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On 0− Edge labelings in certain graphs

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Abstract
In this paper we prove that the complete nary pseudo tree, two dimensional cylindrical meshes $P_m \times C_n$, $n \equiv 0(\text{mod}2)$, n− dimensional hypercube Q_n, the graph obtained by at- taching C_m to $mK_{1,n}$($m \equiv 0(\text{mod } 2)$, The circular ladder graph, The friendship graph $C_n(m)$ and the graph $P_m \times P_m \times P_m$ are 0− edge magic graphs.

Keywords: graph labeling, magic labeling, 0-edge magic labeling

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967 [5]. A graph labeling is an assignment of integers to the vertices or edges or both, sub- ject to certain conditions. Labeled graphs serve as useful models for a broad range of applications such as coding theory, x-ray crystallography, radar, as- tronomy, circuit design, communication network addressing and data base management. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, mean labeling, arithmetic labeling etc., have been studied in over 1800 papers [1]. Various results on magic graphs have been studied in the literature [4, 6, 7, 8, 9, 10]. The concept of 0−edge magic labeling was introduced in [2]. They proved that paths, grid graphs, cycles, wheels, binary trees and some flower graphs are 0−edge magic. This concept was generalized as n− edge magic graphs in [3].

In this paper we prove that the complete n− ary pseudo tree, two dimen- sional cylindrical meshes Pm × Cn, n ≡ 0(mod2), n− dimensional hypercube Qn, the graph obtained by attaching Cm to mK1, n(m ≡ 0(mod 2), The circular ladder graph, The friendship graph C(m) and the graph Pm × Pm × Pm are 0− edge magic graphs.

2. Preliminaries

In this section we give the basic notions relevant to this paper. Let G = G(V, E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we deal with labeling with domain either the set of all vertices or the set of all.

Edges or the set of all vertices and edges. We call these labelings as the vertex labeling or the edge labeling or the total labeling respectively.

Definition 2.1 The vertex-weight of a vertex v in G under an edge labeling to be the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels correspond- ing to all the edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or vertex-magic total labeling respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling respectively.

Definition 2.2 The edge-weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e. Under a total labeling, we also add the label of e. Using edge-weight, we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

Definition 2.3 A (p, q)-graph G is said to be (1, 0) edge-magic with the common edge count k if there exists a bijection f: V (G) → {1, 2, . . . , p} such that for all e = (u, v) ∈ E(G), f (u) + f (v) = k. It is said to be (1, 0) edge-antimagic if for all e = (u, v) ∈ E(G), f (u) + f (v) are distinct.

Definition 2.4 A (p, q)-graph G is said to be (0, 1) vertex-magic with the common vertex count k if there exists a bijection f: E(G) → {1, 2, . . . , q} such that for each u ∈ V (G),),e f (e) = k for all e = (u, v) ∈ E(G) with v ∈ V (G). It is said to be (0, 1) vertex-antimagic if for each u ∈ V (G),),e f (e) are distinct for all e = (u, v) ∈ E(G) with v ∈ V (G)

Definition 2.5 A (p, q)-graph G is said to be (1, 1) edge-magic with the common edge count k if there exists a bijection f: V (G) ∪ E(G) → {1, 2, . . . , p + q} such that f (u)+f (v)+f (e) = k for all e = (u, v) ∈ E(G). It is said to be (1, 1) edge-antimagic if f (u) + f (v) + f (e) are distinct for all e = (u, v) ∈ E(G).

3. Main Results

Definition 3.1 A (p, q)-graph G is said to be 0− edge magic if there exists a bijection
f: V(G)→{1,−1} such that for each uv ∈ E(G), f (u)+f(v) = 0

Theorem 3.1 A complete n− ary pseudo tree is 0−edge magic.

Proof: Let G be a complete n− ary pseudo tree of height h. Clearly the vertices in level li are adjacent to some vertices in level (i − 1) and some vertices in level (i + 1) for 1 ≤ i ≤ (h − 1). This is true for the vertices in all the levels except the root level l0 and the leaf level lh. Label the vertices in ith level as −1 when i ≡ 1(mod2) and label the remaining vertices as 1. Therefore, all the vertices in level i have label −1 when i is odd and the vertices in (i − 1)th level have label 1. The edges incident at ith level vertices has the label zero. If the height of the tree h is odd, The (h − 1)th level is even and the vertices at level h is adjacent to the vertices in (h − 1)th level. The edges incident at level h has label zero. Clearly the root vertex is ad- jacent to the first level vertices, the edges incident with root vertex has label zero. Thus all the edges in n− ary pseudo tree has label zero and hence G is zero edge magic.

Theorem 3.2 The two dimensional cylindrical meshes Pm × Cn, n ≡ 0(mod2) is 0−edge magic.

Proof: The two dimensional cylindrical mesh Pm × Cn has mn vertices and 2mn + (m − n) edges. Denote the vertex set as V = {v1, v2, . . . vmn} and the edge set as
E = {vivi+1|1 ≤ i ≤ (mn − 1), i /= nj, j ∈ N} ∪ {vjnvn(j−1)+1|1 ≤ j ≤m} ∪ {vivi+n|1 ≤ i ≤ n(m − 1)}.
Now define a map f: V → {−1, 1} as follows.

$$f(v_i)=\begin{cases}(-1)^i & \text{if } (nj+1)\le i\le n(j+1),\ 1\le j\le (m-1), j\equiv 1(mod\ 2)\\ (-1)^{i+1} & \text{if } (nj+1)\le i\le n(j+1),\ 1\le j\le m, j\equiv 0(mod\ 2)\end{cases}$$

Define the induced map f ∗ : E → N such that
f ∗(vivj) = f (vi) + f (vj)
Clearly by the definition of f, every pair of adjacent vertices has different la- bels. That is, if vivj is an edge, and if f (vi) = 1, then f (vj) = −1, and vice versa.
f ∗(vivj) = f (vi) + f (vj) = 1 − 1 = 0.
Hence, Pm × Cn is 0− edge magic.

Theorem 3.3 The n− dimensional hypercube Qn is 0−edge magic.

Proof: The graph Qn has 2n vertices and n2n−1 edges.
Denote the vertices as $\{v_{11}, v_{12}, v_{13}, v_{14}, v_{21}, v_{22}, v_{23}, v_{24}, . . .v_{2n-21}, v_{2n-22}, v_{2n-23}, v_{2n-24}\}$.

Denote the edge set as $E = \{v_{ij}v_{(i+1)j} | 1 \le j \le 4, 1 \le i \le 2^{n-2} - 1\} \cup \{v_{2n-2j}, v_{ij} | 1 \le j \le 4\} \cup \{v_{lk}, v_{l(k+1)} | 1 \le l \le 2^{n-2}, 1 \le k \le 3\} \cup \{v_{i4}v_{i1} | 1 \le i \le 2^{n-2}\}$.

Define a map f: V → {1, −1} such that $f(v_{ij}) = (-1)^{i+j}$.
Clearly, the vertices v_{ij} where (i+j) ≡ 1(mod2), are adjacent to the vertices v_{lk} where (l + k) ≡ 0 (mod 2) and vice versa.
Define the induced mapf ∗: E → N such that $f*(v_{ij}, v_{kl}) = f(v_{ij}) + f(v_{kl})$.
Clearly, for every edge (vij , vkl), (k + l) is even whenever (i + j) is odd and (k + l) is odd whenever (i + j) is even.
f^*(vij , vkl) = f (vij) + f (vkl) = (−1)i+j + (−1)k+l = −1 + 1 = 0.
Thus Qn is 0− edge magic.

Theorem 3.4 The graph Cm attached to mK1, n(m ≡ 0(mod 2) is 0−edge magic.

Proof: The graph Cm attached to mK1, n has m(n + 1) + 1 vertices and m(n + 2) edges.
Denote the vertices as
V = {v1, v2, . . .vm, w1, w2, . . . , wm, w11, w12, . . . , w1(n−1), w21, w22, . . . , w2(n−1),
. . . , wm1, wm2, . . . , wm(n−1)} and denote the edge set E as
E = {vmv1} ∪ {vivi+1|1 ≤ i ≤ (m − 1)} ∪ {wi, wij |1 ≤ i ≤ m, 1 ≤ j ≤ (n − 1)} ∪ {viwi|1 ≤ i ≤ m}.

Define a map f: V → {−1, 1} as follows.

$$f(v_i) = (-1)^i,\ 1 \le i \le m;$$
$$f(w_i) = (-1)^{i+1},\ 1 \le i \le m;$$
$$f(w_{ij}) = -f(w_i),\ 1 \le j \le (n-1);$$

Define the induced map $f^*: E \to N$ such that $f^*(uv) = f(u) + f(v)$.
For 1 ≤ i ≤ m − 1, f ∗(vivi+1) = f (vi) + f (vi+1) = 1 − 1 = 0.

$$f^*(v_1, v_m) = -1 + 1 = 0$$
$$f^*(wij, wi) = 1 - 1 = 0.$$

Thus all the edge labels are zero and hence the graph Cm attached to $mK_{1,n}$ is 0− edge magic.

Theorem 3.5 The circular ladder graph is 0−edge magic.

Proof: The circular ladder graph G = {V, E} has 2n vertices and 3n edges. Denote the vertex set as

$V = \{v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}, \ldots, v_{2n}\}$ and the edge set E as $E = \{v_i v_{n+i} | 1 \leq i \leq n\} \cup \{v_j\ v_{j+1} | 1 \leq j \leq (n-1)\} \cup \{v_{n+j}, v_{n+j+1} | 1 \leq j \leq (n-1)\} \cup \{v_1 v_n, v_{n+1} v_{2n}\}$

Define the map $f: V \rightarrow \{1, -1\}$ as follows.

$f(v_i) = (-1)^{i+1}$, $1 \leq i \leq n$;

$f(v_{n+i}) = -f(v_i)$, $1 \leq i \leq n$;

Define the induced map f^* as $f^*(v_{n+j}, v_{n+(j+1)}) = f(v_{n+j}) + f(v_{n+(j+1)})$.

Clearly $n + j + 1$ is odd whenever $n + j$ is even and $n + j + 1$ is even when ever $n + j$ is odd. $f(v_{n+j})$ and $f(v_{n+(j+1)})$ has distinct values.

$f*(v_{n+\ n+(j+1)}) = f(v_{n+j}) + f(v_{n+(j+1)}) = 1-1=0.$

$f^*(v_1\ v_n) = f(v_1) + f(v_n) = 1-1=0$

$(f^*(v_{n+1,}\ v_{2n}) = f(v_{n+1}) + f(v_{2n}) = 1-1=0.$

Thus all the edges has label zero and hence the circular ladder graph is 0−edge magic.

Theorem 3.6 The friendship graph $C_n^{(m)}$ is 0−edge magic.

Proof: The friendship graph $C_n^{(m)}$ has $m(n-1)+1$ vertices and mn edges.

Denote the vertex set as $V = \{v_0, v_{11}, \ldots, v_{1(n-1)}, v_{21}, v_{22}, \ldots, v_{2(n-1)}, \ldots, v_{m1}, v_{m2}, \ldots, v_{m(n-1)}\}$ and edge set E as $E = \{v_{ij}\ v_{i(j+1)} | 1 \leq i \leq m, 1 \leq j \leq m-2\} \cup \{v_0 v_{i1} | 1 \leq i \leq m\} \cup \{v_0 v_{i(n-1)} | 1 \leq i \leq m\}$.

Define the map $f: V \rightarrow \{1,-1\}$ such that

$f(v_0) = 1$, $1 \leq i \leq m, i \equiv 1(\text{mod}2)$; $f(v_{ij}) = (-1)^{i+j+1}$, $1 \leq j \leq (n-1)$;

$f(v_{ij}) = (-1)^{i+j}$; $1 \leq i \leq m$; $i \equiv 0(\text{mod}2), 1 \leq j \leq (n-1)$.

Define the induced function $f^*: E \rightarrow N$ such that $f^*(v_i v_j) = f(v_i) + f(v_j)$

Clearly all the edges has label zero and hence $C_n^{(m)}$ is 0−edge magic.

Theorem 3.7 The graph $P_m \times P_m \times P_m$ is 0−edge magic.

Proof: The graph $P_m \times P_m \times P_m$ has m^3 vertices and $3m^2(m-1)$ edges. Denote the vertices as $V = \{v_{11}, v_{12}, \ldots v_{1m^2}, v_{21}, v_{22}, \ldots v_{2m^2}, v_{31}, v_{32}, \ldots v_{3m^2}, \ldots, v_{m1}, v_{m2}, \ldots, v_{mm^2}\}$

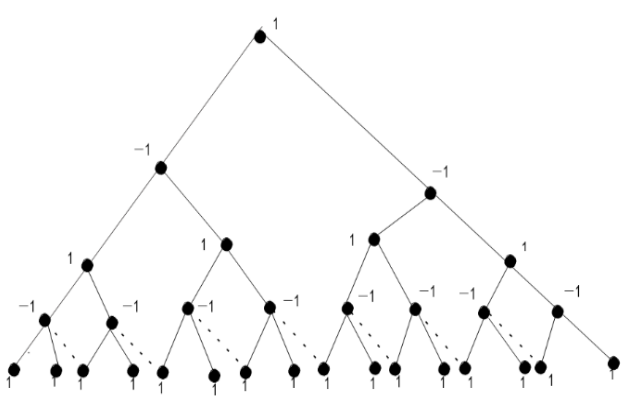

Fig 1: n-ary psudo tree

and the edges as $E = \{v_{ij}\ v_{i(m+j)} | 1 \leq j \leq (m^2-m), 1 \leq i \leq m\} \cup \{v_{ji} v_{(j+1)i} | 1 \leq j \leq (m-1), 1 \leq i \leq m\} \cup \{v_{i(k+j)} v_{i(k+j+1)} | 0 \leq k \leq m^2 - m\}$. Label the vertices of $P_m \times P_m \times P_m$ as follows.

$f(v_{ij}) = (-1)^{(i+j)}$.

Define the induced map $f^*: E \rightarrow N$ such that $f^*(uv) = f(u) + f(v)$.

Clearly, the end vertices of edges are v_{ij}, v_{kl}, in which $(k + l)$ is even whenever $(i+j)$ is odd or $(k+l)$ is odd whenever $(i+j)$ is even. The labels of each vertices is 1 and −1 and hence the edge label is $1 + (-1) = 0$,. $f^*(uv) = f(u) + f(v) = 1 + (-1) = 1 - 1 = 0$. Thus $P_m \times P_m \times P_m$ is 0−edge magic.

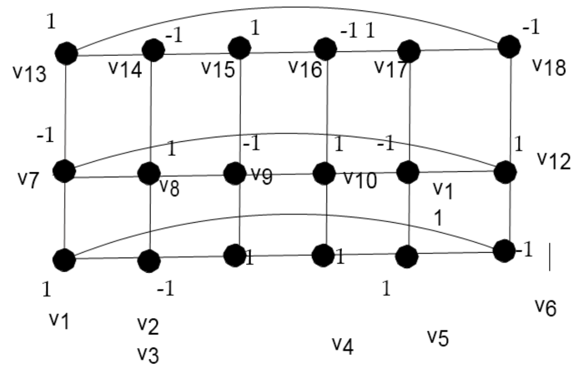

Fig 2: Cylindrical mesh $P_m \times C_n$

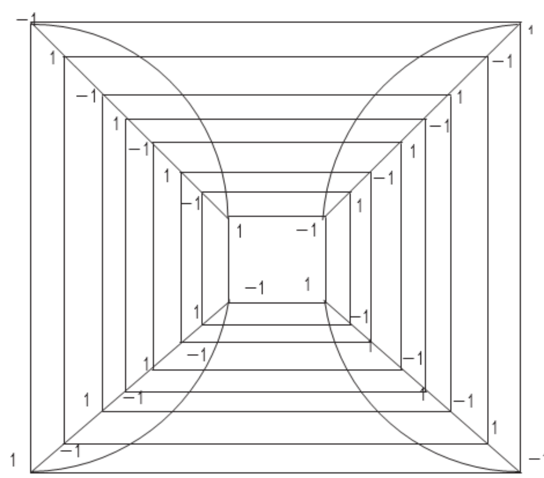

Fig 3: 5-dimensional Hyper cube

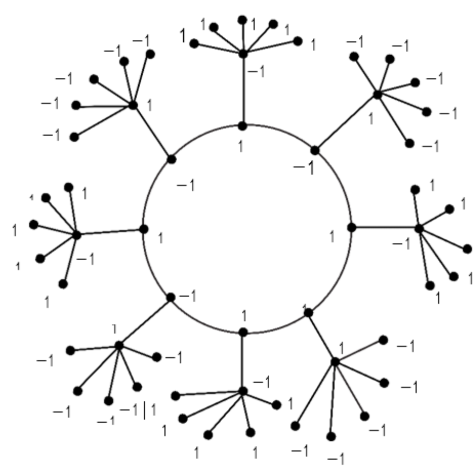

Fig 4: star graph

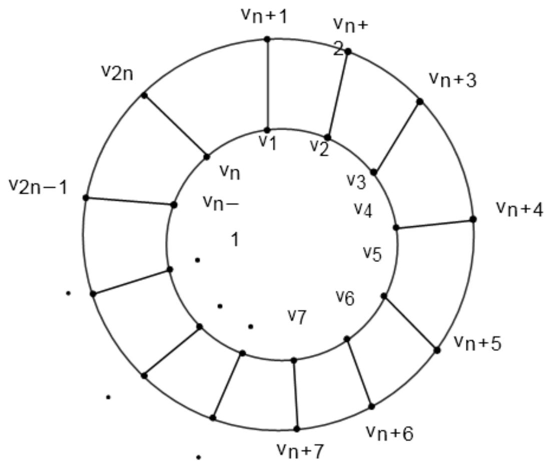

Fig 5: Cirular ladder

4.1 Conclusion

In this paper we have shown that the graphs such as complete n− ary pseudo tree, two dimensional cylindrical meshes Pm × Cn, n ≡ 0(mod 2), n− dimen- sional hypercube Qn, graph obtained by attaching Cm to mK1,n(m ≡ 0(mod 2), circular ladder graph, friendship graph C(m) and the graph Pm × Pm × Pm are 0_n− edge magic graphs.

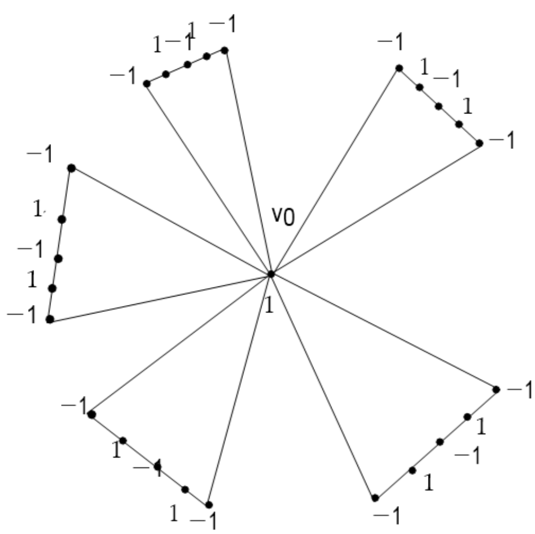

Fig 6: Friendship graph

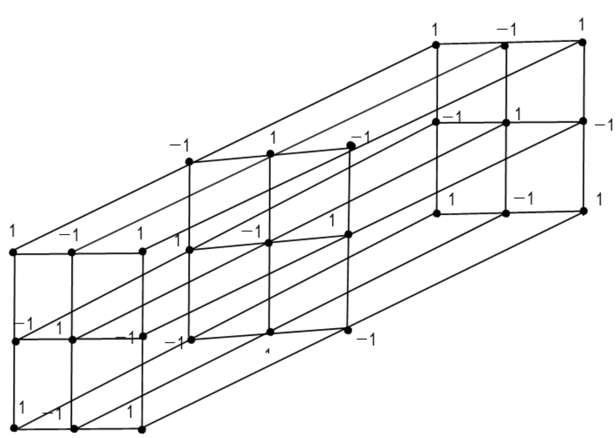

Fig 7: The graph P3 × P3 × P3

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