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M Latha
Ph.D Scholar in Mathematics,
Karpagam University,
Coimbatore-641021

N Anitha
Department of Mathematics,
Karpagam University,
Coimbatore-641021

Correspondence:
M Latha
Ph.D scholar in Mathematics,
Karpagam University,
Coimbatore-641021

Anti (S, Q)-fuzzy Subhemirings of a Hemiring

M Latha, N Anitha

Abstract
In this paper, an attempt has been madeto study the algebraic nature of an anti(S, Q)-fuzzy subhemiring of a hemiring.
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Introduction

There are many concepts of universal algebras generalizing an associative ring(R; +; .). Some of them, in particular, about near rings and several kinds of semirings have been proved very useful. Semirings (also called half rings) are algebras (R; + ;.) which share the same properties as a ring excepting that (R; +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications the theory of automata and formal languages. An algebra (R; +;.) is said to be a semi ring (R; +) and (R; .) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemi ring if it is additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [12], several researchers explored the generalization of the concept of fuzzy sets. The notion of anti-left h-ideals in hemi ring was introduced by Akram. M and K.H. Dar [1].The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N. Palaniappan & K. Arjunan [6]. Osman Kanzanci, Sultan Yamark and Serife Yilmaz in [13] introduced the notion of intuitionistic Q-fuzzification of N-subgroups (sub near-rings) in a near –ring and investigated some related properties. A. Solairaju and R. Nagarajan have given a new structure in construction of Q-fuzzy groups and subgroups [14,[15]. This paper introduces some properties and theorems in (S,Q)-fuzzy subhemirings of a hemiring.

1. Preliminaries

1.1 Definition: A S-norm is a binary operation $S: [0,1] X [0,1] \rightarrow [0,1]$ satisfying the following requirements:
(i) 0 S x = x, 1 S x=1(boundary conditions)
(ii) x S y = y S x (commutativity)
(iii) x S (y S z)= (x S y) S z (associativity)
(iv) If x £ y and w £ z, then x S w £ y S z (monotonicity).

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is function $A: X \times Q \rightarrow [0,1]$.

1.3 Definition: Let (R, +,.) be a hemiring. A Q-fuzzy subset of R is said to be an anti (S,Q)-fuzzy subhemiring(anti Q-fuzzy subhemiring with respect to S-norm) of R if it satisfies the following conditions:
(i) $\mu_A(x+y,q) \leq S((\mu_A(x,q), \mu_A(y,q))$
(ii) $\mu_A(xy,q) \leq S((\mu_A(x,q), \mu_A(y,q))$, for all x and y in R, and q in Q.

1.4 Definition: Let A and B be (S, Q)-fuzzy subsets of sets G and Respectively.
The anti-product of A and B, denoted by AxB is defined as AxB=$\left\{\frac{\langle(x,y),q),\mu_{AXB}(x,y),q)\rangle}{}\text{for all x in G and y in H \&q in Q}\right\}$, where $\mu_{AXB}(x,y),q) = \max\{(\mu_A(x,q),\mu_B(y,q)\}$.

1.5 Definition: Let A be a Q-fuzzy subset in a set S, the anti-strongest relation Q-fuzzy relation on S, that is a Q - fuzzy relation on A is V given by $\mu_V((x,y),q) = \max\{(\mu_A(x,q),\mu_B(y,q)\}$for all x and y in S and q in Q.

1.6 Definition: Let (R, +, .) and $(R',+,.)$ be any two hemirings. Let $f: R \to R'$ be any function and A be an anti (S, Q)-fuzzy subhemiring in R, V be an anti (S, Q)-fuzzy subhemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \inf_{x\in f^{-1}(y)}\mu_A(x,q)$ for all x in R and yin R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an anti(S, Q)-fuzzy subhemiring of a hemiring (R, +, .) and a in R, then the pseudo anti (S, Q)-fuzzy $\text{coset}(aA)^p$ is defined by $((a\mu_A)^P)(x,q) = p^{(a)}S(\mu_A(x,q))$, for every x in R, q in Q and for some p in P.

2. Properties Of Anti (S,Q)-Fuzzy Subhemiring Of A Hemiring

2.1 Theorem: Union of any two anti (S, Q)-fuzzy subhemiring of a hemiring R is an anti (S,Q)-fuzzy subhemiring of R.

Proof: Let A and B be any two anti (S, Q)-fuzzy subhemirings of a hemiring R and x and y in R. Let A=$\{\langle(x,q),\mu_A(x,q)\rangle/ x\in R \,\&q\in Q\}$ and B=$\{\langle(x,q),\mu_B(x,q)\rangle/ x\in R \,\&q\in Q\}$and alsolet$C = AUB = \{\langle(x,q),\mu_C(x,q)\rangle/ x\in R \,\&q\in Q\}$, where $\max\{\mu_A(x,q),\mu_B(x,q)\} = \mu_C(x,q)$,
Now, $\mu_C(x+y,q) \le \max\{S(\mu_A(x,q),\mu_A(y,q), S(\mu_B(x,q),\mu_B(y,q))\} \le S(\mu_C(x,q),\mu_C(y,q))$.
Therefore, $\mu_C(x+y,q) \le S(\mu_C(x,q),\mu_C(y,q))$, forallxandyinRandqinQ.
And,$\mu_C(xy,q) \le \max\{S(\mu_A(x,q),\mu_A(y,q)), S(\mu_B(x,q),\mu_B(y,q))\} \le S(\mu_C(x,q),\mu_C(y,q))$.
Therefore,$\mu_C(xy,q) \le S(\mu_C(x,q),\mu_C(y,q))$, for all x and y in R and q in Q. Therefore C is an anti (S, Q)-fuzzy subhemiring of a hemiring R.

2.2 Theorem: The Union of a family of anti(S,Q)-fuzzy subhemirings of hemiring R is an anti (S,Q)-fuzzy subhemiring of R.

Proof: It is trivial.

2.3 Theorem: If A and B are two anti(S,Q)-fuzzy subhemirings of the hemirings R_1 and R_2 respectively,then anti product AXB is an anti (S,Q)-fuzzy subhemiring of $R_1 X R_2$.

Proof: Let A and B be two anti(S,Q)-fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Letx_1 and x_2 be in R_1, $y_1$and $y_2$be in R_2.Then $((x_1,y_1), q)$ and $((x_2,y_2), q)$ are in$R_1 X R_2$.
Now,$\mu_{AXB}[((x_1,y_1)+(x_2,y_2)),q)] \le \max\{S(\mu_A(x_1,q),\mu_A(x_2,q)), S(\mu_B(y_1,q),\mu_B(y_2,q))\}$
$\le S(\mu_{AXB}(x_1,y_1),q),\mu_{AXB}(x_2,y_2),q))$.Therefore,$\mu_{AXB}[((x_1,y_1)+(x_2,y_2)),q)]$
$\le S(\mu_{AXB}(x_1,y_1),q),\mu_{AXB}(x_2,y_2),q))$. Also, $\mu_{AXB}[((x_1,y_1)(x_2,y_2)),q)] \le$
$\max\{S(\mu_A(x_1,q),\mu_A(x_2,q)), S(\mu_B(y_1,q),\mu_B(y_2,q))\} \le S(\mu_{AXB}(x_1,y_1),q),\mu_{AXB}(x_2,y_2),q))$.
Therefore,$\mu_{AXB}[((x_1,y_1)(x_2,y_2)),q)] \le S(\mu_{AXB}(x_1,y_1),q),\mu_{AXB}(x_2,y_2),q))$. Hence AXB is an anti (S,Q)-fuzzy subhemiring of a hemiring $R_1 X R_2$.

2.4 Theorem: Let A be a Q-fuzzy subset of a hemiring R and V be the anti-strongest fuzzy relation of R. Then A is an anti (S, Q)-fuzzy subhemiring of R if and only if V is an anti (S, Q)-fuzzy subhemiring of RxR.

Proof: Supposethat Aisananti (S, Q)- fuzzy subhemiring of a hemiring R. Then for any
$X = (x_1,x_2) \,\&\, Y = (y_1, y_2)$are in RxR. We have$\mu_V(XY,q) \le \max\{S(\mu_A(x_1y_1,q),\mu_A(x_2y_2,q))$
$\le \max\{S(\mu_A(x_1,q),\mu_A(y_1,q)), S(\mu_A(x_2,q),\mu_A(y_2,q)) \le S(\mu_V((x_1,x_2),q),\mu_V((y_1,y_2),q))$
$\le S(\mu_V(X,q),\mu_V(Y,q)$, forallXandYinRxRandqinQ. Therefore, $\mu_V(XY,q) \le S(\mu_V(X,q),\mu_V(Y,q))$, forallXandYinR x R and q in Q. This proves that V is an anti(S, Q)-fuzzy subhemiring ofa hemiring of R x R. Conversely assume that Visananti (S,Q) – fuzzy subhemiring of Ahemiring of R x R, then for anyX $= (x_1,x_2)$andY $= (y_1,y_2)$are in R x R. wehave
$\max\{S(\mu_A(x_1+y_1,q),\mu_A(x_2+y_2,q)\} = \mu_V(X+Y,q) \le S(\mu_V(X,q),\mu_V(Y,q) = S(\mu_V((x_1,x_2),q),\mu_V((y_1,y_2),q) =$
$S(\max(\mu_A(x_1,q),\mu_A(y_1,q)), \max(\mu_A(x_2,q),\mu_A(y_2,q))$.
If$x_2 = 0$,$y_2 = 0$, we get $\mu_A(x_1+y_1,q) \le S(\mu_A(x_1,q),\mu_A(y_1,q))$for all$x_1$ and y_1 in R.
And, $\max\{S(\mu_A(x_1y_1,q),\mu_A(x_2y_2,q)\} = \mu_V(XY,q) \le S(\mu_V(X,q),\mu_V(Y,q) = S(\mu_V((x_1,x_2),q),\mu_V((y_1,y_2),q) =$
$S(\max\{\mu_A(x_1,q),\mu_A(x_2,q)\}, \max\{\mu_A(y_1,q),\mu_A(y_2,q)\})$.
If $x_2 = 0$,$y_2 = 0$ We get $\mu_A(x_1y_1,q) \le S(\mu_A(x_1,q),\mu_A(y_1,q))$ for all $x_1$and y_1 in R.ThereforeA is an anti (S,Q)-fuzzy subhemiring of R.

2.5 Theorem: If A is an anti (S,Q)-fuzzy subhemiring of a hemiring (R, +, .) if and only if
$\mu_A(x+y,q) \le S(\mu_A(x,q),\mu_A(y,q)$,$\mu_A(xy,q) \le S(\mu_A(x,q),\mu_A(y,q)$ for all x and y in R.

Proof: It is trivial.

2.6 Theorem: If A is an anti (S, Q)-fuzzy subhemiring of a hemiring (R, +, .), then $H = \{x/x \in R : \mu_A(x,q) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

2.7 Theorem: Let A is an anti (S, Q)-fuzzy subhemiring of a hemiring (R, +, .).
If $\mu_A(x+y,q) = 1$, then either $\mu_A(x,q) = 1$ or $\mu_A(y,q) = 1$, for all x and y in R.

Proof: It is trivial.

2.8 Theorem: Let A is an anti (S,Q)-fuzzy subhemiring of a hemiring (R, +, .),then the pseudo anti(S,Q)-fuzzycoset$(aA)^P$is an anti (S,Q)-fuzzysubhemiringof a hemiring R, for every a in R.

Proof: Let A is an anti (S, Q)-fuzzy subhemiring of a hemiring R. For every x and y in R, we have $((a\,\mu_A)^p)(x+y,q) \le p^{(a)}\, S(\mu_A(x,q), \mu_A(y,q)) \in S\left(p^{(a)}\mu_A(x,q), p^{(a)}\mu_A(y,q) = S((a\,\mu_A)^p(x,q), (a\,\mu_A)^p(y,q)\right)$. Therefore,$((a\,\mu_A)^p)(x+y,q) \le S((a\,\mu_A)^p(x,q), (a\,\mu_A)^p(y,q))$.
Now,$((a\,\mu_A)^p)(xy,q) \le p^{(a)}\, S(\mu_A(x,q), \mu_A(y,q)) \in S\left(p^{(a)}\mu_A(x,q), p^{(a)}\mu_A(y,q)\right)$. Therefore,$((a\,\mu_A)^p)(xy,q) \le S((a\,\mu_A)^p(x,q), (a\,\mu_A)^p(y,q))$. Hence$(aA)^P$ is an anti (S,Q)-fuzzy subhemiring of a hemiring R.

2.9 Theorem: Let (R, +, .) and (R′, +, .) be any two hemi rings. The homomorphic image of an anti (S, Q)-fuzzy subhemiring of R is an anti (S, Q)-fuzzy subhemiring of R′.

Proof: Let$f : R \to R'$be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an anti $(S,Q)-$ fuzzy subhemiring of R. Now, for $f(x), f(y)$in R', $\mu_V((f(x) + (f(y)), q)) \le \mu_A(x+y,q) \le S(\mu_A(x,q), \mu_A(y,q)$, which implies that $\mu_V((f(x) + (f(y)), q)) \le S(\mu_V((f(x),q), \mu_V(f(y),q))$.Again,$\mu_V((f(x)f(y)),q)) \le \mu_A(xy,q) \le S(\mu_A(x,q), \mu_A(y,q)$,which implies that $\mu_V((f(x)f(y),q)) \le S(\mu_A(x,q), \mu_A(y,q)$
Hence V is an anti (S,Q)-fuzzy subhemiring of hemiring R′.

2.10 Theorem: Let (R, +, .) and (R′, +, .) be any two hemirings. The homomorphic preimage of an anti (S, Q)-fuzzy subhemiring of R′ is an anti (S, Q)-fuzzy subhemiring o f R.

Proof: Let $f : R \to R'$be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let$V = f(A)$, where V is an anti $(S,Q)-$ fuzzy subhemiring of R′.
Now, for x, y in R, $\mu_A(x+y,q) = \mu_V((f(x) + f(y)), q) \le S(\mu_V(f(x),q), \mu_V(f(y),q) = S(\mu_A(x,q), \mu_A(y,q)$, whichimpliesthat$\mu_A((x+y,q)) \le S(\mu_A(x,q), \mu_A(y,q)$. Again,$\mu_A((xy,q)) = \mu_V((f(x)f(y)),q) \le S(\mu_V(f(x),q), \mu_V(f(y),q) = S(\mu_A(x,q), \mu_A(y,q)$,which implies that $\mu_A((xy,q)) \le S(\mu_A(x,q), \mu_A(y,q)$. Hence A is an anti (S, Q) fuzzy subhemiring of hemiring R.

2.11 Theorem: Let (R, +, .) and (R′, +, .) be any two hemirings. The anti-homomorphic image of an anti (S, Q)-fuzzy subhemiring of R is an anti (S,Q)-fuzzy subhemiring of R′.

Proof: Let$f : R \to R'$be a homomorphism. Then, $f(x+y) = (x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an anti $(S,Q)-$ fuzzy subhemiring of R.
$\mu_V((f(x) + f(y)), q)) \le \mu_A(y+x,q) \le S(\mu_A(y,q), \mu_A(x,q)) = S(\mu_A(x,q), \mu_A(y,q))$, which implies that $\mu_V((f(x) + (f(y)), q)) \le S(\mu_V(f(x),q), \mu_V(f(y),q))$. Again, $\mu_V((f(x)f(y),q)) \le \mu_A(yx,q) \le S((\mu_A(y,q), \mu_A(x,q)) = S((\mu_A(x,q), \mu_A(y,q))$, which implies that $\mu_V((f(x)f(y),q)) \le S(\mu_V(f(x),q), \mu_V(f(y),q))$. Hence V is an anti (S, Q)- fuzzy subhemiring of hemiring **R′**.

2.12 Theorem: Let (R, +, .) and (R′, +, .) be any two hemirings. The anti-homomorphic preimage of an anti (S, Q)-fuzzy subhemiring of R′ is an anti (S,Q)-fuzzy subhemiring of R.

Proof: Let $V = f(A)$ where V is an anti (S, Q)fuzzy subhemiring of R′. Let x and y in R. Then $\mu_A((x+y,q)) = \mu_V((f(x) + (f(y)), q) \le S(\mu_V(f(y),q), \mu_V(f(x),q) = S(\mu_A(x,q), \mu_A(y,q))$, which implies that, $\mu_A((x+y,q)) \le S(\mu_A(x,q), \mu_A(y,q)$.Again$\mu_A((xy,q)) = \mu_V((f(x)f(y),q) \le S(\mu_V(f(y),q), \mu_V(f(x),q) = S(\mu_A(x,q), \mu_A(y,q))$,which implies that $\mu_A((xy,q)) \le S((\mu_A(x,q), \mu_A(y,q))$.Hence A is an anti (S,Q) fuzzy subhemiring of hemiring R.

In the following Theorem∘ is the composition operation of functions

2.13 Theorem: Let A be an anti (S, Q)-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemi ring R onto H. Then A∘ f is an anti (S, Q)-fuzzy subhemiring of R.

Proof: Let xand y in R. Then we have, $(\mu_A \circ f)\big((x+y, q)\big) = \mu_A((f(x), q) + (f(y), q) \leq S(\mu_A(f(x), q), \mu_A(f(y), q) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$, which implies that

$(\mu_A \circ f)\big((x+y, q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$. And, $(\mu_A \circ f)\big((xy, q)\big) = \mu_A\big((f(x), q)(f(y), q)\big)$
$\leq S\big((\mu_A(f(x), q), \mu_A(f(y), q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$,

which implies that $(\mu_A \circ f)\big((xy, q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$. Therefore A ∘f is an anti (S, Q) fuzzy subhemiring of hemiring R.

2.14 Theorem: Let A bean anti (S, Q)-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then A∘ f is an anti (S, Q)-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $(\mu_A \circ f)\big((x+y, q)\big) = \mu_A((f(y), q) + (f(x), q) \leq S\big(\mu_A(f(x), q), \mu_A(f(y), q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$, which implies that
$(\mu_A \circ f)\big((x+y, q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$. And, $(\mu_A \circ f)\big((xy, q)\big) = \mu_A((f(y), q)(f(x), q) \leq S\big((\mu_A(f(x), q), \mu_A(f(y), q)\big) \leq \big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$, which implies that $(\mu_A \circ f)\big((xy, q)\big) \leq S\big((\mu_A \circ f)(x, q), (\mu_A \circ f)(y, q)\big)$.Therefore A∘f is an anti (S, Q) fuzzy subhemiring of hemiring R.

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