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On $WI_{\hat{g}}$- Continuous and WI_{*g}-Continuous Functions in Ideal Topological Spaces

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Abstract
In this paper we introduce and study the notions of $wI_{\hat{g}}$-continuous and wI_{*g}-continuous, $wI_{\hat{g}}$-irresolute and wI_{*g}-irresolute in ideal topological spaces, and also we studied their properties.

Keywords: $wI_{\hat{g}}$-closed, wI_{*g}-closed, $wI_{\hat{g}}$-continuous, wI_{*g}-continuous, $wI_{\hat{g}}$-irresolute, wI_{*g}-irresolute.

Introduction

Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett [5] once again investigated applications of topological ideals. The notion of I_g-closed sets was first by Dontchev.et.al [3] in 1999. Navaneethakrishnan and Joseph [9] further investigated and characterized I_g-closed sets and I_g-open sets by the use of local functions. The notion of I_{*g}-closed sets was introduced by Ravi.et.al [10] in 2013. Recently the notion of $wI_{\hat{g}}$-closed sets and wI_{*g}-closed sets was introduced and investigated by Maragathavalli.et.al [8]. In this paper, we introduce the notions of $wI_{\hat{g}}$-continuous and wI_{*g}-continuous functions in ideal topological spaces.

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) $A \in I$ and $B \subseteq A$ implies $B \in I$, (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset $A \subseteq X$, $A^*(I,\tau) = \{x \in X : A \cap U \notin I$ for every $U \in \tau(X, x)\}$ is called the local function of A with respect to I and τ [6]. We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$ called the *-topology, finer than τ is defined $cl^*(A) = A \cup A^*$ [11]. If $A \subseteq X$, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition 1.1. A subset A of a topological space (X, τ) is called
1. g-closed [7], if cl (A) $\subseteq$ U whenever $A \subseteq U$ and U is open in (X, τ).
2. $\hat{g}$-closed [12], if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is semi open in (X, τ).

Definition 1.2. A subset A of a topological space (X, τ, I) is called
1. I_g-closed [9], if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2. $I_{\hat{g}}$-closed [1], if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
3. $wI_{\hat{g}}$- closed [8], if $int(A^*) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
4. wI_{*g}-closed [8], if $int(A^*) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in X.
5. *g-closed [10], if $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

Definition 1.3. A function $f: (X, \tau, I) \to (Y, \sigma)$ is said to be
1. g-continuous [2], if for every open set $V \in \sigma$, $f^{-1}(V)$ is g-open in (X, τ).
2. $\hat{g}$-continuous [12], if for every open set $V \in \sigma$, $f^{-1}(V)$ is $\hat{g}$-open in (X, τ).

Definition 1.4. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be I_g-continuous [4], if $f^{-1}(V)$ is I_g-closed in (X, τ, I) for every closed set V in (Y, σ).

2. $wI_{\hat{g}}$-continuous and wI_{*g}-continuous.

Definition 2.1: A function f : (X, τ, I) → (Y, σ) is Said to be

1. Weakly $I_{\hat{g}}$-continuous (briefly $wI_{\hat{g}}$-continuous) if $f^{-1}(V)$ is weakly $I_{\hat{g}}$-closed set in (X, τ, I) for every closed set V in (Y, σ).
2. Weakly I_{*g}-continuous (briefly wI_{*g}-continuous) if $f^{-1}(V)$ is weakly I_{*g}-closed set in (X, τ, I) for every closed set V in (Y, σ).

Definition 2.2: A function f : (X, τ, I_1) → (Y, σ, I_2) is Said to be

(i) $wI_{\hat{g}}$-irresolute if $f^{-1}(V)$ is $wI_{\hat{g}}$-closed in (X, τ, I_1) for every $wI_{\hat{g}}$-closed set V in (Y, σ, I_2).

(ii) wI_{*g}-irresolute if $f^{-1}(V)$ is wI_{*g}-closed in (X, τ, I_1) for every wI_{*g}-closed set V in (Y, σ, I_2).

Theorem 2.3: Ever continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be a continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is closed set in (X, τ, I). Since every closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.4: Let X = Y = {a, b, c}, τ = {φ, {b}, {b,c}, X}, σ = {φ, {c}, Y} and I = {φ, {b}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not continuous.

Theorem 2.5: Ever continuous function is wI_{*g}-continuous.

Proof: Let f be a continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is closed set in (X, τ, I). Since every closed set is wI_{*g}-closed. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.6: In example 2.4, let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not continuous.

Theorem 2.7: Ever $I_{\hat{g}}$-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be a $I_{\hat{g}}$-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $I_{\hat{g}}$-closed set in (X, τ, I). Since every $I_{\hat{g}}$-closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.8: Let X = Y = {a, b, c, d}, τ = {φ, {a,b}, {a,b,c}, X}, σ = {φ, {a,b}, {a}, Y} and I = {φ, {a}}. Let the function f : (X, τ, I) → (Y, σ) is defined by f(a) = b, f(b) = c, f(c) = a, f(d) = d. Then the function f is $wI_{\hat{g}}$-continuous but not $I_{\hat{g}}$-continuous.

Theorem 2.9: Ever $\hat{g}$-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an $\hat{g}$-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $\hat{g}$-closed set in (X, τ, I). Since every $\hat{g}$-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.10: Let X = Y = {a, b, c, d}, τ = {φ, {b}, {a,b,c}, X}, σ = {φ, {c}, {a,c}, Y} and I = {φ, {c}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not $\hat{g}$-continuous.

Theorem 2.11: Ever g-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be a g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is g-closed set in (X, τ, I). Since every g-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.12: Let X = Y = {a, b, c, d}, τ = {φ, {b},{c}, {b,c}, X}, σ = {φ, {c}, X} and I = {φ, {b}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not g-continuous.

Theorem 2.13: Ever I_{*g}-continuous function is wI_{*g}-continuous.

Proof: Let f be an I_{*g}-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is I_{*g}-closed set in (X, τ, I). Since every I_{*g}-closed set is wI_{*g}-closed, hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.14: Let X = Y = {a, b, c, d}, τ = {φ, {a,b},{c,d}, X}, σ = {φ, {c,d}, Y} and I = {φ, {d}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not I_{*g}-continuous.

Theorem 2.15: Ever g-continuous function is wI_{*g}-continuous.

Proof: Let f be a g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is g-closed set in (X, τ, I). Since every g-closed set is wI_{*g}-closed set. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.16: Let X = Y = {a, b, c, d}, τ = {φ, {a,b},{a,b,c}, X}, σ = {φ, {d}, {c,d}, Y} and I = {φ, {a}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not g-continuous.

Theorem 2.17: Ever I_g-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an I_g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is I_g-closed set in (X, τ, I). Since every I_g-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.18: In example 2.16, let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not I_g-continuous.

Theorem 2.19: Ever I_g-continuous function is wI_{*g}-continuous.

Proof: Let f be a I_g-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is I_g-closed set in (X, τ, I).

Since every I_g-closed set is wI_{*g}-closed set. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.20: Let X = Y = {a, b, c, d}, τ = {φ, {b}, {a,b,c}, X}, σ = {φ, {a}, {a,c,d}, Y} and I = {φ, {d}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not I_g-continuous.

Theorem 2.21: Ever wI_{*g}-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be a wI_{*g}-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Since every wI_{*g}-closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.22: Let X = Y = {a, b, c, d}, τ = {φ, {d},{a,b,c}, X}, σ = {φ, {a}, Y} and I = {φ, {b}}. Let the function f : (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not wI_{*g}-continuous.

Theorem 2.23: A map f: (X, τ, I) → (Y, σ) is $wI_{\hat{g}}$-continuous iff the inverse image of every closed set in (Y, σ) is $wI_{\hat{g}}$- closed in (X, τ, I).

Proof: Necessary: Let v be a closed set in (Y, σ). Since f is $wI_{\hat{g}}$- continuous, $f^{-1}(v^C)$ is $wI_{\hat{g}}$- closed in (X, τ, I). But $f^{-1}(v^C) = X - f^{-1}(v)$. Hence $f^{-1}(v)$ is $wI_{\hat{g}}$- closed in (X, τ, I).

Sufficiency: Assume that the inverse image of every closed set in (Y, σ) is $wI_{\hat{g}}$- closed in (X, τ, I). Let v be a closed set in (Y, σ). By our assumption $f^{-1}(v^C) = X - f^{-1}(v)$ is $wI_{\hat{g}}$-closed in (X, τ, I), which implies that $f^{-1}(v)$ is $wI_{\hat{g}}$- closed in (X, τ, I). Hence f is $wI_{\hat{g}}$- continuous.

Remark 2.24:

(i) The union of any two $wI_{\hat{g}}$- continuous function is $wI_{\hat{g}}$continuous.

(ii) The intersection of any two $wI_{\hat{g}}$- continuous function is need not be $wI_{\hat{g}}$- continuous.

Theorem 2.25: Let f:(X, τ, I_1) → (Y, σ, I_2) and g:(Y, σ, I_2) →(Z, η, I_3) be any two functions. Then the following hold.

(i) g ∘ f is $wI_{\hat{g}}$ - continuous if f is $wI_{\hat{g}}$ continuous and g is continuous.

(ii) g ∘ f is $wI_{\hat{g}}$ - continuous if f is $wI_{\hat{g}}$ irresolute and g is $wI_{\hat{g}}$ continuous.

(iii) g ∘ f is $wI_{\hat{g}}$ - irresolute if f is $wI_{\hat{g}}$ irresolute and g is irresolute.

Proof:

(i) Let v be a closed set in Z. Since g is continuous, $g^{-1}(v)$ is closed in Y. $wI_{\hat{g}}$-continuous of f implies, $f^{-1}(g^{-1}(v))$ is $wI_{\hat{g}}$ -closed in X and hence g ∘ f is $wI_{\hat{g}}$ -continuous.

(ii) Let v be a closed set in Z. Since g is $wI_{\hat{g}}$ -continuous, $g^{-1}(v)$ is $wI_{\hat{g}}$ -closed in Y. Since f is $wI_{\hat{g}}$ -irresolute, $f^{-1}(g^{-1}(V))$ is $wI_{\hat{g}}$ -closed in X. Hence g ∘ f is $wI_{\hat{g}}$ -continuous.

(iii) Let v be a $wI_{\hat{g}}$ -closed in Z. Since g is $wI_{\hat{g}}$ - irresolute, $g^{-1}(v)$ is $wI_{\hat{g}}$ -closed in Y. Since f is $wI_{\hat{g}}$ -irresolute, $f^{-1}(g^{-1}(v))$ is $wI_{\hat{g}}$ -closed in X. Hence g ∘ f is $wI_{\hat{g}}$ -irresolute.

Theorem 2.26: Let X = A ∪ B be a topological space with topology τ and Y be a topological space with topology σ. Let f:(A, τ/A) → (Y, σ) and g:(B, τ/B) → (Y, σ) be $wI_{\hat{g}}$-continuous maps such that f(x) = g(x) for every x ∈ A ∩ B. Suppose that A and B are $wI_{\hat{g}}$-closed sets in X. Then the combination α: (X, τ, I) → (Y, σ) is $wI_{\hat{g}}$ - continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$ where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. But C is $wI_{\hat{g}}$ -closed in A and A is be $wI_{\hat{g}}$ -closed in X and so C is $wI_{\hat{g}}$ -closed in X. Since we have proved that if B ⊆ A ⊆ X, B is $wI_{\hat{g}}$-closed in A and A is $wI_{\hat{g}}$ -closed in X, then B is $wI_{\hat{g}}$ -closed in X. Also C ∪ D is $wI_{\hat{g}}$ -closed in X. Therefore $\alpha^{-1}(F)$ is $wI_{\hat{g}}$-closed in X. Hence α is $wI_{\hat{g}}$ -continuous.

Theorem 2.27: A map f: (X, τ, I) → (Y, σ) is wI_{*g}-continuous iff the inverse image of every closed set in (Y, σ) is wI_{*g}- closed in (X, τ, I).

Proof: Necessary: Let v be a closed set in (Y, σ). Since f is wI_{*g}- continuous, $f^{-1}(v^C)$ is wI_{*g}- closed in (X, τ, I). But $f^{-1}(v^C) = X - f^{-1}(v)$. Hence $f^{-1}(v)$ is wI_{*g}- closed in (X, τ, I).

Sufficiency: Assume that the inverse image of every closed set in (Y, σ) is wI_{*g}- closed in (X, τ, I). Let v be a closed set in (Y, σ). By our assumption $f^{-1}(v^C) = X - f^{-1}(v)$ is wI_{*g}-closed in (X, τ, I), which implies that $f^{-1}(v)$ is wI_{*g}- closed in (X, τ, I). Hence f is wI_{*g}- continuous.

Remark 2.28:

(i) The union of any two wI_{*g}- continuous function is wI_{*g}-continuous.

(ii) The intersection of any two wI_{*g}- continuous function is need not be wI_{*g}- continuous.

Theorem 2.29: Let f:(X, τ, I_1) → (Y, σ, I_2) and g:(Y, σ, I_2) →(Z, η, I_3) be any two functions. Then the following hold.

(i) g ∘ f is wI_{*g} - continuous if f is wI_{*g} continuous and g is continuous.

(ii) g ∘ f is wI_{*g} - continuous if f is wI_{*g} irresolute and g is wI_{*g} continuous.

(iii) g ∘ f is wI_{*g} - irresolute if f is wI_{*g} irresolute and g is irresolute.

Proof:

(i) Let v be a closed set in Z. Since g is continuous, $g^{-1}(v)$ is closed in Y. wI_{*g}-continuous of f implies, $f^{-1}(g^{-1}(v))$ is wI_{*g}-closed in X and hence g ∘ f is wI_{*g} -continuous.

(ii) Let v be a closed set in Z. Since g is wI_{*g} -continuous, $g^{-1}(v)$ is wI_{*g} -closed in Y. Since f is wI_{*g} -irresolute, $f^{-1}(g^{-1}(V))$ is wI_{*g} -closed in X. Hence g ∘ f is wI_{*g} -continuous.

(iii) Let v be a wI_{*g} -closed in Z. Since g is wI_{*g} - irresolute, $g^{-1}(v)$ is wI_{*g} -closed in Y. Since f is wI_{*g} -irresolute, $f^{-1}(g^{-1}(v))$ is wI_{*g} -closed in X. Hence g ∘ f is wI_{*g} - irresolute.

Theorem 2.30: Let X = A ∪ B be a topological space with topology τ and Y be a topological space with topology σ. Let f:(A, τ/A) → (Y, σ) and g:(B, τ/B) → (Y, σ) be wI_{*g}-

continuous maps such that f(x) = g(x) for every x ∈ A ∩ B. Suppose that A and B are wI_{*g}-closed sets in X. Then the combination α: (X, τ, I) → (Y, σ) is wI_{*g} - continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ = C ∪ D where C = $f^{-1}(F)$ and D = $g^{-1}(F)$. But C is wI_{*g} -closed in A and A is be wI_{*g} -closed in X and so C is wI_{*g} - closed in X. Since we have proved that if B ⊆ A ⊆ X, B is wI_{*g}-closed in A and A is wI_{*g} -closed in X, then B is wI_{*g} - closed in X. Also C ∪ D is wI_{*g}-closed in X. Therefore $\alpha^{-1}(F)$ is wI_{*g}-closed in X. Hence α is wI_{*g} -continuous.

References

1. J Antony Rex Rodrigo, O Ravi, A Naliniramalatha. $\hat{g}$-closed sets in ideal topological spaces, Methods of Functional Analysis and Topology, 2011; 17(3):274-280.
2. Balachandran K, Sundaram P, Maki H. On generalized continuous maps in topological spaces. Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 1991; 12:5-13.
3. Dontchev J, Ganster M, Noiri T. Unified approach of generalized closed sets via topological ideals, Math. Japan, 1999; 49:395-401.
4. V Indhumathi, S Krishnaprakash, N Rajamani. Strongly I-locally closed sets and decompositions of *-continuity, Acta Math. Huger, (to appear).
5. Jankovic D, Hamlett TR. New topologies from old via ideals, Amer. Math. Monthly, 1990; 97(4):295-310.
6. Kuratowski Topology, Academic press, Newyork, 1966, I.
7. Levine N. Generalized closed sets in topology. Rend. Circ. Mat. Palermo 1970; 19:89-96.
8. Maragathavalli S, Suresh N, Revathi A. Weakly $I_{\hat{g}}$-closed sets and weakly I_{*g}-closed sets in ideal topological spaces, (communicated).
9. Navaneethakrishnan M, Paulraj joseph J. g-closed sets in ideal topological spaces, Acta math Hunger, 2008; 119:365-371.
10. O Ravi, S Tharmar, S Sangeetha, J Antony Rex Rodrigo. *g-closed sets in ideal topological spaces, Jordan journal of Mathematics and Statistics (IJMS) 2013; 6(1):1-13.
11. R Vaidyanathaswamy. Set topology, Chelsea, Publishing company, Newyork, 1960.
12. Veerakumar MKRS. $\hat{g}$-closed sets in topological spaces, Math, Soc, 2003; 18:99-112.