ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(12): 829-835
www.allresearchjournal.com
Received: 14-09-2015
Accepted: 16-10-2015

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Energy Method to Analysis the vibration of orthotropic circular plate with Bi-dimensional Thickness Variation

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Abstract
Vibration of circular plate with bi-dimensional varying thickness and temperature are investigated in the present study. Using the separation of variables method, the differential equation has been solved for vibration of visco-elastic orthotropic circular plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two term deflection function. The frequencies corresponding to the first two modes of vibrations are obtained for a circular plate for different values of taper constant and thermal gradient.

Keywords: plate, frequency, taper constant, thickness, thermal effect

Introduction

Plates of variable thickness are often encountered in engineering applications and their use in machine design, nuclear reactor technology, naval structures and acoustical components is quite common. The consideration of visco-elastic behavior of the plate material together with the variation in thickness of the structural components not only ensure the reduction in the rate and size but also meets the desirability of high strength in various technological situations of aerospace industry, ocean engineering and electronic and optical equipments. The research in the field of vibration is quite mesmerizing and continuously acquiring a great attention of scientists and design engineers because of its unbounded effect on human life. In the engineering we cannot move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. Study of effect of vibration can't be restricted only in the field of science but, our day to day life is also affected by it.

Recently, Leissa [1] has given the solution for rectangular plate of variable thickness. N. Bhardwaj, A.P Gupta, K.K Choong, C.M Wang & Hiroshi Ohmori [2] discussed about transverse vibrations of clamped and simply-supported circular plates with two dimensional thickness variations. Anukul De and D. Debnath [3] studied about vibration of orthotropic circular plate with thermal effect in exponential thickness and quadratic temperature Distribution. Khanna, A., Kaur, N., & Sharma, A. K. [4] have discussed effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate. Sharma, S. K., & Sharma, A. K. [5] have discussed the mechanical vibration of orthotropic rectangular plate with 2d linearly varying thickness and thermal Effect. Khanna, A., & Sharma, A. K. [6] have solved the problem on vibration analysis of visco-elastic square plate of variable thickness with thermal gradient. Kumar Sharma, A., & Sharma, S. K. [7] have discussed the vibration computational of visco-elastic plate with sinusoidal thickness variation and linearly thermal effect in 2d. Khanna, A., Kumar, A., & Bhatia, M. [8] has investigated the computational prediction on two dimensional thermal effects on vibration of visco-elastic square plate of variable thickness. Khanna, A., & Sharma, A. K. [9] studied natural vibration of visco-elastic plate of varying thickness with thermal effect. Kumar Sharma, A., & Sharma, S. K. [10] discussed free vibration analysis of visco-elastic orthotropic rectangular plate with bi-parabolic thermal effect and bi-linear thickness variation. Sharma, S. K. & Sharma, A. K [11] discussed effect of bi-parabolic thermal and thickness variation on vibration of visco-elastic orthotropic rectangular plate. Khanna, A., & Sharma, A. K. [12]

analyzed a computational prediction on vibration of square plate by varying thickness with bi-dimensional thermal effect. Khanna, A., & Sharma, A. K. [13] discussed effect of thermal gradient on vibration of visco-elastic plate with thickness variation. Khanna, A., & Sharma, A. K [14] have studies the mechanical vibration of visco-elastic plate with thickness variation. Khanna, A., Kaur, N., & Sharma, A. K. [15] discussed about effect of varying poisson ratio on thermally induced vibrations of non-homogeneous rectangular plate.

In the aeronautical field, analysis of thermally induced vibrations in circular plates of variable thickness has a great interest due to their utility in aircraft wings. So, it is essentially required to have the knowledge of vibration for a designer. Here, present investigation is to study the vibrations of circular plate with bi-dimensional varying thickness and temperature. Rayleigh-Ritz's method has been applied to derive the frequency equation of the plate. All results are illustrated with Graphs.

Equation of Transverse Motion

The equation of motion for a circular plate of radius a is governed by the equation [3]

$$r\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial}{\partial r}(rM_r) - M_\theta\right)\right] = \rho h\frac{\partial^2\omega}{\partial t^2} \qquad (1)$$

The resultant moments of M_r and M_θ for a polar visco-elastic material of plate are

$$\left.\begin{aligned} M_r &= -\widetilde{D}\mathrm{D_r}\left(\frac{\partial^2\omega}{\partial r^2} + \frac{v}{r}\frac{\partial\omega}{\partial r}\right) \\ M_\theta &= -\widetilde{D}\mathrm{D_\theta}\left(\frac{1}{r}\frac{\partial\omega}{\partial r} + v\frac{\partial^2\omega}{\partial r^2}\right) \end{aligned}\right\} \qquad (2)$$

where

$$D_r = \frac{E_r h^3}{12(1-v_\theta v_r)} \; and \; D_r = \frac{E_\theta h^3}{12(1-v_\theta v_r)} \qquad (3)$$

and $\widetilde{D}$ is the visco-elastic operator.

The deflection ω can be sought in the form of product of two functions as follows:

$$w(r,\theta,t) = W(r,\theta)T(t) \qquad (4)$$

where $\mathrm{W}(r,\theta)$ is the deflection function and $\mathrm{T}(t)$ is the time function. Using equations (2) and (4) in (1) one gets

$$D_r\frac{\partial^4 w}{\partial r^4} + \frac{2}{r}\left(D_r + r\frac{\partial D_r}{\partial r}\right)\frac{\partial^3 w}{\partial r^3} + \frac{1}{r^2}\left[-\overline{D_\theta} + r(2+v_\theta)\frac{\partial D_r}{\partial r} + r^2\frac{\partial^2 D_r}{\partial r^2}\right]\frac{\partial^2 w}{\partial r^2} + \frac{1}{r^3}\left[\overline{D_\theta} + r\frac{\partial\overline{D_\theta}}{\partial r} + r^2\frac{\partial^2 D_r}{\partial r^2}\right]\frac{\partial w}{\partial r} + \bar{\rho}h\frac{\partial^2 w}{\partial t^2} = 0 \qquad (5)$$

Substituting dimensionless quantities, $R = \frac{r}{a}, H = \frac{h}{a}, \rho = \frac{\bar{\rho}}{a}, D_R = \frac{D_r}{a^3}, D_\theta = \frac{\overline{D_\theta}}{a^3} \; and \; \overline{W} = \frac{w}{a}$

These equations are expressions for transverse motion of a circular plate with variable thickness.

Equation (1) will be transformed in to the following form [3]

$$D_R\frac{\partial^4\overline{W}}{\partial R^4} + \frac{2}{r}\left(D_R + R\frac{\partial D_R}{\partial R}\right)\frac{\partial^3\overline{W}}{\partial R^3} + \frac{1}{R^2}\left[-D_\theta + R(2+v_\theta)\frac{\partial D_R}{\partial R} + r^2\frac{\partial^2 D_R}{\partial R^2}\right]\frac{\partial^2\overline{W}}{\partial R^2} + \frac{1}{R^3}\left[D_\theta + R\frac{\partial D_\theta}{\partial R} + R^2\frac{\partial^2 D_R}{\partial R^2}\right]\frac{\partial\overline{W}}{\partial R} + a^3\rho H\frac{\partial^2\overline{W}}{\partial t^2} = 0$$

Analysis of Equation of Motion

Assuming a steady temperature field in the radial and circumference direction for a circular plate as [2]

$$T = T_0(1-R)(1-\cos\theta) \qquad (6)$$

where T denotes the temperature excess above the reference temperature and T_0 denotes thereference temperature.

The temperature dependence of the modulus of elasticity for most structural material is given as

$$E_R(T) = E_1(1-\gamma T), E_\theta(T) = E_2(1-\gamma T) \qquad (7)$$

where E_0 is the value of Young's modulus at the reference temperature, i.e. T =0 and γ is the slope of variation E of with T.

The module variation, in view of expressions (6) and (7), becomes

$$E_R(r) = E_1[1-\alpha(1-R)(1-\cos\theta)], E_\theta(r) = E_2[1-\alpha(1-R)(1-\cos\theta)] \qquad (8)$$

where $\alpha = \gamma T_0$ $(0 \le \alpha < 1)$ is a parameter known as thermal gradient.

The expression for the maximum strain energy V and maximum kinetic energy T in the plate, when it vibrates with the mode shape $W(r,\theta)$ are given as [3]

$$\left.\begin{aligned} V &= \frac{1}{2}\int_0^a\int_0^{2\pi}\left[D_R\left\{\left(\frac{\partial^2 W}{\partial R^2}\right) + 2v_\theta\frac{\partial^2 W}{\partial R^2}\left(\frac{1}{R}\frac{\partial W}{\partial R}\right)\right\} + D_\theta\left(\frac{1}{R}\frac{\partial W}{\partial R}\right)^2\right]R\,dR\,d\theta \\ &\text{and} \\ T &= \frac{1}{2}p^2\int_0^a\int_0^{2\pi}\rho H\,W^2 R\,dR\,d\theta \end{aligned}\right\} \qquad (9)$$

It is assumed that the thickness varies in linearly in two dimensional as [2]

$$H = H_o\,F(R,\theta) \qquad (10)$$

where $(H_0 = H|_{\cdot R=0})$ and $F(R,\theta) = (1-\beta_1 R)(1-\beta_2\,Cos\,\theta)$

Assume that $W(r,\theta) = W_1(r)cos\theta$ (11)

Using equations (6) and (10) in equations (9), one gets

$$V = \frac{\pi a^3 E_0 H^3{}_0}{24(1-v^2)}\int_0^1 \left(1-\alpha(1-R)(1-cos\,\theta)\right)\left((1-\beta_1 R)(1-\cos\theta)\right)^3 \left\{\left[\left(\frac{d^2\bar{W}}{dR^2}\right) + 2v_\theta \frac{d^2\bar{W}}{dR^2}\left(\frac{1}{R}\frac{d\bar{W}}{dR}\right) + \left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)^2\right]\right\} RdR$$

and

$$T = \frac{\pi a^8 p^2 \rho\, H_0}{2}\int_0^1 [(1-\beta_1 R)(1-\beta_2\cos\theta)]R\bar{W}^2 dR \quad (12)$$

Solutions and Frequency Equation

Rayleigh-Ritz technique requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that

$$\delta(V - T = 0) \quad (13)$$

For arbitrary variation of W satisfying relevant geometric boundary conditions. Circular plate clamped at the edges r=a i.e. R=1 the boundary conditions are

$$\bar{W} = \frac{d\bar{W}}{dR} = 0 \quad \text{at} \quad R = 1 \quad (14)$$

and the corresponding two terms of deflection function is taken as

$$\bar{W}(R) = C_1(1-R^2) + C_2(1-R)^3 \quad (15)$$

where$C_1$and $C_2$are undetermined coefficients.
Now using equations (13) in equation (14), one has

$$\delta(V_1 - \lambda^2 T_1) = 0 \quad (16)$$

Where

$$V_1 = \int_0^1 \left(1-\alpha(1-R)(1-cos\,\theta)\right)\left((1-\beta_1 R)(1-\cos\theta)\right)^3 \left\{\begin{bmatrix}\left(\frac{d^2\bar{W}}{dR^2}\right) + 2v_\theta \frac{d^2\bar{W}}{dR^2} \\ \left(\frac{1}{R}\frac{d\bar{W}}{dR}\right) + \left(\frac{1}{R}\frac{d\bar{W}}{dR}\right)^2\end{bmatrix}\right\} RdR$$

And

$$T = \int_0^1 [(1-\beta_1 R)(1-\beta_2\cos\theta)]R\bar{W}^2\, dR \quad (17)$$

Where,

$$l = \frac{\pi a^5 E_0 H_0{}^3}{24(1-v^2)} \quad (18)$$

Equation (16) involves the unknowns C_1 and C_2 arising due to substitution of $\bar{W}(R)$ from (15). These unknowns are to be determined from equation (16), for which

$$\frac{\partial}{\partial C_n}(V_1 - \lambda^2 T_1) = 0,\ for\, n = 1,2. \quad (19)$$

Equation (16) simplifies to the form

$$b_{n1}C_1 + b_{n2}C_2 = 0 \quad for\ n = 1,2. \quad (20)$$

Where,$b_{n1}\ and\ b_{n2}$ $(n = 1,2)$ involve the parametric constant and frequency parameter.
For a non-trivial solution, the determinant of the coefficient of equation (20) must be zero. Thus, one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \quad (21)$$

On solving (21) one gets a quadratic equation in λ^2, so it will give two roots.

Result and Discussion

Frequencies for the first two modes of vibrations are computed for circular plate whose thickness varies linearly in two directions. Different values of thermal gradient α and taper constants β_1 and β_2, has been considered. All results are presented in graphical form.

Figure 1 contains numerical results for frequency parameter λ for different values of thermal gradient α form 0.0 to 1 and taper constants

a) $\beta_1 = \beta_2 = 0.0$
b) $\beta_1 = \beta_2 = 0.4$
c) $\beta_1 = \beta_2 = 0.8$

It can be seen from the tables that as thermal gradient α increases, frequency parameter decreases for both the modes of vibration.

Figure 2 contains numerical results for frequency parameter λ for different values of thermal gradient α form 0.0 to 1 and taper constants

a) $\beta_1 = 0.0$, $\beta_2 = 0.2$
b) $\beta_1 = 0.2$, $\beta_2 = 0.4$
c) $\beta_1 = 0.4$, $\beta_2 = 0.8$

It can be seen from the tables that as thermal gradient α increases, frequency parameter also decreases for both the modes of vibration.

Figure 3 contains numerical results for frequency parameter λ for different values of thermal gradient β_1 form 0.0 to 1 and taper constants

a) $\alpha = \beta_2 = 0.0$
b) $\alpha = \beta_2 = 0.4$
c) $\alpha = \beta_2 = 0.8$

It can be seen from the tables that as thermal gradient β_1 increases, frequency parameter increases for both the modes of vibration.

Figure 4 contains numerical results for frequency parameter λ for different values of thermal gradient β_2 form 0.0 to 1 and taper constants

a) $\alpha = \beta_1 = 0.0$
b) $\alpha = \beta_1 = 0.4$
c) $\alpha = \beta_1 = 0.8$

It can be seen from the tables that as thermal gradient β_1 increases, frequency parameter also increases for both the modes of vibration.

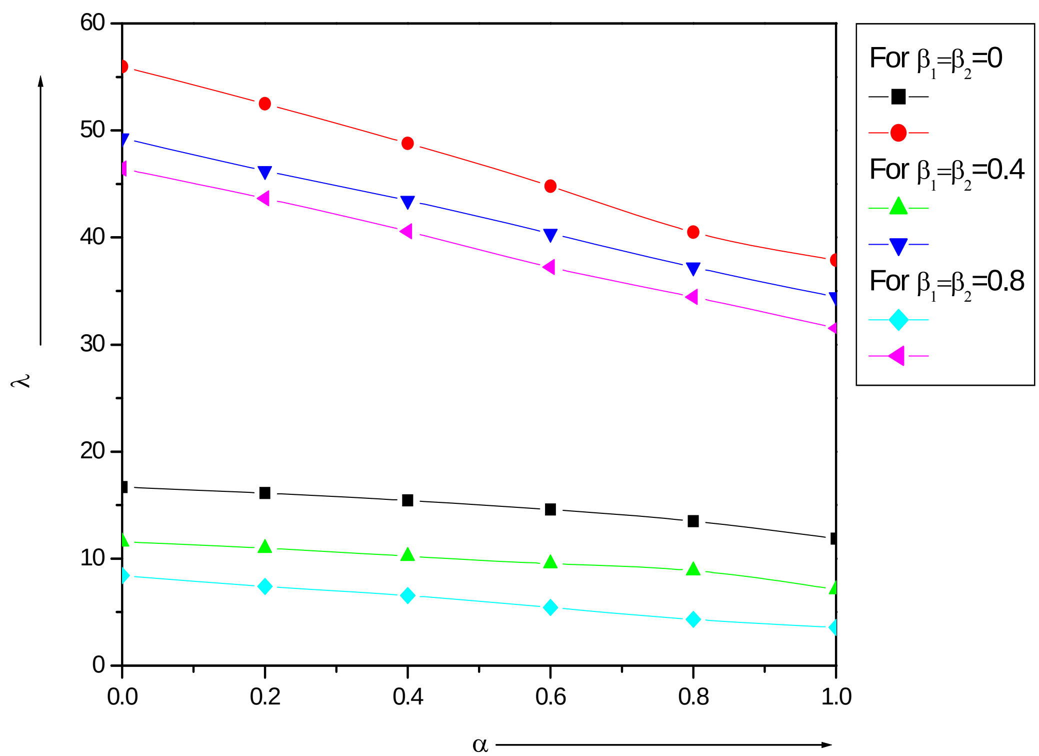

Fig 1 Frequency parameter vs thermal gradient

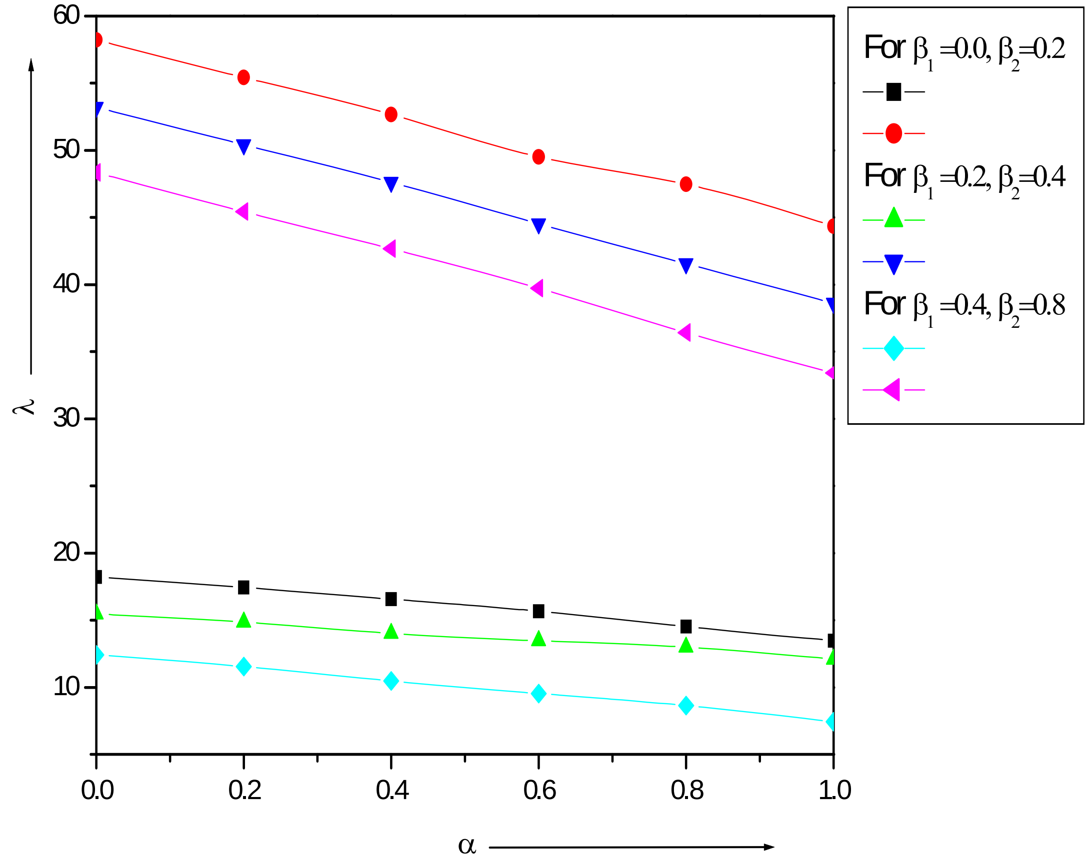

Fig 2: Frequency parameter vs thermal gradient

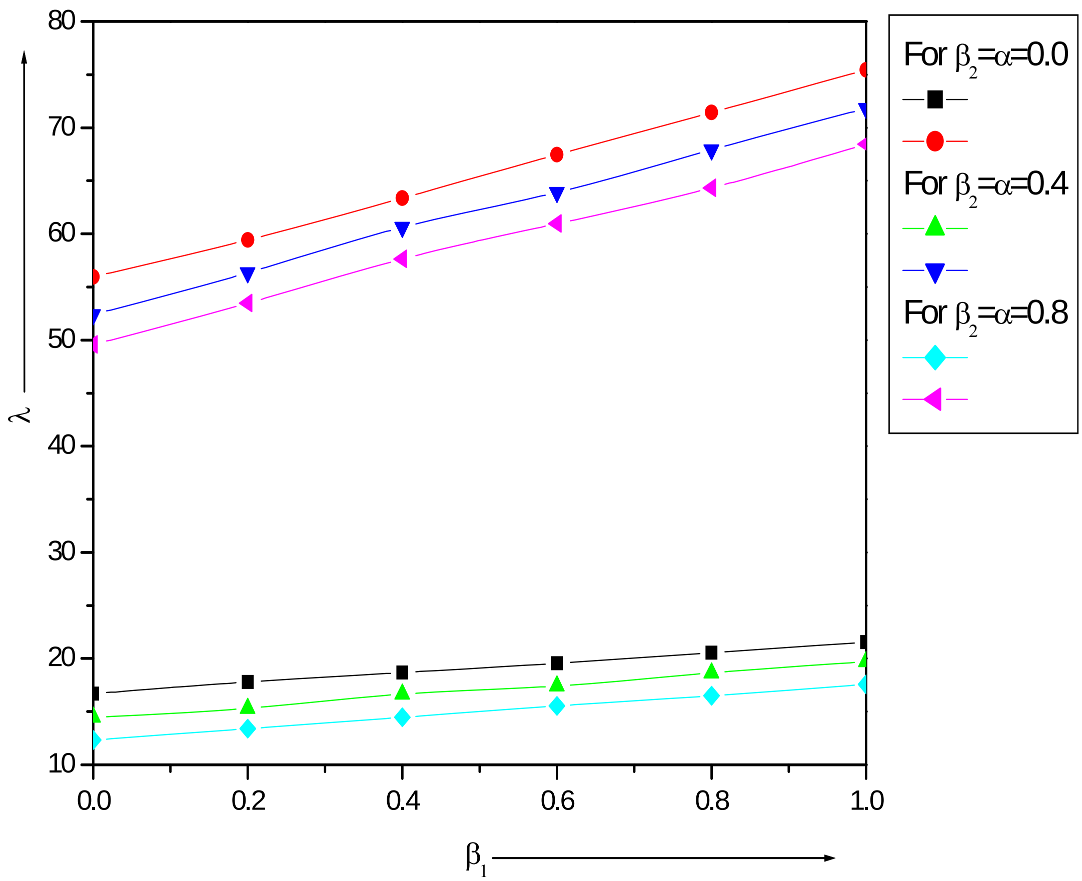

Fig 3: Frequency parameter vs taper constant β_1

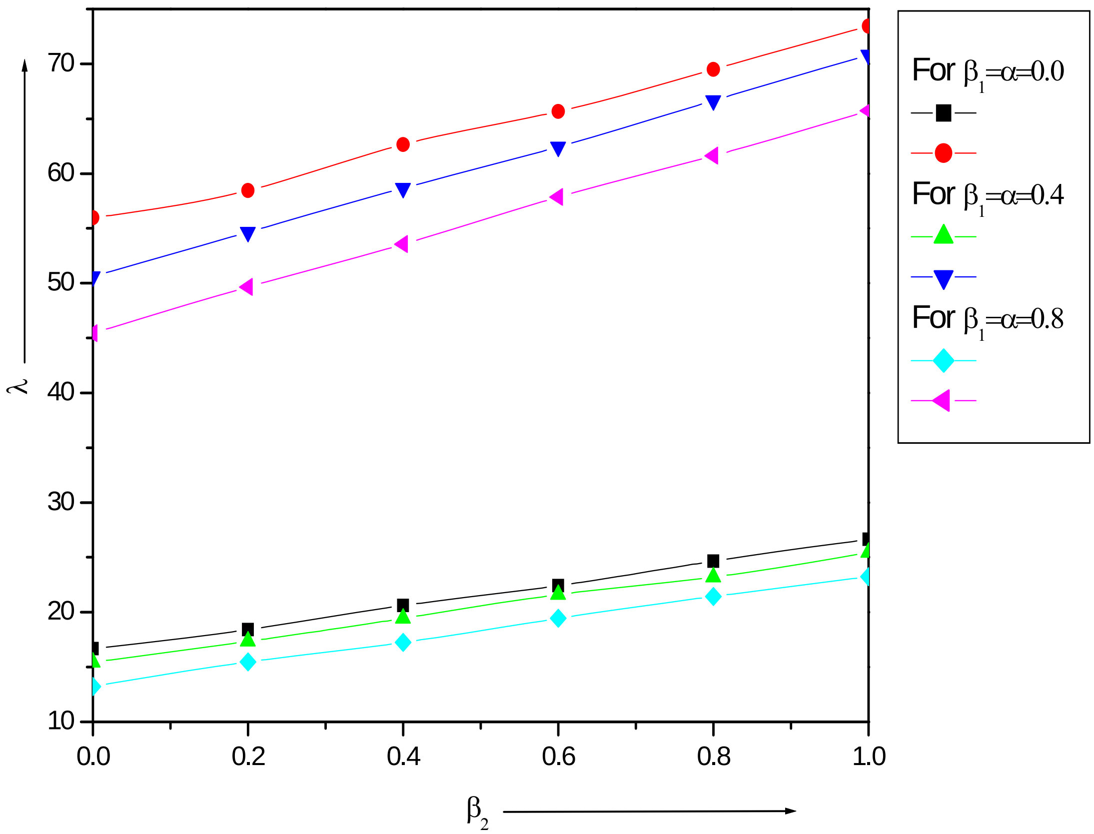

Fig 4: Frequency parameter vs taper constant β_2

Conclusion

This paper has devoted to study the effect of material on the fundamental frequencies of circular plate whose thickness varies linear in two directions based on classical plate theory. Different values of taper constant and thermal gradient have been considered. It is observed on increasing the value of thermal gradient, frequency parameter decreases whereas on increasing thermal gradient the value of frequency parameter increases.

Thus engineers can obtained a change in the frequencies of a plate by a proper choice of various plate parameters considered here and fulfill their requirements.

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