International Journal of Applied Research 2015; 1(11): 511-518

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(11): 511-518
www.allresearchjournal.com
Received: 02-08-2015
Accepted: 05-09-2015

RS Wali
Department of Mathematics,
Bhandari Rathi College,
Guledagudd-587 203,
Karnataka State, India.

Prabhavati S Mandalageri
Department of Mathematics,
K.L.E'S, S.K. Arts College &
H.S.K. Science Institute,
Hubballi-31, Karnataka State,
India.

Correspondence
Prabhavati S Mandalageri
Department of Mathematics,
K.L.E'S, S.K. Arts College &
H.S.K. Science Institute,
Hubballi-31, Karnataka State,
India.

On αrω–closed and αrω–open maps inTopological Spaces

RS Wali, Prabhavati S Mandalageri

Abstract
The aim of this paper is to introduce new type αrω–closed maps and αrω–open maps, αrω*–closed maps and αrω*–open maps. We also obtain some properties of αrω–closed maps and αrω–open maps.

Keywords: αrω–closed maps, αrω*-closed maps and αrω–open maps, αrω*- open maps.

Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. Norman Levine introduced the notion of generalized closed sets. After him different mathematicians worked and studied on different versions of generalized closed sets and related topological properties. Generalized closed mappings were introduced and studied by Malghan [1]. wg-closed maps and rwg-closed maps were introduced and studied by Nagavani [2]. Regular closed maps, gpr-closed maps and rw-closed maps have been introduced and studied by Long [3], Gnanambal [4] and R S Wali [5] respectively. In this paper, a new class of maps called α regular ω–closed (briefly, αrω–closed) maps, αrω*- closed maps have been introduced and studied their relations with various generalized closed maps. We prove that the composition of two αrω–closed maps need not be αrω–closed map. We also obtain some properties of αrω–closed maps.We give the definitions of some of them which are used in our present study.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.

We recall the following definitions and results.

Definition 2.1: A subset A of a topological space (X, τ) is called

1) Semi-open set [6] if A ⊆cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.
2) Pre-open set [7] if A ⊆int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
3) α-open set [8] if A ⊆int(cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.
4) Semi-preopen set [9] (=β-open [10] if A⊆cl(int(cl(A)))) and a semi-pre closed set (=β-closed) if int(cl(int(A))) ⊆A.
5) Regular open set [11] if A = int(clA)) and a regular closed set if A = cl(int(A)).
6) Regular semi open set [12] if there is a regular open set U such that U ⊆ A ⊆cl(U).
7) Regular α-open set [13] (briefly,rα-open) if there is a regular open set U such that U ⊆ A ⊆ αcl(U).

Definition 2.2: A subset A of a topological space (X,τ) is called

1. Generalized pre regular closed set(briefly gpr-closed)[4] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

2. ωα- closed set [14] if αcl(A) ⊆ U whenever A⊆ U and U is ω-open in X.
3. α regular ω- closed (briefly αrω -closed) set[15] if αcl(A) ⊆ U whenever A⊆U and U is rω-open in X.
4. Regular generalized α-closed set (briefly, rgα-closed)[13] if αcl (A)⊆U whenever A⊆U and U is regularα-open in X.
5. Generalized closed set(briefly g-closed)[16] if cl(A) ⊆ U whenever A⊆ U and U is open in X.
6. Generalized semi-closed set(briefly gs-closed)[17] if scl(A) ⊆ U whenever A⊆U and U is open in X.
7. Generalized semi pre regular closed (briefly gspr-closed) set [18] if spcl(A)⊆U whenever A⊆U and U isregular open in X.
8. Strongly generalized closed set [18](briefly,g*-closed) if Cl(A)⊆U whenever A⊆U and U is g-open in X.
9. α-generalized closed set(briefly αg -closed)[19] if αcl(A)⊆U whenever A⊆U and U is open in X.
10. ω-closed set[20] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
11. weakely generalized closed set(briefly, wg-closed)[2] if cl(int(A))⊆U whenever A⊆U and U is open in X.
12. Regular weakly generalized closed set (briefly, rwg-closed)[2] if cl(int(A)) ⊆ U whenever A⊆U and U is regular open in X.
13. Semi weakly generalized closed set (briefly, swg-closed)[2] if cl(int(A))⊆UwheneverA ⊆U and U is semi open in X.
14. Generalized pre closed (briefly gp-closed) set [21] if pcl(A) ⊆U whenever A⊆U and U is open in X.
15. Regular ω-closed (briefly rω -closed) set [5] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
16. g*-pre closed (briefly g*p-closed) [22] if pcl(A)⊆ U whenever A⊆ U and U is g-open in X
17. Generalized regular closed (briefly gr–closed) set [23] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
18. Regular generalized weak (briefly rgw-closed) set [24] if cl(int(A)) ⊆ U whenever A ⊆ U and U is regular semi open in X.
19. Weak generalized regular–α closed (briefly wgrα-closed) set [25] if cl(int(A)⊆ U whenever A⊆ U and U is regular α-open in X.
20. Regular pre semi–closed (briefly rps-closed) set [26] if spcl(A) ⊆U whenever A ⊆U andU is rg- open in X.
21. Generalized pre regular weakly closed (briefly gprw-closed) set [27] if pcl(A)⊆U whenever A⊆ U and U is regular semi-open in X.
22. α-generalized regular closed(briefly αgr-closed) set [28] if αcl(A) ⊆U whenever A ⊆U and U is regular open in X.
23. R*-closed set [29] if rcl(A) ⊆U whenever A⊆ U and U is regular semi-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map f: (X,τ)→(Y,σ) is said to be
1. Regular-continuous(r-continuous) [30] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
2. Completely–continuous [30] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
3. g–continuous[31] if $f^{-1}(V)$ is g–closed in X for every closed subset V of Y
4. αrω–continuous [32] if $f^{-1}(V)$ is αrω–closed in X for every closed subset V of Y.
5. Strongly αrω–continuous [32] if $f^{-1}(V)$ is closed set in X for every αrω–closed set V in Y.

Definition 2.4: A map f: (X,τ)→(Y,σ) is said to be
1. α–irresolute [7] if $f^{-1}(V)$ isα-closed in X for every α-closed subset V of Y.
2. αrω–irresolute [32] if $f^{-1}(V)$ is αrω–closed in X for every αrω-closed subset V of Y.
3. αg–irresolute [19] if $f^{-1}(V)$ is αg-closed in X for every αg-closed subset V of Y.
4. Irresolute [3] if $f^{-1}(V)$ is semi-closed in X for every semi-closed subset V of Y.
5. Contra irresolute [7] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V ofY.
6. contrar–irresolute[1] if $f^{-1}(V)$ is regular-open in Xfor everyregular-closed subset V of Y
7. Contra continuous [38] if $f^{-1}(V)$ is open in X for every closed subset V of Y.
8. rω*-open(resprω*-closed) [20] map if f(U) is rω-open (resp rω-closed) in Y for every rω-open (resp rω-closed)subsetU ofX.
9. α*-quotient map[34] if f is α-irresolute and $f^{-1}(V)$ is an α-open set in (X,τ) implies V is an open set in (Y,σ**).**

Lemma 2.5 see [15]
1. Every closed (resp regular-closed, α-closed) set is αrω-closed set in X.
2. Every αrω-closed set is αg-closed set
3. Every αrω-closed set is αgr-closed (resp ωα-closed, gs-closed, gspr-closed, wg-closed, rwg-closed, gp-closed, gpr-closed) set in X

Lemma 2.6 see [15]: If a subset A of a topological space X, and
1. If A is regular open and αrω-closed then A is α-closed set in X
2. If A is open and αg-closed then A is αrω-closed set in X
3. If A is open and gp-closed then A is αrω-closed set in X
4. If A is regular open and gpr-closed then A is αrω-closed set in X
5. If A is open and wg-closed then A is αrω-closed set in X
6. If A is regular open and rwg-closed then A is αrω-closed set in X
7. If A is regular open and αgr-closed then A is αrω-closed set in X
8. If A is ω-open and ωα-closed then A is αrω-closed set in X

Lemma 2.7: see [15] If a subset A of a topological space X, and
1. If A is semi-open and sg-closed then it is αrω-closed.
2. If A is semi-open and ω-closed then it is αrω-closed.
3. A is αrω-open iff U ⊆αint(A), whenever U is rω-closed and U ⊆A.

Definition 2.8: A topological space (X,τ) is called
1. an α-space [8] if every α-closed subset of X is closed in X.

2. $T_{½}$ space [1] if every g-closed set is closed.
3. $_{½}T_{\alpha}$-space[19] if every αg-closed set is α-closed.
4. $T_{\alpha r\omega}$ space [32] if every αrω-closed set is closed.

Definition 2.9 A map f : (X, τ) → (Y, σ) is said to be
1. α–closed[8] if f(F) is α–closed in Y for every closed subset F ofX.
2. αg–closed[19] if f(F) is αg–closed in Y for every closed subset F ofX.
3. wg–closed[2] if f(V) is wg–closed in Y for every closed subset V ofX
4. rwg–closed[2] if f(V) is rwg–closed in Y for every closed subset V ofX
5. gs–closed[17] if f(V) is gs–closed in Y for every closed subset V ofX
6. gp–closed[21] if f(V) is gp–closed in Y for every closed subset V ofX
7. gpr–closed[4] if f(V) is gpr–closed in Y for every closed subset V ofX
8. αgr–closed[28] if f(V) is αgr–closed in Y for every closed subset V ofX
9. ωα–closed[14] if f(V) is ωα–closed in Y for every closed subset V ofX
10. gspr–closed[18]if f(V) is gspr–closed in Y for every closed subset V ofX
11. g–closed[31] if f(V) is g–closed in Y for every closed subset V ofX
12. ω–closed[20] if f(V) is ω–closed in Y for every closed subset V of X
13. rgα– closed[13] if f(V) is rgα–closed in Y for every closed subset V ofX
14. gr–closed[23] if f(V) is gr–closed in Y for every closed subset V ofX
15. g*p–closed[21] if f(V) is g*p–closed in Y for every closed subset V ofX
16. rps–closed[26] if f(V) is rps–closed in Y for every closed subset V ofX
17. R*– closed[29] if f(V) is R*–closed in Y for every closed subset V ofX
18. gprw–closed[27] if f(V) is gprw–closed in Y for every closed subset V ofX.
19. wgrα–closed[25] iff(V) is wgrα–closed in Y for every closed subset V ofX.
20. swg–closed[2] if f(V) is swg–closed in Y for every closed subset V ofX.
21. rω–closed[5] if f(V) is rw–closed in Y for every closed subset V ofX.
22. rgw–closed[24] if f(V) is rgw–closed in Y for every closed subset V of X.
23. regular closed if f(F) is closed in Yfor every regular closed set F ofX
24. Contra closed [35] if f(F) is closed in Yfor every open set F ofX.
25. Contra regular closed if f(F) is r-closed in Y for every open set F of X.
26. Contra semi-closed [36] if f(F) is s-closed in Y for every open set F of X.

Definition 2.10A map f : (X, τ) → (Y, σ) is said to be
1. g-open [31] if f(U) is g-open in (Y, σ) for every open set U of (X, τ),
2. w-open [20] if f(U) is w-open in (Y, σ) for every open set U of (X, τ),
3. wg-open [2] if f(U) is wg-open in (Y, σ) for every open set U of (X, τ),
4. rwg-open [2] if f(U) is rwg-open in (Y, σ) for every open set U of (X, τ),
5. gpr-open [4] if f(U) is gpr-open in (Y, σ) for every open set U of (X, τ),
6. regular open if f(U) is open in (Y, σ) for every regular open set U of (X, τ).

3 αrω–closed Maps and αrω–open Maps :

We introduce the following definition

Definition 3.1 A map f **:** (X, τ) → (Y, σ) is said to be **α regular ω-closed** (briefly, αrω–closed) if the image of every closed set in (X, τ) is αrω–closed in (Y, σ).

Theorem 3.2 Every closed map is αrω–closed map, but not conversely.

Proof. The proof follows from the definitions and fact that every closed set is αrω–closed.

Theorem 3.3 Every α-closed map is αrω–closed map but not conversely.

Proof. The proof follows from the definitions and fact that every α-closed set is αrω–closed.

The converse of the above Theorem need not be true, as seen from the following example.

Example3.4 Let X=Y={a,b,c,d}, τ ={X, ϕ, {a}, {b}, {a,b}, {a,b,c} } and σ ={Y, ϕ, {a},{b}, {a,b}, {a,b,c}} Let map f: X→Y defined by f(a)=c, f(b)= a, f(c)=b, f(d)=d, then f is αrω–closed map but not closed map and not α–closed map, as image of closed set F= {c,d} in X, then f(F)={b,d} in Y, which is not α–closed, not closed set in Y.

Theorem 3.5 If f: (X, τ) → (Y, σ) is contra-r-closed and αrω–closed map then f is α-closed map.

Proof: Let V be a closed set in (X, τ). Then f (V) is regular open and αrω–closed. By Lemma 2.6, f (V) is α–closed.Thus f is α-closed map.

Theorem 3.6 If a map f: X→Y is closed, Then the following holds.

i) If f is αrω–closed map, then f is αg–closed map.
ii) If f is αrω–closed map, then f is wg– closed map (resp gs– closed map, rwg– closed map, gp– closed map, gspr–closed map, gpr– closed map, ωα–closed map, αgr–closed map).

Proof.(i) The proof follows from the definitions and fact that every αrω–closed set is αg-closed.
(ii) Similarly we can prove (ii).

The converse of the above Theorem need not be true, as seen from the following example.

Example 3.7 Let X=Y={a,b,c}, τ= {X, ϕ, {a},{b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by f(a)=b, f(b)=a, f(c)=c then f is αg–closed map, wg–closed map, gs–closed map, gp–closed map, gspr–closed map, gpr–closed map, rwg–closed map, αgr–closed map but not αrω–closed map as image closed set F= {b,c} in X, then f(F)={a,c} in Y, which is not αrω–closed set in Y.

Remark 3.8 The following examples shows that αrω-closed maps are independent of pre–closed, β–closed, g–closed, ω–closed, rω–closed, swg–closed, rgw–closed, wgrα–closed, rgα–closed, gprw–closed, g*p–closed, gr–closed, R*–closed, rps–closed, semi-closed maps.

Example 3.9 Let X=Y={a,b,c}, τ = {X, ϕ,{a},{b,c}} σ ={Y, ϕ,{a},{b,c}}, Let map f: X→Y defined by, f(a)=b, f(b)=a, f(c)=c then f is pre–closed map, β–closed map, g– closed map, ω– closed map, rω–closed map, swg–closed map, rgw–closed map, wgrα–closed map, rgα–closed map, gprw–closed map, g*p– closed map, gr–closed map, R*–closed map, rps–closed map but f is not αrω–closed map, as closed set F= {b,c} in X, then f(F)={a,c} in Y, which is not αrω–closed set in Y.

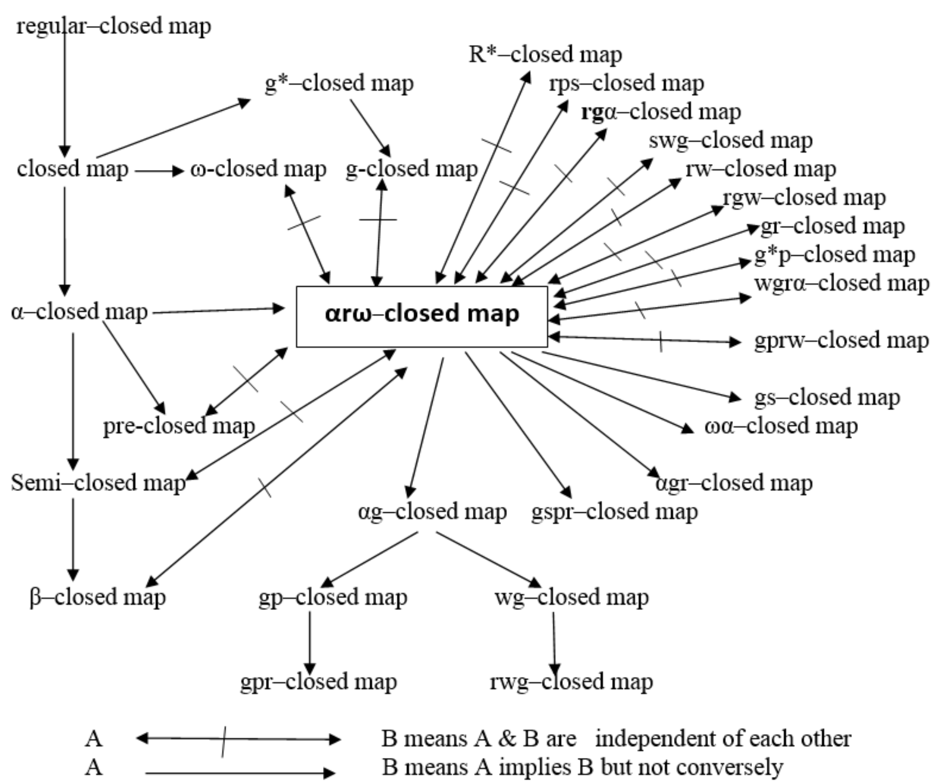
Example 3.10 X={a,b,c}, Y={a,b,c,d} τ ={X, ϕ, {a}}, σ = {Y, ϕ, {a}, {b},{a,b},{a,b,c}}, Let mapf: X→Y defined by, f(a)=b, f(b)=a, f(c)=dthen f is αrω– closed map but f is not gprw–closed map, rps–closed map, wgrα–closed map,rgw–closed map, rgα–closed map, swg–closed map, pre–closed map, R*–closed map,rω–closed map, ω-closed map, as closed set F= {b,c} in X, then f(F)={a,d} in Y, which is not gprw-closed (resp rps–closed, wgrα–closed, rgw–closed, rgα–closed, swg–closed, pre–closed, R*–closed, rω–closed, ω-closed) set in Y.

Example 3.11 X=Y={a,b,c,d}, τ = {X, ϕ {a}, {b}, {a,b}, {a,b,c}} σ ={Y, ϕ, {a},{b},{a,b},{a,b,c}} Let map f: X→Y defined by, f(a)=b, f(b)=a, f(c)=d, f(d)=c then f is αrω–closed map but f is gr–closed map, g–closed map, g*p–closed map, as closed set F={d} in X, then f(F)={c} in Y, which is not gr–closed (resp g–closed, g*p–closed) set in Y.

Example 3.12 X={a,b,c}, Y={a,b,c,d} τ ={X, ϕ,{a}}, σ = {Y, ϕ,{a},{b},{a,b},{a,b,c}}, Let mapf: X→Y defined by, f(a)=b, f(b)=a, f(c)=d then f is αrω–closed map but f is not semi–closed map, β–closed map, as closed set F= {a,b,c} in X, then f(F)={a,b,d} in Y which is not semi-closed (resp β–closed) set in Y.

Example 3.13 X={a,b,c}, Y={a,b,c,d} τ ={X, ϕ, {a}}, σ = {Y, ϕ, {a}, {b}, {a,b},{a,b,c}}, Let map f: X→Y defined by f(a)=d, f(b)=b, f(c)=c then f is semi–closed map, β–closed map but f is not αrω–closed map, as closed set F= {b,c} in X, then f(F)={b,c} in Y, which is not αrω–closed set in Y.

Remark3.14 From the above discussion and know results we have the following implications. **(Fig)**



Theorem 3.15 If f: (X, τ) → (Y, σ) is contra closed and αg–closed map then f is αrω–closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) isopen and αg–closed. By Lemma 2.6, f(V) is αrω–closed. Thus f is αrω–closed map.

Theorem 3.16 If f: (X, τ) → (Y, σ) is contra closed and wg–closed map then f is αrω–closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) is open and wg–closed. By Lemma 2.6, f(V) is αrω–closed. Thus f is αrω–closed map.

Theorem 3.17 If f: (X, τ) → (Y, σ) is contra regular closed and rwg–closed map then f is αrω–closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) is regular open and rwg–closed. By Lemma 2.6,f(V) is αrω–closed. Thus f is αrω–closed map.

Theorem 3.18 If f: (X, τ) → (Y, σ) is contra semi–closed and ω–closed map then f is αrω–closed map.

Proof: Let V be a closed set in (X, τ). Then f(V) is semi–open and ω–closed. By Lemma 2.7, f(V) is αrω–closed. Thus f is αrω–closed map.

Theorem 3.19 If a mapping f: (X, τ) → (Y, σ) is αrω–closed, then αrω–cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof. Suppose that f is αrω–closed and A ⊆X. Then cl(A) is closed in X and so f(cl(A)) is αrω–closed in (Y, σ). We have f(A) ⊆ f(cl(A)), by Theorem 5.2(iv) in [33], αrω–cl(f(A)) ⊆ αrω–cl(f(cl(A))) → (i). Since f(cl(A)) is αrω– closed in (Y, σ), αrω–cl(f(cl(A))) = f(cl(A)) → (ii), by the Theorem 5.3 in [33]. From (i) and (ii), we have αrω–cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Corollary 3.20 If a mapping f: (X, τ) → (Y, σ) is a αrω–closed, then the image f(A) of closed set A in (X, τ) is $\tau_{\alpha r\omega}$–closed in (Y, σ).

Proof. Let A be a closed set in (X, τ). Since f is αrω–closed, by above Theorem 3.19,αrω–cl(f(A))⊆ f(cl(A))→(i). Also cl(A)=A, as A is a closed set and so f(cl(A)) = f(A) →(ii). From (i) and (ii), we have αrω–cl(f(A)) ⊆ f(A). We know that f(A) ⊆ αrω–cl(f(A)) and so αrω–cl(f(A)) =f(A). Therefore f(A) is $\tau_{\alpha r\omega}$–closed in (Y, σ).

Theorem 3.21 Let (X, τ) be any topological spaces and (Y, σ) be a topological space where "αrω–cl(A) = α-cl(A) for every subset A of Y " and f : (X, τ) →(Y, σ) be a map, then the following are equivalent.
(i) f is αrω–closed map.
(ii) αrω–cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof. (i) ⇒(ii) Follows from the Theorem 3.19.
(ii) ⇒(i) Let A be any closed set of (X, τ). Then A=cl(A) and so f(A)=f(cl(A))⊇αrω–cl(f(A)) by hypothesis. We have f(A) ⊆ αrω–cl(f(A)), by Theorem 5.2(ii) in [33]. Therefore f(A) = αrω–cl(f(A)). Also f(A) = αrω–cl(f(A)) = α-cl(f(A)), by hypothesis. That is f(A) = α-cl(f(A)) and so f(A) is α-closed in (Y, σ). Thus f(A) is αrω–closed set in (Y,σ) and hence f is αrω–closed map.

Theorem 3.22 A map f : (X, τ) → (Y, σ) is αrω–closed if and only if for each subset S of (Y, σ) and each open set U containing $f^{-1}(S)$ ⊆U, there is a αrω–open set V of (Y, σ) such that S ⊆V and $f^{-1}(V)$ ⊆ U.

Proof. Suppose f is αrω–closed. Let S ⊆ Y and U be an open set of (X, τ) such that $f^{-1}(S)$ ⊆U. Now X − U is closed set in (X, τ). Since f is αrω–closed, f(X − U) is αrω–closed set in (Y, σ). Then V = Y − f(X − U) is a αrω–open set in (Y, σ). Note that $f^{-1}(S)$ ⊆ U implies S ⊆ V and $f^{-1}(V)$ =X − f^{-1}(f(X − U)) ⊆ X − (X − U) = U. That is $f^{-1}(V)$⊆ U.

For the converse, let F be a closed set of (X, τ). Then $f^{-1}((f(F))^c)$ ⊆ F^c and F^c is an open in (X, τ). By hypothesis, there exists a αrω–open set V in (Y, σ) such that $f(F)^c$ ⊆ V and $f^{-1}(V)$ ⊆ F^c and so F ⊆ $(f^{-1}(V))^c$. Hence V^c ⊆ f(F) ⊆ f($((f^{-1}(V))^c)$ ⊆V^c which implies f(F) = V^c. Since V^c is αrω–closed, f(F) is αrω–closed. Thus f(F) is αrω–closed in (Y, σ) and therefore f is αrω–closed map.

Remark 3.23 The composition of two αrω–closed maps need not be αrω–closed map in general and this is shown by the following example.

Example 3.24 Let X = Y = Z = {a, b, c}, τ={X, ϕ,{a},{a,b}} , σ = {Y, ϕ, {a}} and η = {Z, ϕ, {a}, {c}, {a, c}}. Define f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are the identity maps. Then f and g are αrω–closed maps, but their composition g ∘ f : (X, τ) → (Z, η) is not αrω–closed map, because F = {c} is closed in (X, τ), but g ∘ f(F) =g ∘ f({c}) = g(f({c})) = g({c}) = {c} which is not αrω–closed in (Z, η).

Theorem 3.25 If f: (X, τ) → (Y, σ) is closed map and g: (Y, σ) → (Z, η) is αrω–closed map, then the composition g ∘ f: (X, τ) → (Z, η) is αrω–closed map.

Proof. Let F be any closed set in (X, τ). Since f is closed map, f(F) is closed set in (Y, σ). Since g is αrω–closed map, g(f(F)) is αrω–closed set in (Z, η). That is g ∘ f(F) = g(f(F)) is αrω–closed and hence g ∘ f is αrω–closed map.

Remark 3.26 If f: (X, τ) → (Y, σ) is αrω–closed map and g: (Y, σ) → (Z, η) is closed map, then the composition need not be αrω–closed map as seen from the following example.

Example 3.27 Let X = Y = Z = {a, b, c},τ ={X,ϕ,{a},{a,b}}, σ = {Y, ϕ, {a}} and η = {Z, ϕ, {a}, {c}, {a, c}}. Define f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are the identity maps. Then f is αrω–closed map and g is a closed map. But their composition g ∘ f: (X, τ) → (Z, η) is not αrω–closed map, since for the closed set {c} in (X, τ), but g ∘ f({c}) = g(f({c})) = g({c}) = {c} which is not αrω–closed in (Z, η).

Theorem 3.28 If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is αrω–closed maps and (Y, σ) be a $T_{\alpha r\omega}$-space then g ∘ f: (X, τ) → (Z, η) is αrω–closed map.

Proof. Let A be a closed set of (X, τ). Since f is αrω–closed, f(A) is αrω–closed in (Y, σ). Then by hypothesis, f(A) is closed. Since g is αrω–closed, g(f(A)) is αrω–closed in (Z, η) and g(f(A)) = g ∘ f(A). Therefore g ∘ f is αrω–closed map.

Theorem 3.29 Iff: (X, τ) → (Y, σ) is g-closed, g: (Y, σ) → (Z, η) be αrω–closed and (Y, σ) is $T_{1/2}$-space then their composition g ∘ f: (X, τ) → (Z, η) is αrω–closed map.
Proof. Let A be a closed set of (X, τ). Since f is g-closed, f(A) is g-closed in (Y, σ). Since (Y, σ) is $T_{1/2}$-space, f(A) is closed in (Y, σ). Since g is αrω–closed, g(f(A)) is αrω–closed in (Z, η) and g(f(A)) = g ∘ f(A). Therefore g ∘ f is αrω–closed map.

Theorem 3.30 Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be two mappings such that their composition g∘f: (X, τ) → (Z, η) be αrω–closed mapping. Then the following statements are true.
(i) If f is continuous and surjective, then g is αrω–closed.

(ii) If g is αrω–irresolute and injective, then f is αrω–closed.
(iii) If f is g-continuous, surjective and (X, τ) is a $T_{1/2}$-space, then g is αrω–closed.
(iv) If g is strongly αrω–continuous and injective, then f is αrω–closed.

Proof.
(i) Let A be a closed set of (Y, σ). Since f is continuous, $f^{-1}(A)$ is closed in (X, τ) and since g ∘ f is αrω–closed, (g ∘ f)($f^{-1}(A)$) is αrω–closed in (Z, η). That is g(A) is αrω–closed in (Z, η), since f is surjective. Therefore g is αrω–closed.
(ii) Let B be a closed set of (X, τ). Since g ∘ f is αrω– closed, g ∘ f(B) is αrω–closed in (Z, η). Since g is αrω–irresolute, g^{-1}(g ∘ f(B)) is αrω–closed set in (Y, σ). That is f(B) is αrω–closed in (Y, σ), since f is injective. Therefore f is αrω–closed.
(iii) Let Cbe a closed set of (Y, σ). Since f is g-continuous, $f^{-1}(C)$ is g-closed set in (X, τ). Since (X, τ) is a $T_{1/2}$-space, $f^{-1}(C)$ is closed set in (X, τ). Since g ∘ f is αrω–closed, (g∘f)($f^{-1}(C)$) is αrω–closed in (Z, η). That is g(C) is αrω–closed in (Z, η), since f is surjective. Therefore g is αrω–closed.
(iv) Let D be a closed set of (X, τ). Since g ∘ f is αrω–closed, (g ∘ f)(D) is αrω–closed in (Z, η). Since g is strongly αrω–continous, g^{-1}((g ∘f)(D) is closed set in (Y, σ). That is f(D) is closed set in (Y, σ), since g is injective. Therefore f is closed.

Theorem 3.31 If f: (X, τ) → (Y, σ) is an open, continuous, αrω–closed surjection and cl(F) = F for every αrω–closed set in (Y, σ), where X is regular, then Y is regular.

Proof. Let U be an open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈X such that f(x) = p. Since X is regular and f is continuous, there is an open set V in X such that x ∈ V ⊆cl(V) ⊆ $f^{-1}(U)$. Here p ∈ f(V) ⊆ f(cl(V)) ⊆ U → (i). Since f is αrω–closed, f(cl(V)) is αrω–closed set contained in the open set U. By hypothesis, cl(f(cl(V))) = f(cl(V)) and cl(f(V)) = cl(f(cl(V))) → (ii). From (i) and (ii), we have p ∈ f(V) ⊆ cl(f(V)) ⊂ U and f(V) is open, since f is open. Hence Y is regular.

Theorem 3.32 If a map f: (X, τ) → (Y, σ) is αrω–closed and A is closed set of X, then f_A: (A, τ_A) → (Y, σ) is αrω–closed.

Proof. Let F be a closed set of A. Then F = A ∩ E for some closed set E of (X, τ) and so F is closed set of (X, τ). Since f is αrω–closed, f (F) is αrω–closed set in (Y, σ). But f(F) = $f_A(F)$ and therefore f_A: (A, τ_A) → (Y, σ) is αrω–closed.

Theorem 3.33 If f: (X,τ)→(Y,σ) is a continuous, αrω–closed map from a normal space (X,τ) onto a space (Y,σ) then (Y,σ) is α-normal.
Proof: Let A and B be two disjoint closed sets of (Y,σ). Let A and B are disjoint closed sets of (Y,σ). Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets of (X,τ), since f is continuous. Therefore there exists open sets U and V such that$f^{-1}(A)$ ⊆ U and $f^{-1}(B)$ ⊆ V, since X is normal. Using theorem 3.23, there exists αrω–open sets C, D in (Y,σ) such that A ⊆ C, B ⊆ D, $f^{-1}(C)$ ⊆ U and $f^{-1}(D)$ ⊆ V. Since A and B are closed, A and B are α-closed and rω–closed. By the definition of αrω–open, C is αrω–open if and only if A ⊆ α-int(C) whenever A ⊆ C and A is rω-closed, we get A ⊆ α-int(C) Thus A ⊆ α-int(C) and B ⊆ α-int(D). Hence Y is α-normal.

Theorem 3.34 If f: (X,τ)→(Y,σ) is an α-irresolute, (Y,σ) is a$_{1/2}T_\alpha$-space then fis an αrω-irresolute map.
Proof: Let U be αrω-closed in (Y,σ) then U is αg-closed. Since (Y,σ) is a $_{1/2}T_\alpha$-space, U is α-closed. Since f is α-irresolute, $f^{-1}(U)$ is α-closed. Hence $f^{-1}(U)$ is αrω-closed. Thus f is αrω-irresolute map.

Theorem 3.35 If f: (X,τ)→(Y,σ) is an αg-irresolute where (X,τ) is a discrete space then f is an αrω-irresolute map.
Proof: Let U be αrω-closed in (Y,σ). Then U is αg-closed. Since fis αg-irresolute and (X,τ) is discrete, $f^{-1}(U)$ is αg-closed and open. Hence $f^{-1}(U)$ is αrω-closed. Thus f is αrω-irresolute map.

Theorem 3.36 If f: (X,τ)→(Y,σ) is a α*-quotient map and (Y,σ) is $_{1/2}T_\alpha$-space then (Y,σ) is a $T_{\alpha r\omega}$-space.
Proof: Let U be αrω-closed in (Y,σ). Then U is αg-closed in (Y,σ). Since (Y,σ) is $_{1/2}T_\alpha$ – space then U is α-closed in (Y,σ), Since f is α-irresolute,$f^{-1}(U)$ is α-closed in (X,τ). Since f is α*-quotient mapif f is α-irresolute, $f^{-1}(U)$ is α-closed in (X,τ) then U is closed in (Y,σ). Thus (Y,σ) is a$T_{\alpha r\omega}$-space.

Analogous to αrω–closed maps, we define αrω–open map as follows.

Definition 3.37 A map f: (X, τ) → (Y, σ) is called a **αrω–open map** if the image f(A) is αrω–open in (Y, σ) for each open set A in (X, τ).

From the definitions we have the following results.

Theorem 3.38 (i) Every open map is αrω–open but not conversely.
(ii) Every α-open map is αrω–open but not conversely.
(iii) Every αrω–open map is αg-open but not conversely.
(iv) Every αrω–open map is rwg-open but not conversely.
(v) Every αrω–open map is gpr-open but not conversely.

Theorem 3.39 For any bijection map f: (X, τ) → (Y, σ), the following statements are equivalent:
i) f^{-1}: (Y, σ) → (X, τ) is αrω–continuous.
ii) f is αrω–open map and
iii) f is αrω–closed map.

Proof. (i) ⇒ (ii) Let U be an open set of (X, τ). By assumption, $(f^{-1})^{-1}(U)$ =f(U) is αrω–open in (Y, σ) and so f is αrω–open.

(ii) ⇒(iii) Let F be a closed set of (X, τ). Then F^c is open set in (X, τ). By assumption, f(F^c) is αrω–open in (Y, σ). That is f(F^c) = $f(F)^c$ is αrω–open in (Y, σ) and therefore f(F) is αrω–closed in (Y, σ). Hence f is αrω–closed.

(iii) ⇒(i) Let F be a closed set of (X, τ). By assumption, f(F) is αrω–closed in (Y, σ). But f(F) = $(f^{-1})^{-1}(F)$ and therefore f^{-1} is continuous.

Theorem 3.40 If a map f: (X, τ) → (Y, σ) is αrω–open, then f(int(A)) ⊆ αrω–int(f(A)) for every subset A of (X, τ).

Proof. Let f: (X, τ) → (Y, σ) be a open map and A be any subset of (X, τ). Then int(A) is open in (X, τ) and so f(int(A)) is αrω–open in (Y, σ). We have f(int(A)) ⊆ f(A). Therefore by Theorem 5.15 (iii) in [33], f(int(A)) ⊆ αrω–int(f(A)).

Theorem 3.41 If a map f : (X, τ) → (Y, σ) is αrω–open, then for each neighbourhood U of x in (X, τ), there exists a αrω–neighbourhood W of f(x) in (Y, σ) such that W ⊆ f(U).

Proof. Let f : (X, τ) → (Y, σ) be a αrω–open map. Let x ∈X and U be an arbitrary neighbourhood of x in (X, τ). Then there exists an open set G in (X, τ) such that x ∈ G ⊆ U. Now f(x) ∈f(G) ⊆ f(U) and f(G) is αrω–open set in (Y, σ), as f is a αrω–open map. By Theorem 6.7 in [33], f(G) is αrω–neighbourhood of each of its points. Taking f(G) = W, W is a αrω–neighbourhood of f(x) in (Y, σ) such that W ⊆ f(U).

Theorem 3.42 A map f: (X, τ) → (Y, σ) is αrω–open if and only if for any subset S of (Y, σ) and any closed set of (X, τ) containing $f^{-1}(S)$, there exists a αrω–closed set K of (Y, σ) containing S such that $f^{-1}(K)$ ⊆ F.

Proof. Suppose f is αrω–open map. Let S ⊆ Y and F be a closed set of (X, τ) such that $f^{-1}(S)$ ⊆ F. Now X −F is an open set in (X, τ). Since f is αrω–open map, f(X − F) is αrω–open set in (Y, σ). Then K = Y − f(X − F) is a αrω–closed set in (Y, σ). Note that $f^{-1}(S)$ ⊆ F implies S ⊆ K and $f^{-1}(K)$ = X − f^{-1}(X − F) ⊆ X − (X − F) = F. That is $f^{-1}(K)$ ⊆ F.
For the converse, let U be an open set of (X, τ). Then $f^{-1}((f(U))^c) \subset U^c$ and U^c is a closed set in (X, τ). By hypothesis, there exists a αrω–closed set K of (Y, σ) such that $(f(U))^c$ ⊆ K and $f^{-1}(K) \subseteq U^c$ and so U ⊆ $(f^{-1}(K))^c$. Hence K^c ⊆ f(U) ⊆ $f((f^{-1}(K)))^c \subseteq K^c$ which implies f(U) = K^c. Since K^c is a αrω–open, f(U) is αrω–open in (Y, σ) and therefore f is αrω–open map.

Theorem 3.43 If a function f: (X, τ) → (Y, σ) is αrω–open, then f^{-1}(αrω–cl(B)) ⊆ cl(f^{-1}(B)) for each subset B of (Y, σ).

Proof. Let f: (X, τ) → (Y, σ) be a αrω–open map and B be any subset of (Y, σ). Then f^{-1}(B) ⊆cl(f^{-1}(B)) and (f^{-1}(B)) is closed set in (X, τ). By above Theorem 3.44., there exists a αrω–closed set K of (Y, σ) such that B ⊆ K and f^{-1}(K) ⊆ cl(f^{-1}(B)). Now αrω–cl(B) ⊆ αrω–cl(K) = K, by Theorems 5.2 and 5.3 in [33], as K is αrω–closed set of (Y, σ). Therefore f^{-1}(αrω–cl(B)) ⊆ f^{-1}(K) and so f^{-1}(αrω–cl(B)) ⊆ f^{-1}(K) ⊆ cl(f^{-1}(B)). Thus f^{-1}(αrω–cl(B)) ⊆ cl(f^{-1}(B)) for each subset of B of (Y, σ).

We define another new class of maps called αrω∗-closed maps which are stronger than αrω–closed maps.

Definition 3.44 A map f: (X, τ) → (Y, σ) is said to be αrω*-closed map if the image f(A) is αrω–closed in (Y, σ) for every αrω–closed set A in (X, τ).

Theorem 3.45 Every αrω*-closed map is αrω–closed map but not conversely.

Proof. The proof follows from the definitions and fact that every closed set is αrω–closed.

The converse of the above Theorem is not true in general as seen from the following example.

Example 3.46 Let X = Y = {a, b, c}, τ = {X, ϕ, {a}} and σ ={Y, ϕ, {a},{b},{a, b}} and f : (X, τ) → (Y, σ) be the identity map. Then f is αrω–closed map but not αrω*-closed map. Since {b} is αrω–closed set in (X, τ), but its image under f is {b}, which is not αrω–closed in (Y, σ).

Theorem 3.47 If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are αrω*-closed maps, then their composition g ∘ f: (X, τ) → (Z, η) is also αrω∗-closed.

Proof. Let F be a αrω–closed set in (X, τ). Since f is αrω*–closed map, f(F) is αrω–closed set in (Y, σ). Since g is αrω∗–closed map, g(f(F)) is αrω–closed set in (Z, η). Therefore g ∘ f is αrω*–closed map.

Analogous to αrω*-closed map, we define another new class of maps called αrω*–open maps which are stronger than αrω–open maps.

Definition 3.48 A map f: (X, τ) → (Y, σ) is said to be αrω∗–open map if the image f(A) is αrω–open set in (Y, σ) for every αrω–open set A in (X, τ).

Remark 3.49 Since every open set is a αrω–open set, we have every αrω∗–open map is αrω–open map. The converse is not true in general as seen from the following example.

Example 3.50 Let X = Y = {a, b, c}, τ = {X, ϕ {a}} and σ = {Y, ϕ, {a}, {b}, {a, b}}. Let f: (X, τ) → (Y, σ) be the identity map. Then f is αrω–open map but not αrω∗–open map, since for the αrω–open set {a, c} in (X, τ), f({a, c}) = {a, c} which is not αrω–open set in (Y, σ).

Theorem 3.51 If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are αrω∗-open maps, then their composition g ∘ f:(X, τ) → (Z, η) is also αrω∗-open.
Proof. Proof is similar to the Theorem 3.47.

Theorem 3.52 For any bijection map f :(X, τ)→(Y, σ), the following statements are equivalent:
i) f^{-1}: (Y, σ) → (X, τ) is αrω irresolute.
ii) f is αrω∗–open map
iii) f is αrω∗–closed map.

Proof: Proof is similar to that of Theorem3.39.

Acknowledgment
The Authors would like to thank the referees for useful comments and suggestions.

References
1. Malghan SR.Generalized Closed Maps, J Karnatk Univ. Sci.,1982; 27:82-88.
2. Nagaveni N. Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D. Thesis, Bharathiar University,Coimbatore, 1999.
3. Long PE, Herington LL. Basic Properties of Regular Closed Functions,Rend. Cir. Mat. Palermo1978; 27:20-28.

4. Gnanambal Y.,On generalized pre regular closed sets in topological spaces, Indian J Pure. Appl. Math.1997; 28(3):351-360.
5. Benchalli S.S, Wali R.S.,on rω- Closed sets is Topological Spaces, Bull, Malays, Math, sci, soc30, 2007, 99-110.
6. Levine N. Semi-open sets and semi-continuity in topological spaces1963; 70:36-41.
7. Mashhour AS, Abd El-Monsef ME, El-Deeb S N. On pre-continuous and weak pre continuous mappings,Proc. Math. Phys.Soc. Egypt1982; 53:47-53.
8. Jastad ON. On some classes of nearly open sets, Pacific J Math.1965; 15:961-970
9. Abd El-Monsef ME, El-Deeb S N, Mahmoud RA. β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ1983; 12:77-90.
10. Andrijevic D. Semi-preopen sets, Mat. Vesnik,1986; 38(1):24-32.
11. Stone M. Application of the theory of Boolean rings to general topology, Trans. Amer. Math.Soc.1937;41:374-481.
12. Cameron DE. Properties of s-closed spaces, pracAmerMath, soc 1978;72:581-586
13. Vadivel A, K vairamamanickam. rgα-Closed sets & rgα-open sets in Topological Spaces, Int J of math, Analysis. 2009; 3(37):1803-1819
14. Benchalli SS, Patil P.G, Rayanagaudar T.D. ωα-Closed sets is Topological Spaces, The Global. J Appl. Math and Math.Sci. 2009; 2:53-63.
15. Wali R.S, Mandalgeri P. S , On α Regular ω-closed sets in topological spaces, Int. J ofMath Archive. 2014; 5(10):68-76
16. Levine N. Generalized closed sets in topology,Rend. Circ Mat. Palermo 1970; 19(2):89-96.
17. Arya S P, T M Nour, Chatcterizations of s-normal spaces, Indian J Pure Appl, Math. 1990;21:717-719
18. Pushpalatha A. Studies on generalizations of mapping in topological spaces, PhD Thesis,Bharathiar university, Coimbatore,2000.
19. Maki H, Devi R, Balachandran K. Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math1994; 15:51-63.
20. Sundaram P, Sheik John M. On w-closed sets in topology, Acta Ciencia Indica2000; 4:389-39
21. Maki H, Umehara J, Noiri T. Every Topological space is pre $T_{½}$mem Fac sci, Kochi univ,Math1996; 17:33-42.
22. Veera Kumar MKRS. g*-pre closed sets, Acts Ciencia indica 2002; 28(1):51-60.
23. Bhattacharya S. on generalized regular closed sets, Int JContemp.Math science., 201(6), 145-152
24. Mishra S. On regular generalized weakly (rgw)closed sets in topological spaces, Int. J of Math Analysis 2012; 6(30):1939-1952
25. Jayalakshmi A, JanakiC. on wgrα-closed sets in Topological Spaces, Int J ofmaths. 2012; 3(6):2386-2392.
26. ShlyaIsac Mary T, P Thangavelv.on Regular pre-semi closed sets in topological spaces, KBM J of Math Sc & comp Applications. 2010;1:9-17
27. Joshi V, GuptaS, Bhardwaj N,kumar R. on Generalised pre Regular weakly(gprw)-closed set in sets in Topological Spaces, int math foruro 2012;7-40:1981-1992
28. Veerakumar MKRS. On α-generalizedregular closed sets, Indian J of Math. 2002; 44(2):165-181
29. Janaki C, Renu Thomas.on R*-Closed sets in Topological Spaces, Int J of Math Archive. 2012; 3(8):3067-3074
30. Arya S P, Gupta R. On strongly continuous functions, Kyungpook Math.J.1974; 14:131-143.
31. Balachandran K, Sundaram P,MakiH. On Generalized Continuous Maps in Topological Spaces, Mem. I ac Sci. Kochi Univ. Math., 1991; 12:5-13.
32. Wali R.S., Mandalgeri P.S. . On αrω–Continuous and αrω–Irresolute Maps in Topological Spaces, IOSR-JM 2014; 10(6):14-24
33. Wali R.S. , Mandalgeri P.S. . On α Regular ω-open sets in topological spaces, J of comp & Math Sci. 2014; 5(6):490-499
34. Thivagar M L. A note on quotient mapping, Bull. Malaysian Math. Soc 1991; 14:21-30
35. Baker CW. Contra open and Contra closed functions, Math. Sci1994; 17:413-415.
36. Navalagi G B. "On Semi-pre Continuous Functions and Properties of Generalized Semi-pre Closed in Topology", IJMS 2002; 29(2):85-98.
37. Yasuf Becerem, On strongly α-continuous functions Far East J Math. Sci2000; 1:50
38. Dontchev J. Contra continuous functions and strongly S-closed spaces, Int. J Math. Sci. 1996; 19:15-31.