2$ }
Let E(Tg_n) = { [(u_iu_{i+1}) : $1\leq i \leq n-1$] ∪ [(u_iv_{i-1}) : $2\leq i \leq n-1$] ∪ [(u_iw_{i-1}) : $2\leq i \leq n-1$] }
Define f : V(Tg_n) → {0,1}

Case: 1
When n ≡ 1 (mod 2)
The vertex labeling are,
$f(u_i) = 0, 1\leq i \leq n-1$
$f(u_n) = 1$
$f(w_i) = 1, 1\leq i \leq n-2$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-2 \end{cases}$$

The induced edge labelingare,
$f^*(u_iu_{i+1}) = 0, 1\leq i \leq n-2$
$f^*(u_{n-1}u_n) = 1$
$f^*(u_iw_{i-1}) = 1, 2\leq i \leq n-1$

$$f^*(u_iv_{i-1}) = \begin{cases} 0, 2 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n-1 \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and
$e_f(0) = e_f(1)$ for all n
Therefore, The Graph Tg_n satisfies the conditions
$| v_f(1) – v_f(0) |\leq 1$
$| e_f(1) – e_f(0) |\leq 1$
Hence, Twig Tg_n(n-odd) is Mean Square Cordial Graph.

Case: 2
When n ≡ 0 (mod 2)
The vertex labeling are,
$f(u_i) = 0, 1\leq i \leq n-1$
$f(u_n) = 1$
$f(w_i) = 1, 1\leq i \leq n-2$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-2}{2} \\ 1, \frac{n}{2} \leq i \leq n-2 \end{cases}$$

The induced edge labelingare,
$f^*(u_iu_{i+1}) = 0, 1\leq i \leq n-2$
$f^*(u_{n-1}u_n) = 1$
$f^*(u_iw_{i-1}) = 1, 2\leq i \leq n-1$

$$f^*(u_iv_{i-1}) = \begin{cases} 0, 2 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n-1 \end{cases}$$

Here, $v_f(1) = v_f(0)$for all n and
$e_f(1) = e_f(0) + 1$ for all n
Therefore, The Graph Tg_n satisfies the conditions
$| v_f(1) – v_f(0) |\leq 1$
$| e_f(1) – e_f(0) |\leq 1$
Hence, Twig Tg_n (n-even) is Mean Square Cordial Graph.
For example, The Mean Square Cordial Labeling ofTg_6, Tg_7 are shown in figures 3.9 and 3.10

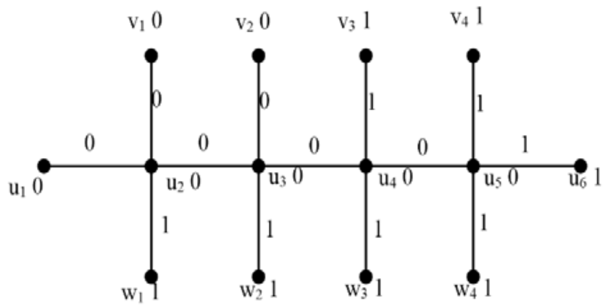

Fig 3.9

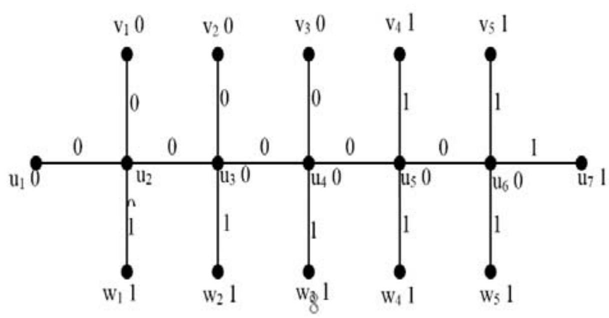

Fig 3.10

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