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Determination of optimal labour demand when supply has random Fluctuations

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Abstract
In any organization or industry manpower is an important input. Usually the optimal level of supply of labour is determined taking into account the demand for the same. But there are some industries like construction work, the demand for labour can be decided taking into account the level of supply of the same. In this paper the optimal demand for labour size is determined under the assumption the supply of the same is of random nature. Numerical illustration is also provided.

Keywords: Demand, Supply of labour, optimal size.

Introduction
In any production oriented or marketing organisation or industry the planning of the work schedule is based on the availability of several factors like raw material, fuel, machinery and manpower. Manpower plays a vital role since without necessary level of manpower the activities cannot be carried out. In many areas of human activity the availability of suitable level of manpower is not always assured. For example the availability of suitable manpower in contract industry is highly fluctuating. It is in other words it can be called as random in character. Hence optimal level of demand of manpower can be determined by suitable planning of work schedule. The work plan or schedule should be decided in such a way that the demand for manpower is optimal in the sense that the shortage or excess of manpower is minimized. The determination of optimal level of supply of a product, raw material when demand is of random character is discussed in Hanssmann (1961) [1]. A stochastic inventory model to determine optimal inventory raw materials in between two machines in series has been discussed by Ramachandran and Sathiyamoorthi (1981) [2]. Lack of Memory Property (LMP). This kind of a distribution with SCBZ property has been discussed by Raja Rao and Talwalker (1990) [3]. The determination of optimal manpower reserve when demand for manpower has fluctuations has been discussed by Arivazhagan *et al*. (2010) [4]. In this paper it is proposed to determine the optimal demand for manpower utility, under the assumption that the supply of manpower is highly fluctuating and hence it can be depicted as a random variable.

2. Assumptions
- The supply of labour in the work location is a random variable.
- The work schedule can be pre-determined such that the excess of manpower available and also shortage of manpower is a minimum.
- The supply of manpower planning in work area is not a control variable and it is a random variable.
- The non-utilization of the labour procured involves a loss and the shortage of manpower also results in loss.

3. Notations
X: The level of demand for labour which is the control variable, since the work schedule can be suitably determined.

S: The supply of labor which is a random variable with p.d.f f(.) and c.d.f F(.)
C_1: The cost due to shortage of manpower.
C_2: The cost due to non-utilization of labor or delay due to non-utilization of available labor.

4. Results

The expected cost of utilization of manpower is denoted as

$$E(C) = c_1 \int_0^X (X-S)\ f(s)\ ds + c_2 \int_X^\infty (S-X)\ f(s)\ ds \quad (1)$$

$$= I_1 + I_2$$

To find the optimal X (demand) or (optimal utilization of manpower)

the condition is $\frac{dE(C)}{dx} = 0$ (2)

Using Leibnitz rule for the differentiation of an integral given as

$$\frac{d}{dx}\int_{\varphi(x)}^{\Psi(x)} f(x,t)dt = \Psi'(x)f[x,\Psi(x)] - \varphi'(x)f[x,\varphi(x)] + \int_{\varphi(x)}^{\Psi(x)} \frac{d}{dx}f(x,t)dt$$

$$\frac{dI_1}{dX} = c_1 \int_0^X (1)\ f(s)\ ds = c_1\,[F(x)] \quad (3)$$

$$\frac{dI_2}{dX} = c_2\,[0-(1)(0) + \int_X^\infty (-1)\ f(s)\ ds] = -c_2\,[1-F(x)] \quad (4)$$

$$\frac{dE(C)}{ds} = c_1\,[F(x)] - c_2 + c_2\,[F(x)] = 0$$

$$F(x)(c_1+c_2) - c_2 = 0$$

$$F(\hat{X}) = \frac{C_2}{C_1+C_2} \quad (5)$$

Any value of X, given the distribution of X, c_1 and c_2, the optimal value of X can be determined

Let S ~ exp(θ)

$$F(\hat{X}) = 1 - e^{-\theta\hat{x}} = \frac{c_2}{c_1+c_2}$$

$$e^{-\theta\hat{x}} = 1 - \frac{c_2}{c_1+c_2} = \frac{c_1}{c_1+c_2}$$

Taking logarithm both sides

$$-\theta\hat{x} = \log_e\left(\frac{c_1}{c_1+c_2}\right)$$

and hence

$$\hat{x} = \frac{1}{\theta}\log_e\left(\frac{c_1+c_2}{c_1}\right)$$

5. Application of the Model

The optimal solution is derived under the assumption that the supply of labor follows Erlang 2 distribution. If may be observed that Erlang distribution doesn't satisfy the so called Setting the Clock Back to Zero property which is a slightly modified form of the expression for expected cost is given as

$$E(C) = c_1 \int_0^X (X-S)\ f(s)\ ds + c_2 \int_X^\infty (S-X)\ f(s)\ ds$$

Where $c_2 = Cost\ due\ to\ excess\ of\ labour$

$c_1 = Cost\ due\ to\ shortage\ of\ manpower$

$Erlang\ 2\ distribution\ with\ p.d.f$

$f(x) = \mu^2\, x\, e^{-\mu x}$

$S \sim E_2\ Which\ doesnot\ satisfy\ the\ SCBZ\ property$

$$F(\hat{x}) = \frac{c_2}{c_1+c_2}$$

$The\ optimal\ solution\ is\ feasible\ oly\ when\ c_2 > c_1$

$In\ the\ case\ of\ E_2\ the\ c.d.f$

$$F(x) = \int_0^X \mu^2\, t\, e^{-\mu t}\, dt$$

$$F(x) = \mu^2 \int_0^X t\, e^{-\mu t}\, dt$$

$$u = t, dv = e^{-\mu t}dt, u' = 1, v = \frac{e^{-\mu t}}{-\mu}, v_1 = \frac{e^{-\mu t}}{\mu^2}$$

$$\int udv = uv - u'v_1 + u''v_2 - \cdots \ldots \ldots$$

$$\mu\left[(-te^{-\mu t})_0^x - \left(\frac{e^{-\mu t}}{\mu}\right)_0^x\right]$$

$$\mu\left[-xe^{-\mu x} - \left[\frac{e^{-\mu x}}{\mu} - \frac{1}{\mu}\right]\right]$$

$$\mu\left[-xe^{-\mu x} - \frac{e^{-\mu x}}{\mu} + \frac{1}{\mu}\right]$$

$$-x\mu e^{-\mu x} - \frac{\mu e^{-\mu x}}{\mu} + \frac{\mu}{\mu}$$

$$1 - e^{-\mu x} - x\mu e^{-\mu x}$$

$$F(x) = 1 - e^{-\mu x}(1 + x\mu) \quad On\ simplification$$

$$F(\hat{x}) = \frac{c_2}{c_2 + c_1}$$

$$1 - e^{-\mu x}(1 + x\mu) = \frac{c_2}{c_2 + c_1}$$

$$e^{-\mu x}(1 + x\mu) = 1 - \frac{c_2}{c_2 - c_1}$$

$$e^{-\mu x} = \frac{c_2}{c_2 + c_1} \cdot \frac{1}{(1 + x\mu)}$$

$$-\mu x = log\left(\frac{c_1}{c_1 + c_2} \cdot \frac{1}{(1 + x\mu)}\right)$$

$$\mu x = -log_e\left(\frac{c_1}{(c_1 + c_2)(1 + x\mu)}\right)$$

$$x = \frac{1}{\mu} log_e\left(\frac{c_2}{(c_1 + c_2)(1 + \hat{x}\mu)}\right)^{-1}$$

$$\hat{x} = \frac{1}{\mu} log_e\left(\frac{(c_1 + c_2)(1 + \hat{x}\mu)}{c_1}\right) \qquad (6)$$

The optimal value of x namely $\hat{x}$ is obtained by solving equation (6)

Numerical illustration

A numerical illustration is provided to establish the validity of model.

Table 1

$\mu = 0.1, c_1 = 100, c_2 = 400$	
$\hat{x}$	R. H .S
1	7.404
2	7.782
3	8.129
4	8.451
5	8.751
6	9.031
7	9.294
8	9.542
9	9.777
10	10.000

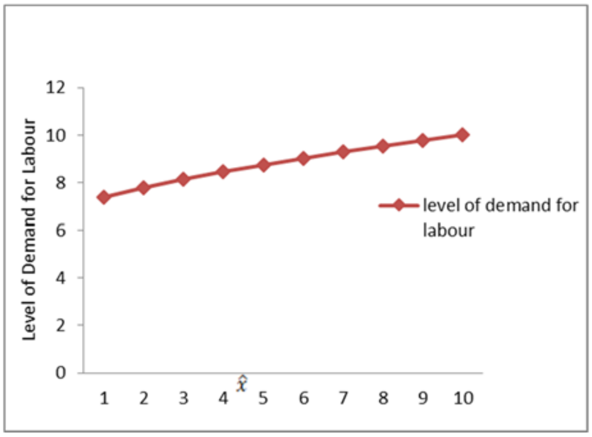

Fig 1

Conclusion

It is observed that when the random variable S follows Erlang 2 distribution, the optimal size of demand for labour to be fixed is 10, when μ=0.1, C_1 =100 and C_2= 400. The optimal size of demand for labour can be found out given the value of μ, C_1 and C_2. Similar remits can be obtained when the supply follows some other distribution.

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