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Cotangent similarity measure of rough fuzzy sets and its application on wireless network

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Abstract
Similarity measure plays a vital role in pattern recognition and medical diagnosis. In this paper, a new rough fuzzy cotangent similarity measure between two rough fuzzy sets is formulated. Moreover, considering the importance of each element, the weighted rough fuzzy cotangent similarity (WRFS) is proposed. We also provide a numerical example to show the effectiveness and the flexibility of the proposed method.

Keywords: Cotangent similarity measure, rough fuzzy, wireless network

1. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. So the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own role in solving the difficulties.

The notion of rough set theory was proposed by Z. Pawlak [21]. The concept of rough set theory is an extension of crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful tool for dealing with uncertainty or imprecision information. In recent years the concept of rough fuzzy set was found to be more useful in decision making problems.

Fuzzy sets and rough sets are two different concepts and both are capable of handling uncertainty and incomplete information. The new concept called the rough fuzzy sets seems to be very interesting and applicable in realistic problems. Similarity measure is an excellent method for decision making in real life situations. Many methods have been proposed to measure the degree of similarity between fuzzy sets. Wang [17] first introduced the concept of similarity measure of fuzzy sets and give a computation formula. Since then, the similarity measure of fuzzy sets has successfully been applied in many fields, such as pattern recognition, medical diagnose, fuzzy clustering, fuzzy neural networks, fuzzy reasoning, and fuzzy control. So far, there have been many discussions on similarity measures [2-20], and propose many formula to calculate similarity measure.

Application of rough fuzzy sets in decision making problems is a recent topic to solve real life problems in an imprecise environment. In this paper, Rough set theory has been combined with fuzzy sets in dealing with uncertainty decision making. This paper proposes a general decision-making framework based on the rough fuzzy set model. We have applied the notion of similarity between two rough fuzzy sets to obtain the solution of a decision problem in an imprecise environment.

2. Preliminaries:

Definition [9]: Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in a fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples A = $\{(x, \mu_A(x)) : x \in X\}$

Definition [9]: Let A = $\{(x, \mu_A(x)) : x \in X\}$ and B = $\{(x, \mu_B(x)) : x \in X\}$ be two fuzzy sets in X. Then their union $A \vee B$, intersection $A \wedge B$ and complement A^c are also fuzzy sets with the membership functions defined as follows:

(i) $\mu_{A \vee B}(x)$ = max $\{\mu_A(x), \mu_B(x)\}, \forall x \in X$

(ii) $\mu_{A \wedge B}(x)$ = min $\{\mu_A(x), \mu_B(x)\}, \forall x \in X$

(iii) $\mu_{A^c}(x)$ = 1 - $\mu_A(x)$, $\forall x \in X$

Further,

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\forall x \in X$

(b) A = B iff $\mu_A(x) = \mu_B(x)$, $\forall x \in X$

Definition [24]: Suppose (*U*, *R*) is an approximation space, where, *U* is a finite nonempty domain, *R* inidicates an equivalence relation on *U*, and $[x]_R$ represents the equivalence classes including $x \in U$ based on the equivalence relation *R*. If *X* is a nonempty subset on U, then *R*-lower approximation and *R*-upper approximation of *X* on *U* are separately defined by

$\underline{R}X = \{x \in U / [x]_R \subseteq X\}$

$\overline{R}X = \{x \in U / [x]_R \cap X \neq \phi\}$

If $\underline{R}X$ = $\overline{R}X$, we say that *X* is definable; otherwise, we say *X* is rough, and the set pair ($\underline{R}X$, $\overline{R}X$) is called the rough set of *X*.

Definition [24] suppose (*U*, *R*) is an approximation space. *R* is the equivalence relation, and *A* is a fuzzy set on *U*. Then, the lower approximation $\underline{A}R$ and the upper approximation $\overline{A}R$ of *A* on (*U*, *R*) are a pair of fuzzy sets, and the membership functions are respectively defined as

$\underline{A}R(x)$ = inf $\{A(y) / y \in [x]_R\},\ x \in U$

$\overline{A}R(x)$ = sup $\{A(y) / y \in [x]_R\},\ x \in U$

If $\underline{A}R$ = $\overline{A}R$, we say that *A* is definable; otherwise, we say that *A* is rough, and the set pair $\left(\underline{A}R, \overline{A}R\right)$ is called the rough fuzzy set of *A*.

3. Rough fuzzy cotangent similarity measure

Definition 3.1: Let A = $<\left\langle (\underline{\mu}_A(x_i)), (\overline{\mu}_A(x_i)\right\rangle>$ and B = $<\left\langle (\underline{\mu}_B(x_i)), (\overline{\mu}_B(x_i)\right\rangle>$ be two rough fuzzy sets in $X = \{x_1, x_2, x_3, ...x_n\}$ Now we present the rough cotangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Therefore, a new cotangent similarity measure between rough fuzzy sets is proposed in 3-D vector space can be presented as:

$$COT_{RFS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left\langle Cot\left(\frac{\pi}{4}\left[1 + \left|\rho\mu_A(x_i)) - \rho\mu_B(x_i)\right|\right]\right)\right\rangle$$

Where,

$$\rho\mu_A(x_i) = \left(\frac{\underline{\mu}_A(x_i) + \overline{\mu}_A(x_i)}{2}\right)$$

$$\rho\mu_B(x_i) = \left(\frac{\underline{\mu}_B(x_i) + \overline{\mu}_B(x_i)}{2}\right)$$

Proposition 3.2:

The defined cotangent similarity measure $COT_{RFS}(A,B)$ between two rough fuzzy sets A and B satisfies the following conditions:

(i) $0 \leq COT_{RFS}(A,B) \leq 1$

(ii) $COT_{RFS}(A,B) = COT_{RFS}(B,A)$

(iii) $COT_{RFS}(A,B)$ = 1 iff A = B

(iv) If C is RFS in Y and A⊂B⊂C then, $COT_{RFS}(A,C) \leq COT_{RFS}(A,B)$, and $COT_{RFS}(A,C) \leq COT_{RFS}(B,C)$

Proof:

(i) Since, $\frac{\pi}{4} \leq \left(\frac{\pi}{4}\left[1 + \left|\rho\mu_A(x_i)) - \rho\mu_B(x_i)\right|\right]\right) \leq \frac{\pi}{2}$, it is obvious that the cotangent function $COT_{RFS}(A,B)$ are within 0 and 1.

(ii) It is obvious.

(iii) When A = B, then obviously $COT_{RFS}(A,B)$ = 1.

On the other hand if $COT_{RFS}(A,B)$ =1, then,

$\rho\mu_A(x_i) = \rho\mu_B(x_i)$

$\underline{\mu}_A(x_i) = \underline{\mu}_B(x_i)$

$\overline{\mu}_A(x_i) = \overline{\mu}_B(x_i)$

This implies that A = B.

(iv)Suppose A⊂B⊂C then we can write $\underline{\mu}_A(x_i) \leq \underline{\mu}_B(x_i) \leq \underline{\mu}_C(x_i)$,

$\overline{\mu}_A(x_i) \leq \overline{\mu}_B(x_i) \leq \overline{\mu}_C(x_i)$.

The cotangent function is decreasing function within the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Hence we can write $COT_{RFS}(A,C) \leq COT_{RFS}(A,B)$, and $COT_{RFS}(A,C) \leq COT_{RFS}(B,C)$.

Definition 3.3: Weighted rough fuzzy cotangent similarity measure

If we consider the weights of each element *x*i, a weighted rough fuzzy cotangent similarity measure between rough neutrosophic sets *A* and *B* can be defined as follows:

$$COT_{WRFS}(A,B) = \sum_{i=1}^{n} w_i\left\langle Cot\left(\frac{\pi}{4}\left[1 + \left|\rho\mu_A(x_i)) - \rho\mu_B(x_i)\right|\right]\right)\right\rangle$$

$w_i \in [0, 1]$, i = 1, 2, ……..n and $\sum_{i=1}^{n} w_i = 1$.

If we take $w_i = \frac{1}{n}$, i = 1, 2, 3………n, then $COT_{WRFS}(A,B) = COT_{RFS}(A,B)$.

Proposition 3.4: The weighted rough fuzzy cotangent similarity measure $COT_{WRFS}(A,B)$ between two rough neutrosophic sets *M* and *N* also satisfies the following properties:

(i) $0 \leq COT_{WRFS}(A,B) \leq 1$

(ii) $COT_{WRFS}(A,B) = COT_{WRFS}(B,A)$

(iii) $COT_{WRFS}(A,B) = 1$ iff A = B

(iv) If C is WRFS in Y and $A \subset B \subset C$ then, $COT_{WRFS}(A,C) \leq COT_{WRFS}(A,B)$, and $COT_{WRFS}(A,C) \leq COT_{WRFS}(A,B)$.

Proof:

(i) Since, $\frac{\pi}{4} \leq \left(\frac{\pi}{4}\left[1 + \left|\rho\mu_A(x_i)) - \rho\mu_B(x_i)\right|\right]\right) \leq \frac{\pi}{2}$, and $\sum_{i=1}^{n} w_i = 1$ it is obvious that the weighted cotangent function $COT_{WRFS}(A,B)$ are within 0 and 1. That is,

(ii) $0 \leq COT_{WRFS}(A,B) \leq 1$

(iii) It is obvious.

(iv) Here $\sum_{i=1}^{n} w_i = 1$. When *A* = B, then obviously $COT_{WRFS}(A,B) = 1$. On the other hand if $COT_{WRFS}(A,B) = 1$ then,

(v) $\rho\mu_A(x_i) = \rho\mu_B(x_i)$

(vi) $\underline{\mu}_A(x_i) = \underline{\mu}_B(x_i)$

(vii) $\overline{\mu}_A(x_i) = \overline{\mu}_B(x_i)$

(viii) This implies that A = B.

(ix) If $A \subset B \subset C$ then we can write $\underline{\mu}_A(x_i) \leq \underline{\mu}_B(x_i) \leq \underline{\mu}_C(x_i)$, $\overline{\mu}_A(x_i) \leq \overline{\mu}_B(x_i) \leq \overline{\mu}_C(x_i)$.

The cotangent function is decreasing function within the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

Here, $\sum_{i=1}^{n} w_i = 1$.

Hence we can write $COT_{WRFS}(A,C) \leq COT_{WRFS}(A,B)$, and $COT_{WRFS}(A,C) \leq COT_{WRFS}(A,B)$.

4. Example on Wireless Networks

We consider a wireless network problem from practical point of view for the illustration of the proposed approach. A mobile phone also known as a wireless phone, cell phone is a very small portable radio telephone, which is used to communicate over long distances without wires. The modern mobile phones of today also support a wide variety of other services such as text messaging, MMS, email, Internet access, short-range wireless communications (Bluetooth, Whatsapp), business applications, gaming, and photography. So there is no communication without mobile phone, which is used to provide the worldwide service in business, health care, advertisements etc. So we keep so many SIMs with us for different usages. Some providers will give you a very good messaging offer with fair price; some other will be good in internet offers. At the same time lack of security, service facility, cost, speed and the short range of signal may lead us to some practical problems. Since there are many advantages and disadvantages, it is a difficult task to adopt the best wireless network. Here we consider the rough fuzzy cotangent similarity measure as a suitable tool for the decision making. The proposed similarity measure among the users and the Net Works will provide the proper decision. Now we present the example as follows:

Let P = {P_1, P_2, P_3} be a set of users and Q = {Cost, Range, Special Offers, Net Access, Customer Satisfaction} be a set of features and R = {AIRTEL, BSNL, AIRCEL, DOCOMO, IDEA} be a set of wireless networks. Our task is to determine the users and to determine the best network in rough fuzzy set environment.

Table 1: (Relation 1) the relation between the users and the features:

	Cost	Range	Special Offers	Net Access	Customer Satisfaction
P_1	$\langle 0.1, 0.5 \rangle$	$\langle 0.3, 0.3 \rangle$	$\langle 0.5, 0.7 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.1, 0.7 \rangle$
P_2	$\langle 0.2, 0.4 \rangle$	$\langle 0.4, 0.8 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.6, 0.8 \rangle$
P_3	$\langle 0.2, 0.8 \rangle$	$\langle 0.7, 0.9 \rangle$	$\langle 0.1, 0.5 \rangle$	$\langle 0.2, 0.4 \rangle$	$\langle 0.3, 0.7 \rangle$

Table 2: (Relation 2) the relation between the features and the wireless Networks

	AIRTEL	BSNL	AIRCEL	DOCOMO	IDEA
Cost	$\langle 0.6, 0.8 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.1, 0.5 \rangle$	$\langle 0.5, 0.9 \rangle$	$\langle 0.3, 0.7 \rangle$
Range	$\langle 0.3, 0.5 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.4, 0.8 \rangle$	$\langle 0.5, 0.7 \rangle$	$\langle 0.2, 0.6 \rangle$
Special Offers	$\langle 0.4, 0.6 \rangle$	$\langle 0.6, 0.8 \rangle$	$\langle 0.1, 0.8 \rangle$	$\langle 0.6, 0.6 \rangle$	$\langle 0.4, 0.8 \rangle$
Net Access	$\langle 0.6, 0.8 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.7, 0.9 \rangle$	$\langle 0.3, 0.5 \rangle$
Customer Satisfaction	$\langle 0.3, 0.9 \rangle$	$\langle 0.4, 0.8 \rangle$	$\langle 0.4, 0.6 \rangle$	$\langle 0.3, 0.7 \rangle$	$\langle 0.2, 0.4 \rangle$

Table 3: The relation between the users and the wireless Networks by using WRFS

	AIRTEL	BSNL	AIRCEL	DOCOMO	IDEA
P_1	0.7114	0.8031	0.8642	0.6972	0.8869
P_2	0.7548	0.7920	0.8869	0.7151	0.7086
P_3	0.6652	0.7698	0.8067	0.6960	0.7406

From table 3 we conclude that the users P_1 and P_2 prefers AIRCEL and the user P_3 adopts IDEA for their wireless Networks.

Conclusion

In this paper, we have proposed rough fuzzy cotangent similarity measure of rough fuzzy sets and proved some of their basic properties. We have presented an application of rough cotangent similarity measure of rough fuzzy sets in wireless network problems. We hope that the proposed concept can be applied in solving realistic multi-attribute decision making problems.

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