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On ternary quadratic equation $5x^2 - 2y^2 = 3z^2$

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Abstract
The Ternary Quadratic Diophantine Equation given by $5x^2 - 2y^2 = 3z^2$ is analyzed for its patterns of non-zero integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.

Keywords: Ternary, Quadratic, Integral solutions.

Introduction
The theory of diophantine equations offers a rich variety of fascinating problems [1-5]. For an extensive review of sizable literature and various problems, one may refer [6-20]. This communication concerns with yet another interesting ternary quadratic equation $5x^2 - 2y^2 = 3z^2$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions have been presented.

Notations Used
- $T_{m,n}$-Polygonal number of rank n with size m.
- P_n^k - Pentagonal number of rank n with size k.
- SqP_n - Square Pyramidal number of rank n.

Method Of analysis
The Ternary Quadratic Diophantine Equation to be solved for its non-zero distinct integral solution is

$$5x^2 - 2y^2 = 3z^2 \quad (1)$$

On substitution of linear transformations$(u \neq v \neq 0)$

$$x = u + 2v, y = u + 5v \quad (2)$$

In (1) leads to $u^2 = 10v^2 + z^2$ (3)

The corresponding solutions of (3) is the form

$$\left.\begin{aligned} v &= 2mn \\ u &= 10m^2 + n^2 \\ z &= 10m^2 - n^2 \end{aligned}\right\} \quad (4)$$

In view of (4), the solutions of (1) can be written as

$$x = 10m^2 + n^2 + 4mn$$
$$y = 10m^2 + n^2 + 10m$$
$$z = 10m^2 - n^2$$

Instead of (2), using the transformations $x = u - 2v, y = u - 5v$, in (1), we get again (3) only. Thus, the integer solutions of (1) are obtained as

$$x = 10m^2 + n^2 - 4mn$$
$$y = 10m^2 + n^2 - 10mn$$
$$z = 10m^2 - n^2$$

A few interesting properties observed are as follows:

1. $x(\mathrm{m},3)+\mathrm{z}(\mathrm{m},3)-168\mathrm{T}_{3,\mathrm{m}}+\mathrm{T}_{130,\mathrm{m}} \equiv 0 \pmod{72}$
2. $\mathrm{y}(\mathrm{m},2)+\mathrm{z}(\mathrm{m},2)-\mathrm{T}_{202,\mathrm{m}}+\mathrm{T}_{162,\mathrm{m}} \equiv 0 \pmod{40}$
3. $x(\mathrm{m},1)-\mathrm{T}_{78,\mathrm{m}}+\mathrm{T}_{58,\mathrm{m}} \equiv 1 \pmod{6}$
4. $x(\mathrm{m},2)-\mathrm{T}_{110,\mathrm{m}}+\mathrm{T}_{90,\mathrm{m}} \equiv 4 \pmod{2}$
5. $\mathrm{y}(2,3\mathrm{n})-\mathrm{T}_{106,\mathrm{n}}+\mathrm{T}_{88,\mathrm{n}} \equiv 40 \pmod{51}$
6. $\mathrm{z}(\mathrm{m},4)-\mathrm{T}_{230,\mathrm{m}}+\mathrm{T}_{210,\mathrm{m}} \equiv -16 \pmod{10}$
7. $\mathrm{y}(\mathrm{m},\mathrm{n})-x(\mathrm{m},\mathrm{n}) \equiv 6\mathrm{mn}$
   a) $\mathrm{y}(\mathrm{A},\mathrm{A}+1)-\mathrm{x}(\mathrm{A},\mathrm{A}+1) \equiv 12\mathrm{t}_{3,\mathrm{A}}$
   b) $\mathrm{y}\big(\mathrm{A},\mathrm{A}(\mathrm{A}+1)\big)-\mathrm{x}\big(\mathrm{A},\mathrm{A}(\mathrm{A}+1)\big) \equiv 12P_A^5$
   c) $\mathrm{y}(\mathrm{A},(\mathrm{A}+1)(\mathrm{A}+2))-\mathrm{x}(\mathrm{A},(\mathrm{A}+1)(\mathrm{A}+2)) \equiv 36P_A^3$

Pattern II

Equation (3) is equivalent to

$$z^2=u^2-10v^2$$

Assume that $\quad z = \mathrm{a}^2-10\,\mathrm{b}^2$ (5)

Sub (5) in the above equation,

$(u+\sqrt{10}v)(u-\sqrt{10}v)=(a+\sqrt{10}b)^2(a-\sqrt{10}b)^2$ (6)

Equating the rational and irrational factors in (6), we get

$$u=u(a,b)=a^2+10b^2$$
$$v=v(a,b)=2ab$$

From which we obtained

$$x=a^2+4ab+10b^2$$
$$y=a^2+10ab+10b^2$$
$$z=a^2-10b^2$$

A few interesting properties observed are as follows

1. $x(3,b)-t_{170,b}+t_{150,b} \equiv 9 \ (mod\ 22)$
2. $x(4a,1)-t_{104,a}+t_{72,a} \equiv 10 \ (mod\ 32)$
3. $y(3a,2)-t_{146,a}+t_{128,a} \equiv 40 \ (mod\ 69)$
4. $y(2,2b)-80t_{3,b}-4 \equiv 0$
5. Each of the following expressions represents a Nasty numbers

(a) $y(a,b)+z(a,b)$

(b) $y(a,a)+z(a,a)$

(c) $x(a,1)-y(a,1)$

Pattern III

Equation (3) can be written as

$10v^2+z^2=1*u^2$ (7)

Assume that $\mathrm{u} = 10\mathrm{a}^2+\mathrm{b}^2$ (8)

Write 10 as $10=\big(\mathrm{i}\sqrt{10}\big)(-\mathrm{i}\sqrt{10})$ (9)

Use (8) and (9) in (7) and employing the method of factorization. Define

$$\big(z+i\sqrt{10}v\big)\big(z-i\sqrt{10}v\big)=\frac{(9+2i\sqrt{10})(9-2i\sqrt{10})}{}$$

$\big(z+i\sqrt{10}v\big)=\frac{1}{11}\{\big(9+2i\sqrt{10}\big)\big(b+i\sqrt{10}\,a\big)^2\}$ (10)

Equating the real and imaginary parts in (10)

$z=\frac{1}{11}\,(9b^2-90a^2-40ab)$ (11)

$v=\frac{1}{11}\,(18ab+2b^2-20a^2)$ (12)

Our interest is to obtained the integer solutions, so that the values of zand v are integers for suitable choices of the parameters a and b.

Put a=11A, b=11B in (8), (11) and (12), we get

$u=1210A^2+121B^2$ (13)

$v=22B^2+198AB-220A^2$ (14)

$z=99B^2-990A^2-440AB$ (15)

Substituting (13) and (15) in (2), the corresponding integer solutions of (1) are given by

$$\begin{aligned} x &= 770A^2+165B^2+396AB \\ y &= 110A^2+231B^2+990AB \\ z &= 99B^2-990A^2-440AB \end{aligned} \qquad (16)$$

Thus equation (16) represents non-zero distinct integral solution of (1) on two parameters.
A few interesting properties observed are as follows

1. $7y(B(B+1),B) - x(B(B+1),B) + 13068P_B^5 - T_{2894,B} \equiv 0(mod1445)$
2. $z\big((B+1)(B+2),B\big) + 9y\big((B+1)(B+2),B\big) - 50820P_B^3 - T_{3962,B} \equiv 0(mod1979)$
3. $y[A,(A+1)(2A+1) + z(A,(A+1)(2A+1) - 2x\big(A,(A+1)(2A+1)\big) + T_{4842,A} + 1452SqP_A \equiv 0(mod2419)$
4. $x(1,B) - 7y(1,B) + 2972T_{3,B} - T_{82,B} \equiv 0\ (mod5009)$

Conclusion
To conclude, one may search for other patterns of solutions and their corresponding properties.

Reference

1. Dickson LE. History of Theory of numbers, Chelsea Publishing Company, New York, 1952, 2.
2. Mordell LJ. Diophantine Equations, Academic press, London, 1969.
3. Andre Weil, Number Theory: An approach through history: from hammurapi to legendre / Andre weil: Boston (Birkhauser Boston, 1983.
4. Nigel Smart P. The algorithmic Resolutions of Diophantine equations, Cambridge university press, 1999.
5. Smith DE. History of mathematics Dover publications, New York, 1953, I(II).
6. Gopalan MA. Note on the Diophantine equation $x^2 + axy + by^2 = z^2$ Acta Ciencia Indica 2000; XXVIM(2):105-106.
7. Gopalan MA. Note on the Diophantine equation$x^2 + xy + y^2 = 3z^2$ Acta Ciencia Indica 2000; XXVIM(3):265-266.
8. Gopalan MA, Ganapathy R, Srikanth R. on the Diophantine equation $z^2 = Ax^2 + By^2$, Pure and Applied Mathematika Sciences 2000; LII(1-2):15-17.
9. Gopalanand MA, Anbuselvi R. On Ternary Quadratic Homogeneous Diophantine equation $x^2 + Pxy + y^2 = z^2$, Bulletin of Pure and Applied Sciences 2005; 24E(2):405-408.
10. Gopalan MA, Vidhyalakshmi S, Krishnamoorthy A. Integral solutions Ternary Quadratic$ax^2 + by^2 = c(a+b)z^2$, Bulletin of Pure and Applied Sciences 2005; 24E(2):443-446.
11. Gopalan MA, Vidhyalakshmiands S. Devibala, Integral solutions of $ka(x^2 + y^2) + bxy = 4k\alpha^2 z^2$, Bulletin of Pure and Applied Sciences 2006; 25E(2):401-406.
12. Gopalan MA, Vidhyalakshmiands S. Devibala, Integral solutions of $7x^2 + 8y^2 = 9\,z^2$, Pure and Applied Mathematika Sciences, 2007; LXVI(1-2):83-86.
13. Gopalan MA, Vidhyalakshmi S. An observation on $kax^2 + by^2 = cz^2$, Acta Cienica Indica 2007; XXXIIIM(1):97-99.
14. Gopalan MA, Manjusomanath, Vanitha N. Integral solutions of $kxy + m(x+y) = z^2$, Acta Cienica Indica 2007; XXXIIIM(4):1287-1290.
15. Gopalan MA, Kaliga Rani J. Observation on the Diophantine Equation $y^2 = Dx^2 + y^2$, Impact J Sci Tech. 2008; 2(2):91-95.
16. Gopalan MA, Pondichelvi V. On Ternary Quadratic Equation $x^2 + y^2 = z^2 + 1$, Impact J.Sci. Tech,Vol (2), No:2,2008,55-58.
17. Gopalan MA, Gnanam A. Pythagorean triangles and special polygonal numbers, International Journal of Mathematical Science. 2010; 9(1-2):211-215.
18. Gopalan MA, Vijayasankar A. Observations on a Pythagorean Problem, Acta Cienica Indica 2010; XXXVIM(4):517-520.
19. Gopalan MA, Pandichelvi V. Integral Solutions of Ternary Quadratic Equation $Z(X-Y) = 4XY$, Impact J Sci Tech. 2011; 5(1):01-06.
20. Gopalan MA, Kaligarani J. On Ternary Quadratic Equation $X^2 + Y^2 = Z^2 + 8$, Impact J Sci Tech. 2011; 5(1):39-43.