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On the ternary quadratic diophantine equation $8(x^2 + y^2) + 8(x + y) + 4 = 25z^2$

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Abstract
The ternary quadratic diophantine equation represented by
$8(x^2 + y^2) + 8(x + y) + 4 = 25z^2$ is analyzed for its non-zero distinct integer solutions. A few interesting properties between the solutions and special figurate numbers are obtained.

Keywords: Ternary quadratic, integer solutions, figurate numbers.

2010 Mathematics subject classification: 11D09.

NOTATIONS USED

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$SO_n = n(2n^2 - 1)$$

$$P_n^5 = \frac{n^2(n+1)}{2}$$

$$Pr_n = n(n+1)$$

1. Introduction

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-21] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $8(x^2 + y^2) + 8(x + y) + 4 = 25z^2$ representing non- homogeneous cubic equation with three unknowns for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

2. Method of analysis

The ternary quadratic Diophantine equation to be solved is

$$8(x^2 + y^2) + 8(x + y) + 4 = 25z^2 \quad (1)$$

Introducing the linear transformations

$$x = u + v;\ y = u - v \quad (2)$$

In (1) , it is written as

$$(4u + 2)^2 + 16v^2 = 25z^2 \quad (3)$$

Note that (3) is the form of well known Pythagorean equation. Using the standard solutions of the Pythagorean equation, the values of u, v and z are given by

$$u=\frac{\alpha\beta-1}{2}\quad ,v=\frac{\alpha^2-\beta^2}{4}\quad ,z=\frac{\alpha^2+\beta^2}{5} \tag{4}$$

where $\alpha>\beta>0$

As our interest is on finding integer solutions, it is possible to choose α, β so that u, v and z are integers. Substituting the values of u and v in (2), the integer values for x and y are obtained. Thus, one may obtain integer solutions for (1). A few examples are presented below in table 1.

Table 1: Examples

α	β	x	Y	z
3^n	3^{n-1}	$\frac{7*9^{n-1}-1}{2}$	$\frac{-(9^{n-1}+1)}{2}$	$2*9^{n-1}$
5^{n+1}	5^n	$\frac{(17*5^{2n}-1)}{2}$	$\frac{-(7*5^{2n}+1)}{2}$	$26*5^{2n-1}$
$4n+3$	$2n-1$	$7n^2+8n$	n^2-6n-4	$4n^2+4n+2$
$4n+5$	$2n+5$	$7n^2+20n+12$	$n^2+10n+12$	$4n^2+12n+10$

It is worth to note that another solution pattern for (1) is obtained by solving (3) through the method of factorization which is illustrated below.

Write 25 as

$$25=(4+3i)(4-3i) \tag{5}$$

Assume $z=z(a,b)=a^2+b^2$ (6)

where a, b are non-zero distinct integers.

Using (5) and (6) in (3), it is written in the factorizable form as

$$(4u+2+i4v)(4u+2-i4v)=(4+3i)(4-3i)(a+ib)^2(a-ib)^2$$

which is equivalent to the system of equations

$$(4u+2+i4v)=(4+3i)(a+ib)^2 \tag{7}$$

$$(4u+2+i4v)=(4-3i)(a-ib)^2 \tag{8}$$

Equating the real and imaginary parts in either (7) or (8), we have

$$u=u(a,b)=a^2-b^2-\left(\frac{3ab+1}{2}\right) \tag{9}$$

$$v=v(a,b)=\frac{3}{4}(a^2-b^2)+2ab \tag{10}$$

It is seen that u and v are integers when a and b are both odd. Taking a = 2m+1, b = 2n+1 in (9), (10) and (6) we have,

$$u=u(m,n)=4m^2-4n^2+m-7n-6mn-2$$
$$v=v(m,n)=3m^2-3n^2+7m+n+8mn+2$$
$$z=z(m,n)=4m^2+4n^2+4m+4n+2 \tag{11}$$

In view of (2),

$$x=x(m,n)=7m^2-7n^2+8m-6n+2mn \tag{12}$$

$$y=y(m,n)=m^2-n^2-6m-8n-14mn-4 \tag{13}$$

Thus(12), (13) and (11) represent the integer solutions to (1). A few interesting properties between the solutions, special polygonal and pyramidal numbers are as follows.

- $x(n,1)-y(n,1)-6Pr_n\equiv 0 \pmod{24}$
- $4y(1,SO_n)+z(1,SO_n)+84SO_n+26=0$
- $2z(1,n)-x(1,n)-y(1,n)-t_{34,n}\equiv 14 \pmod{49}$
- $x(m,n)-7y(m,n)-28\equiv 0 \pmod{50}$
- $x(n,t_{3,n})-7y(n,t_{3,n})-50t_{3,n}-100P_n^5\equiv 28 \pmod{50}$
- $x(Pr_n,1)-7y(Pr_n,1)-150\,Pr_n-78=0$
- $2z(m,1)-x(m,1)-y(m,1)\equiv 0 \pmod{2}$
- $2z(1,n)-x(1,n)-y(1,n)-32t_{3,n}\equiv 14 \pmod{18}$
- $x((n+1)(n+2),n)-7y((n+1)(n+2),n)-100t_{3,n+1}-600P_n^3\equiv 28 \pmod{50}$

In addition (2), the above choices of solutions we have an another choice for solution to (1), which is presented as follows.

Replacingz by 2w in (3). Its written as

$$(2u+1)^2 = 25w^2 - 4v^2$$

which is satisfied by

$$v = u^2 + u$$

$$z = \frac{4u^2 + 4u + 2}{5} \quad (14)$$

Since our interest is to find integer solutions, Choose u in (13) suitably. So that, z is an integer.

After performing numerical evaluations, it is seen that there are 2 choices for u, namely, and thus there are 2 sets of solutions to (1), which are exhibited in the table below.

Table 2. Solutions

u	v	X	Y	z
5n-2	$25n^2 - 15n + 2$	$25n^2 - 10n$	$-25n^2 + 20n - 4$	$20n^2 - 12n + 2$
5n-4	$25n^2 - 35n + 12$	$25n^2 - 30n + 8$	$-25n^2 + 40n - 16$	$20n^2 - 28n + 10$

3. Conclusion

In this paper, we have obtained infinetly many non-zero distinct integer solutions to the ternary quadratic diophantine equation represented by

$$8(x^2 + y^2) + 8(x + y) + 4 = 25z^2$$

As quadratic equations are rich in variety, one may search for their choices of quadratic equation with variables greater than or equal to 3 and determine their properties through special numbers.

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integer solutions for the hyperbola represented by the negative pell equation $y^2 = 105x^2 - 5$, As the binary quadratic Diophantine equation are rich in variety , one may search for the other choices of negative pell equations and determine their integer solutions along with suitable properties.

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