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The effect of laser pulse on dual phase lag thermoelastic medium

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Abstract
In the present paper, we have introduced the two phase lag theory to study the dynamical interactions in a thermoelastic medium under the influence of laser pulse. Normal mode analysis technique is employed onto the non-dimensional field equations to obtain the analytical solution. The numerical estimates of the field variables displacement, stress and temperature are computed for magnesium crystal like material and presented graphically.

Keywords: Two phase lag, thermoelasticity, normal mode analysis, laser pulse

1. Introduction

During heating of a metal film by laser pulse, a thermoelastic wave is generated due to thermal expansion in the near surface region and propagates into the target. Several research works have been devoted to problems involving a laser pulse heat source due to the numerous applications in engineering. The coupled thermoelastic vibrations of a microscale beam resonator induced by laser pulse heating were studied by Sun *et al*. (2008) [8].

The classical theory of thermoelasticity suffers from deficiency of admitting thermal signals propagating with infinite speed. Numerous alternative theories of heat conduction have come forth to overcome this deficiency. Generalized theories proposed by Lord and Shulman (1967) [6] and Green and Lindsay (1972) [2] are two well-known theories of thermoelasticity to overcome this deficiency. After that, providing sufficient basic modifications in governing equations, Green and Naghdi (1991, 1992, 1993) [3, 4, 5] produced an alternative theory which was further divided into three different parts, referred to as GN theory of type I, II, III. The conventional Fourier's equation of heat conduction $\vec{q}(\vec{r},t) = -k\nabla T(\vec{r},t)$ can be used in several particular problems, although this turns out to predict an infinite speed of thermal signal, which is physically unrealistic. (Tzou, 1995) [9] developed dual phase lag theory to overcome this deficiency. The classical Fourier's law $\vec{q} = -k\vec{\nabla} T$ has been replaced by $\vec{q}(P, t+\tau_q) = -k\vec{\nabla} T(P, t+\tau_T)$, where the temperature gradient $\vec{\nabla} T$ at a space variable P at time $t+\tau_T$ corresponds to heat flux vector $\vec{q}$ at the same point at time $t+\tau_q$. The delay time τ_T is supposed to be caused by the microstructural interactions and is called the phase lag of temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of the thermal inertia and is called the phase lag of the heat flux. If $\tau_q = \tau_T = 0$, then the Fourier's law in two phase lag model is identical with the classical Fourier's law. The stability of dual phase lag heat conduction was discussed by (Quintanilla and Racke, 2006) [7]. (Ezzat *et al*. 2012) [1] estimated the effects of two-temperature discrepancy and fractional parameter on the wave propagation in the context of dual-phase-lag magneto-thermoelasticity.

The objects of the present investigation is concerned with the determination of displacement components and force stresses generalized thermoelastic medium with dual phase lag effects subjected to inclined mechanical load under laser pulse effect. Normal mode analysis is adopted to find out the exact expressions of the variables considered.

2. Governing equations
The constitutive relations and field equations in the absence of body force for an isotropic, homogeneous, thermoelastic solid can be expressed as follows:

(i) Constitutive relations

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{j,i} + u_{i,j}\right) - \beta_1\theta\delta_{ij}, \quad (1)$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad (2)$$

(ii) Stress equation of motion

$$\sigma_{ji,j} = \rho\ddot{u}_i, \quad (3)$$

(iii) Equation of heat conduction

$$k^*\left(1 + \tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta = \left(1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E\dot{\theta} + \beta_1 T_0\dot{e}\right) - \dot{Q}. \quad (4)$$

3. Problem formulation
Let us consider a homogeneous, isotropic, generalized thermoelastic medium with laser pulse heat source. The rectangular Cartesian co-ordinates are introduced having origin on the surface (z = 0) and z-axis pointing vertically downwards into the medium. All the quantities related to the medium considered will be functions of the time variable t and the coordinates x and z. Also the boundary plane $(z = 0)$ of the half space is heated uniformly by laser pulse with non-Gaussian temporal profile [Sun *et al*. (2008) [8]]. For a two dimensional problem in the x-z plane, we can write the displacement vector as

$$L(t) = \frac{L_0 t}{t_p^2}\exp\left(-\frac{t}{t_p}\right),$$

where t_p is the time duration of a laser pulse and L_0 is the laser intensity, which is defined as the total energy carried by a laser pulse per unit cross section of the laser beam. in the present problem, we take $t_p = 2ps$ as the time duration. The thermal conduction in the beam can be modeled as a one dimensional problem with energy source $Q(z,t)$ as

$$Q(z,t) = \frac{R_a}{\delta}\exp\left(\frac{z - h/2}{\delta}\right)L(t),$$

where δ is the absorption depth of the heating energy and R_a the absorptivity of the irradiated surface. For a two dimensional problem in the x-z plane, we can write the displacement vector and Microrotation vector as

$$u = u_x = u(x,z,t), \quad v = u_y = 0, \text{ and } w = u_z = w(x,z,t). \quad (5)$$

Substitution of Eqs. (2) and (5) into Eqs. (1)

$$\sigma_{xx} = (2\mu + \lambda)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \beta_1\theta, \quad (6)$$

$$\sigma_{zz} = (2\mu + \lambda)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \beta_1\theta, \quad (7)$$

$$\sigma_{xz} = \sigma_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right). \quad (8)$$

With the aid of expressions (1), (2) and (5), the equations of motion (3), take the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \mu \nabla^2 u + (\mu + \lambda)\frac{\partial^2 u}{\partial x^2} + (\mu + \lambda)\frac{\partial^2 w}{\partial z \partial x} - \beta_1 \frac{\partial \theta}{\partial x}, \tag{9}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = \mu \nabla^2 w + (\mu + \lambda)\frac{\partial^2 w}{\partial z^2} + (\mu + \lambda)\frac{\partial^2 u}{\partial z \partial x} - \beta_1 \frac{\partial \theta}{\partial z}, \tag{10}$$

The governing equations can be put into more convenient forms by introducing the following non-dimensional variables:

$$(x', z') = \frac{w^*}{c_1}(x, z), \quad (u', w') = \frac{\rho w^* c_1}{\beta_1 T_0}(u, w),$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad (t', t'_p, \tau'_T, \tau'_q) = w^*(t, t_p, \tau_T, \tau_q),$$

$$\theta' = \frac{\theta}{T_0}, \tag{11}$$

Where

$$w^* = \frac{\rho C_E c_1^2}{k^*}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}.$$

Using Helmholtz decomposition, the displacement components can be written as

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}, \quad \psi = (-\vec{U})_y, \tag{12}$$

where $q(x, z, t)$ and $\psi(x, z, t)$ are scalar potential functions and $\vec{U}(x, z, t)$ is the vector potential function.
Now, in terms of the dimensionless quantities given in (11), Eqs. (9)-(10), (4) and (6)-(8) with the aid of expressions (12) along with some simplifications, assume the forms (after dropping the primes)

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) q - \theta = 0, \tag{13}$$

$$\left(\nabla^2 - a_1 \frac{\partial^2}{\partial t^2}\right) \psi = 0, \tag{14}$$

$$k^*\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2 \theta = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\frac{\partial}{\partial t}\left(\theta + \delta_0 \nabla^2 q\right)$$
$$-c_2\left(1 - \frac{t}{t_p}\right)\exp\left(\frac{2zt_p - ht_p - 2t\delta}{2t_p\delta}\right), \tag{15}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} + a_1 \frac{\partial w}{\partial z} - \theta, \tag{16}$$

$$\sigma_{zz} = \frac{\partial w}{\partial z} + a_4 \frac{\partial u}{\partial x} - \theta, \tag{17}$$

$$\sigma_{xz} = \sigma_{zx} = a_2\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \tag{18}$$

Where

$$a_1 = \frac{\rho c_1^2}{\mu}, \quad \delta_0 = \frac{\beta_1^2 T_0}{\rho}, \quad a_2 = \frac{\lambda}{\rho c_1^2}, \quad a_3 = \frac{1}{a_1}, \quad c_2 = \frac{R_a L_0 \omega^* c_1}{T_0 \delta t_p^2}.$$

4. Solution of the problem

Solution of the physical quantities can be decomposed in terms of normal modes in the following form

$$\left(u, w, q, \psi, \theta, \sigma_{ij}\right)(x, z, t) = \left(u^*, w^*, q^*, \psi^*, \theta^*, \sigma_{ij}^*\right)(z)\, e^{(\omega t + \iota m x)}, \tag{19}$$

where $u^*(z), w^*(z), q^*(z), \psi^*(z), \phi_2^*(z), \theta^*(z), \sigma_{ij}^*(z)$ are the amplitudes of the functions, ω is the angular frequency, ι is the imaginary unit and *m* is the wave number in *x* direction.

Using (19) in the Eqs. (13)-(15), we obtain the following differential equations

$$\left(D^2 - A_1\right)\psi^*(z) = 0, \tag{20}$$

$$\left(D^4 - A_2 D^2 + A_3\right)\left\{q^*(z), \theta^*(z)\right\} = c_3 \exp\left(\frac{z - \frac{h}{2}}{\delta} - \frac{t}{t_p} - \omega t - \iota m x\right), \tag{21}$$

where $D = \frac{d}{dz}$, $A_1 = m^2 + a_1 w^2$, $A_2 = b_2 + b_3 + b_4$, $A_3 = b_2 b_3 + b_4 m^2$,

$b_4 = b_1$, $b_3 = m^2 \delta_0$, $b_2 = m^2 + b_1$, $b_1 = \frac{\left(1 + \tau_q \omega + \frac{\tau_q^2 \omega^2}{2}\right)\omega}{k^*\left(1 + \tau_T \omega\right)}$, $c_3 = \frac{c_2\left(1 - \frac{t}{t_p}\right)}{k^*\left(1 + \tau_T \omega\right)}$.

Since the intent is that the solutions vanish at infinity so as to satisfy the regularity condition at infinity (which are assumed to be bounded as $z \to \infty$), we can express $\psi^*(x, z, t)$, $q^*(x, z, t)$, $\theta^*(x, z, t)$ in the following forms:

$$\psi(x, z, t) = \left(H_{1i} R_i(m, \omega)\, e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)}, \tag{22}$$

$$q(x, z, t) = \left(\sum_{i=2}^{3} R_i(m, \omega)\, e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)} + \varepsilon_1 f_1(z, t), \tag{23}$$

$$\theta(x, z, t) = \left(\sum_{i=2}^{3} H_{1i} R_i(m, \omega)\, e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)} + \varepsilon_2 f_1(z, t), \tag{24}$$

where λ_1^2 and λ_2^2, λ_3^2 are roots with positive real parts of the characteristic equations (20) and (21) respectively, $R_i(m, \omega)$, $(i = 1, 2, 3)$ are parameters, depending upon *m* and ω, and

$H_{11} = -\left(\frac{a_2}{\lambda_i^2 - b_2}\right)$, $H_{1i} = \left(\lambda_i^2 - b_1\right)(i = 2, 3)$, $\varepsilon_1 = \frac{c_3 \delta_4}{1 - A_2 \delta^2 + A_3 \delta^4}$,

$\varepsilon_2 = \frac{1 - b_1 \delta^2}{\delta^2}$, $f_1(z, t) = \exp\left(\frac{z - \frac{h}{2}}{\delta} - \frac{t}{t_p}\right)$.

Application on normal mode analysis to the expressions for stress components (16), (18) and displacement components (12), in combination with the relations (22)-(24), yields

$$u(x, z, t) = \left(\sum_{i=1}^{3} H_{2i} R_i e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)}, \tag{25}$$

$$w(x,z,t) = \left(\sum_{i=1}^{3} H_{3i} R_i e^{-\lambda_i z} \right) e^{(\omega t + \iota m x)}, \quad (26)$$

$$\sigma_{zx}(x,z,t) = \left(\sum_{i=1}^{3} H_{4i} R_i e^{-\lambda_i z} \right) e^{(\omega t + \iota m x)}, \quad (27)$$

$$\sigma_{zz}(x,z,t) = \left(\sum_{i=1}^{3} H_{5i} R_i e^{-\lambda_i z} \right) e^{(\omega t + \iota m x)}, \quad (28)$$

where $H_{21} = -\lambda_1 H_{11}$, $H_{2i} = \iota m\ (i = 2,3)$, $H_{31} = -\iota m H_{11}$
$H_{3i} = -\lambda_i\ (i = 2,3)$, $H_{41} = \left(a_{10}\lambda_1^2 + a_{11}m^2\right)H_{11} - a_{12}$
$H_{4i} = -\left(a_{10} + a_{11}\right) m \iota \lambda_i\ (i = 2,3)$, $H_{51} = \left(1 - a_9\right)\iota m \lambda_1 H_{11}$
$H_{5i} = \lambda_i^2 - a_9 m^2 - H_{1i}\ (i = 2,3)$.

5. Application: Inclined load acting on the surface

The plane boundary is subjected to an inclined mechanical load P_0 and its inclination with *z*-axis is θ. Then we have $P_1 = P_0 \sin\theta$, $P_2 = P_0 \cos\theta$, where P_1 is the intensity of tangential line load acting at the origin in the positive *x*-direction and P_2 is the intensity of normal line load acting in the positive *z*-direction. Hence the boundary conditions are

$$\sigma_{zz}(x,0,t) = -P_2 \psi_1(x,t), \quad (29)$$

;

$$\sigma_{zx}(x,0,t) = -P_1 \psi_1(x,t), \quad (30)$$

$$\theta(x,0,t) = 0. \quad (31)$$

where $\psi_1(x,t) = \delta(x) H(t)$, $\delta(x)$ is Dirac delta function and $H(t)$ is Heaviside unit step function.

$$\vec{\sigma}_{zz}(m,0,\omega) = \frac{-P_0 \cos\theta}{\omega} \quad (31)$$

$$\bar{\sigma}_{zx}(m,0,\omega) = \frac{-P_0 \sin\theta}{\omega}, \quad (32)$$

$$\bar{\theta}(m,0,\omega) = 0, \quad (33)$$

We arrive at a non-homogeneous system of linear equations which can be written in the matrix form as

$$\begin{bmatrix} 0 & H_{12} & H_{13} \\ H_{41} & H_{42} & H_{43} \\ H_{51} & H_{52} & H_{53} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ 0 \end{bmatrix} \quad (32)$$

Solution of system (32) provides us the values of $R_i\ (i = 1,2,3)$ as:

$$R_1 = \frac{\Delta_1}{\Delta}, \quad R_2 = \frac{\Delta_2}{\Delta}, \quad R_3 = \frac{\Delta_3}{\Delta}, \quad (33)$$

Where

$$\Delta = H_{51}\left(H_{12}H_{43} - H_{13}H_{42}\right) - H_{41}\left(H_{12}H_{53} - H_{13}H_{52}\right),$$

$$\Delta_1 = M_1 \left(H_{42} H_{53} - H_{43} H_{52} \right) - M_2 \left(H_{12} H_{53} - H_{13} H_{52} \right),$$

$$\Delta_2 = -M_1 \left(H_{41} H_{53} - H_{43} H_{51} \right) - M_2 \left(H_{13} H_{51} \right),$$

$$\Delta_3 = M_1 \left(H_{41} H_{52} - H_{42} H_{51} \right) + M_2 \left(H_{12} H_{51} \right),\ M_1 = -\varepsilon_2 f_1(z,t) e^{-(\omega t + \imath m x)},$$

$$M_2 = -\varepsilon_1 f_1(z,t) e^{-(\omega t + \imath m x)}$$

Substitution of (33) into expressions (24) and (28) leads to the expression of field variables as:

$$\theta(x,z,t) = \left(\frac{1}{\Delta} \sum_{i=2}^{3} \Delta_i H_{1i} e^{-\lambda_i z} \right) e^{(\omega t + \imath m x)} + \varepsilon_2 f_1(z,t), \tag{34}$$

$$u(x,z,t) = \left(\frac{1}{\Delta} \sum_{i=2}^{3} \Delta_i H_{2i} e^{-\lambda_i z} \right) e^{(\omega t + \imath m x)} + \varepsilon_1 f_1(z,t) \tag{35}$$

$$w(x,z,t) = \left(\frac{1}{\Delta} \sum_{i=2}^{3} \Delta_i H_{3i} e^{-\lambda_i z} \right) e^{(\omega t + \imath m x)} + \varepsilon_2 f_1(z,t) \tag{36}$$

$$\sigma_{zx}(x,z,t) = \left(\frac{1}{\Delta} \sum_{i=2}^{3} \Delta_i H_{4i} e^{-\lambda_i z} \right) e^{(\omega t + \imath m x)} + \varepsilon_1 f_1(z,t) \tag{37}$$

$$\sigma_{zz}(x,z,t) = \left(\frac{1}{\Delta} \sum_{i=2}^{3} \Delta_i H_{5i} e^{-\lambda_i z} \right) e^{(\omega t + \imath m x)} + \varepsilon_2 f_1(z,t) \tag{38}$$

6. Numerical results and discussions

The dynamical interactions between thermal and mechanical fields in solids have many applications in aeronautics, nuclear reactors and high energy particle accelerators. To understand the interaction phenomena, we have evaluated the numerical results of non-dimensional displacement component *w*, normal stress σ_{zz}, and temperature θ and displayed graphically. For numerical computation, we take the following values of relevant parameters for magnesium crystal like material:

$$\rho = 1.74 \times 10^3\, kg\, m^{-3},\ k = 1.0 \times 10^{10}\, kg\, m^{-1} s^{-2},\ k^* = 2.510\, W m^{-1} k^{-1},$$

$$C_E = 9.623 \times 10^2\, J\, kg^{-1} k^{-1},\ \alpha_t = 2.36 \times 10^{-5}\, K^{-1},\ T_0 = 293\, K,\ \tau_q = 0.2\, s,$$

$$\tau_T = 0.15\, s,\ R_a = 0.5,\ \delta = 0.01,\ L_0 = 10^{11}\, jm^{-1},\ t_p = 2\, ps,\ h = 0.01. \tag{39}$$

Utilizing the above values of parameters, values of the non-dimensional field variables have been evaluated and the results are displayed in the form of the graphs at different positions of *z* at *t* = 0.01 and *x* = 1.0. (Figs. 1-2) depicts the effect of inclination of load on the distribution of the field variables by considering three different values of angle as $\theta = 0^\circ$ (solid line), $\theta = 45^\circ$ (long-dashed line) and $\theta = 90^\circ$ (small-dashed line).

Figure 1 describes the behaviour of normal stress distribution with distance *z* under the different cases considered. The normal stress distribution for TPLMIS theory has small values in comparison to GN-III theory which clearly indicates that phase lags have decreasing effect and these values increase with increase of time *t*. Figure also indicates that the effect of time *t* is more pronounced than the effect of phase lags. However, for all the cases, normal stress predicts the same behaviour.

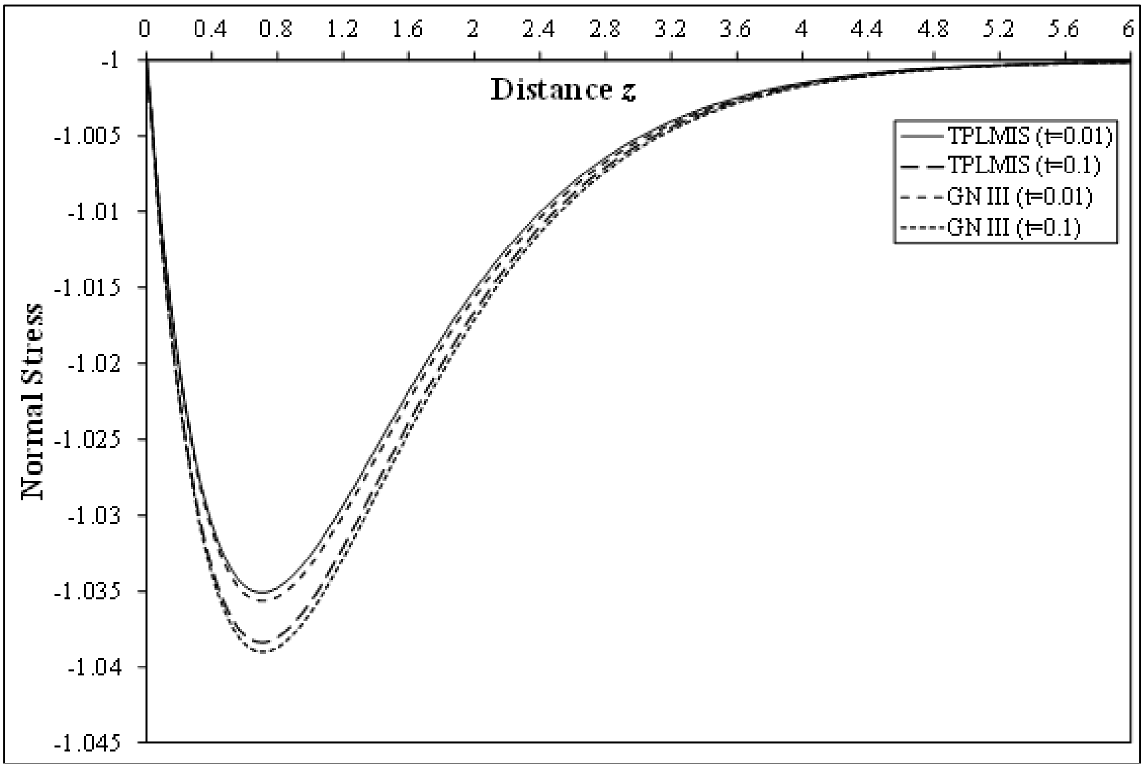

Fig 1

7. Concluding remarks

The work presented in this article provides a mathematical model to obtain the behaviour of normal displacement and temperatures in a homogeneous, isotropic, thermoelastic medium with two dual phase lag theory under the laser pulse effect by using normal mode analysis:

1. All the field variables have non-zero values in a bounded region of space except normal stress. Outside this region, values vanish identically and this means that the region has not felt thermal disturbance yet.
2. We can conclude from the figures that the time *t* plays a significant role in all the studied fields. Change in the value of time *t* causes significant change in all the field quantities. More precisely, the magnitudes of physical fields increase with increase in the values of *t*.

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