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Observations on the ternary quadratic diophantine equation $x^2 + 9y^2 = 50z^2$

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Abstract
General formulas for generating sequences of non-zero integer solutions to the ternary quadratic Diophantine equation $x^2+9y^2=50z^2$ based on its known solutions are determined. Employing the positive integer solutions of the equation under consideration, a special Pythagorean triangle is obtained.

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1. Introduction
The quadratic Diophantine equations with three unknowns offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems of ternary quadratic Diophantine equations representing specific 3 dimensional surfaces, one may refer [3-23].

In this communication, we present general formulas for generating sequences of non-zero integer solutions to the ternary quadratic Diophantine equation $x^2+9y^2=50z^2$ based on its known solution. Employing the positive integer solutions of the equation under consideration, a special Pythagorean triangle is obtained.

2. Method of Analysis
The ternary quadratic Diophantine equation under consideration

$$x^2+9y^2=50z^2 \qquad (1)$$

To start with, it is noted that (1) is satisfied by the following triples:

(2397,7,339),(2469,511,411),(2865,1645,807),(3171,2401,1113),

$([3150P^2+63Q^2+6300PQ],[3150P^2+63Q^2+126PQ],[7350P^2-147Q^2])$

A natural question that arises now is that, whether a general formula for obtaining a sequence of integer solutions based on the given integer solution can be found? The answer to this question is yes and in what follows, we illustrate a method of obtaining the same.

Let (x_0, y_0, z_0) be any given integer solution of (1).

Formula 1:

Let (x_1, y_1, z_1) where, $x_1 = x_0 + 7h, y_1 = y_0, z_1 = h - z_0$) (2)

be the first solutions of (1).

Substituting (2) in (1) and performing a few calculations, it is seen that

$$h = 14x_0 + 100z_0$$

And thus, $x_1 = 99x_0 + 700z_0$; $z_1 = 14x_0 + 99z_0$

which is written in the matrix form as

$$\begin{pmatrix} x_1 \\ z_1 \end{pmatrix} = M \begin{pmatrix} x_0 \\ z_0 \end{pmatrix} \quad \text{where,} \quad M = \begin{pmatrix} 99 & 700 \\ 14 & 99 \end{pmatrix}$$

Proceeding in a similar manner, we have, in general

$$\begin{pmatrix} x_n \\ z_n \end{pmatrix} = M^n \begin{pmatrix} x_0 \\ z_0 \end{pmatrix} \quad (3)$$

Where

$$M^n = \frac{1}{140\sqrt{2}} \begin{pmatrix} 70\sqrt{2}(\alpha^n + \beta^n) & 700(\alpha^n - \beta^n) \\ 14(\alpha^n - \beta^n) & 70\sqrt{2}(\alpha^n + \beta^n) \end{pmatrix}$$

in which $\alpha = 99 + 70\sqrt{2}, \beta = 99 - 70\sqrt{2}$

Thus, the general formula for obtaining a sequence of non-zero distinct integer solutions based on the given solution to (1) is represented by

$$x_n = \frac{1}{140\sqrt{2}}[70\sqrt{2}(\alpha^n + \beta^n)x_0 + 700(\alpha^n - \beta^n)z_0$$

$$y_n = y_0$$

$$z_n = \frac{1}{140\sqrt{2}}[14(\alpha^n - \beta^n)x_0 + 70\sqrt{2}(\alpha^n + \beta^n)z_0$$

Formula 2:

Let $x_1 = 14x_0, y_1 = 14y_0 + 2h, z_1 = h - 14z_0$ be the second solutions of (1).

Repeating the above process, the corresponding general formula representing the solutions to (1) is

$$x_n = 14^n x_0$$

$$y_n = \frac{1}{120\sqrt{2}}[60\sqrt{2}(\alpha^n + \beta^n)y_0 + 200(\alpha^n - \beta^n)z_0$$

$$z_n = \frac{1}{120\sqrt{2}}[36(\alpha^n - \beta^n)y_0 + 60\sqrt{2}(\alpha^n + \beta^n)z_0$$

Formula 3:

Let $x_1 = 5x_0 - h, y_1 = 5y_0 - h, z_1 = 5z_0$ be the Third solutions of (1).

Repeating the above process, the corresponding general formula representing the solutions to (1) is

$$x_n = \frac{1}{2}[(\alpha^n + \beta^n)x_0 + (\beta^n - \alpha^n)y_0$$

$$y_n = \frac{1}{2}[(\beta^n - \alpha^n)x_0 + (\alpha^n + \beta^n)y_0$$

$$z_n = 5^n z_0$$

3. Remarkable observation

Consider the positive integer solution (x,y,z) to (1). Let m,n be two non-zero distinct integers such that $m = x + z, n = z$. Note that m>n>0. Treating m, n as the generators of the Pythagorean triangle $T(\alpha, \beta, \gamma)$ Where, $\alpha = 2mn, \beta = m^2 - n^2, \gamma = m^2 + n^2$, it is seen that,

1. Each of the following expressions is a Nasty Number

a) $6(\alpha - 25\beta + 24\gamma)$ b) $6(26\alpha - \gamma - \frac{100A}{P})$

c) $3(\alpha - \frac{4A}{P})$ d) $6(\beta - \frac{4A}{P})$ e) $6(\frac{\beta + \gamma - \alpha}{2} - \frac{2A}{P})$

2. Each of the expressions $\frac{1}{2}(\alpha + \beta - \gamma) - \frac{2A}{P}$ represents the product xz
3. Each of the expressions $\frac{1}{2}(\beta + \gamma - \alpha), \beta - \frac{2A}{P}$ represents the product $x(x+z)$
4. $\frac{1}{2}(\beta + \gamma - \alpha) + \frac{2A}{P}$ can be expressed as the difference of two squares

In the above A, P represent Area and Perimeter of T respectively.

4. Conclusion

In this paper, we have presented different patterns of integer solutions to the ternary quadratic equation $x^2 + 9y^2 = 50z^2$ representing the cone. As the ternary quadratic Diophantine equations are rich in variety, one may attempt to find integer solutions to other choices of equations along with suitable properties.

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