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Observations on the Quintic Equation with five unknowns $x^4 - y^4 = 37(z^2 - w^2)p^3$

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Abstract
The quintic Diophantine equation with five unknowns $x^4 - y^4 = 37(z^2 - w^2)p^3$ is analysed for its infinitely many non-zero distinct integral solutions. A few interesting properties among the values of x,y,z,w,p and special numbers namely, polygonal, pyramidal, Centered pyramidal, Star, Stella octangular, and Jacobsthal numbers are presented.

Keywords: Quintic equation with five unknowns, integral solutions
MSC 2000 Mathematics Subject Classification: 11D41

Notations Used

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$ - Polygonal number of rank n with size m

$P_n^m = \frac{1}{6}[n(n+1)][(m-2)n + (5-m)]$ - Pyramidal number of rank n with size m

$SO_n = n(2n^2 - 1)$ - Stella Octangular number of rank n

$CP_n^m = m\left[\frac{(n-1)n(n+1)}{6}\right] + n$ - Centered Pyramidal number of rank n with size m

$Pr_n = n(n+1)$ -Pronic number of rank n

$S_n = 6n(n-1) + 1$ - Star number of rank n

$HD_n = \frac{1}{2}(9n^2 - 7n)$ – Hendecagonal number of rank n

$J_n = \frac{1}{3}(2^n - (-1)^n)$ - Jacobsthal number of rank n

1. Introduction

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular quintic equations, homogeneous or non-homogeneous, have aroused the interest of numerous mathematicians since antiquity [1-3]. For illustration, one may refer [4-5] for Quintic equation with three unknowns and [6-12] for Quintic equation with five unknowns. This paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous Quintic equation with five unknowns given by $x^4-y^4=37(z^2-w^2)p^3$. A few relations between the solutions and the special numbers are presented.

Method of Analysis

The Diophantine equation representing the quintic equation with five unknowns under consideration is

$$x^4-y^4=37(z^2-w^2)p^3 \quad (1)$$

Introducing the linear transformations

$$X=u+v,\ y=u-v,\ z=2u+v,\ w=2u-v \tag{2}$$

In (1), we get $u^2+v^2=37p^3$ (3)

Now, we solve (3) through different methods and thus obtain different patterns of solutions to (1).

Pattern I

Assume $p = p(a,b) = a^2+b^2$ (4)

where a and b are non-zero distinct integers.

Write 37 as $(6+i)(6-i)$ (5)

Using (4) and (5) in (3) and applying the method of factorization, it is written as the system of double equations as

$$u+iv=(6+i)(a+ib)^3$$
$$u-iv=(6-i)(a-ib)^3$$

Equating the real and imaginary parts in either of the above equations we have

$$u=u(a,b)=6a^3-18ab^2-3a^2b+b^3$$
$$v=v(a,b)=a^3-3ab^2+18a^2b-6b^3$$

In view of (2), the corresponding solutions of (1) are given by

$$x=x(a,b)=7a^3-21ab^2+15a^2b-5b^3$$
$$y=y(a,b)=5a^3-15ab^2-21a^2b+7b^3$$
$$z=z(a,b)=13a^3-39ab^2+12a^2b-4b^3$$
$$w=w(a,b)=11a^3-33ab^2-24a^2b+8b^3$$
$$p=p(a,b)=a^2+b^2$$

Properties:

A few interesting properties observed are as follows:

1. $4\left(x^2(a,b)-y^2(a,b)\right)^2=\left(z^2(a,b)-w^2(a,b)\right)^2$
2. $x(a,a+1)+w(a,a+1)+p(a,a+1)+18CP_a^{14}+53t_{6,a}$ $\equiv 1(mod120)$
3. $x(a,a)+y(a,a)+z(a,a)+12CP_a^{23}\equiv 0(mod34)$
4. $z(a,a)+w(a,a)+p(a,a)+28SO_a-2Pr_a\equiv 0(mod30)$
5. $y(a,1)+z(a,1)+w(a,1)-6CP_a^{29}+S_a+6HD_a\equiv 12(mod91)$
6. $x(a,1)+z(a,1)+p(a,1)-10SO_a-4t_{16,a}\equiv -8(mod26)$

Pattern II

Write 37 as (1+6i)(1-6i) (6)

Following the procedure similar to pattern I, the corresponding non-zero distinct integral solutions of (1) are found to be

$$x=x(a,b)=7a^3-21ab^2-15a^2b+5b^3$$
$$y=y(a,b)=-5a^3+15ab^2-21a^2b+7b^3$$
$$z=z(a,b)=8a^3-24ab^2-33a^2b+11b^3$$
$$w=w(a,b)=-4a^3+12ab^2-39a^2b+13b^3$$
$$p=p(a,b)=a^2+b^2$$

Properties

1. $x(a,a)+y(a,a)+z(a,a)+36CP_a^{11}$ $\equiv 0(mod30)$
2. $x(a,1)+p(a,1)+y(a,1)-3CP_a^{4}+5t_{16,a}$ $\equiv 13(mod37)$
3. $p(2^n,1)=3J_{2n}+2$
4. $w(1,b)-x(1,b)-y(1,b)-2t_{20,b}-13b+6$ is a cubic integer
5. $11y(1,b)-7z(1,b)-9p(1,b)+120$ is a perfect square

Pattern III

Rewrite (3) as $u^2+v^2=37p^3*1$ (7)

Write 1 as $1=\frac{1}{25}(3+4i)(3-4i)$ (8)

Applying a similar analysis presented as in pattern I and performing a few calculations, the corresponding non-zero distinct integral solutions of (1) are given by

$$x=x(a,b)=1025a^3-3075ab^2-975a^2b+325b^3$$
$$y=y(a,b)=-325a^3+975ab^2-3075a^2b+1025b^3$$
$$z=z(a,b)=1375a^3-4125ab^2-3000a^2b+1000b^3$$
$$w=w(a,b)=25a^3-75ab^2-5100a^2b+1700b^3$$
$$p=p(a,b)=25a^2+25b^2$$

Properties

1. $\frac{1}{5}[z(2b,b)+w(2b,b)+p(2b,b)+26900CP_b^{6}]$ is a perfect square
2. $x(a,1)+z(a,1)-w(a,1)-750CP_a^{19}-250t_{11,a}$ $\equiv -375(mod4625)$
3. $x(a,a)+y(a,a)-z(a,a)-300CP_a^{13}\equiv 0(mod350)$
4. $y(a,2a)+13w(a,2a)-11100CP_a^{25}\equiv 0(mod35150)$

Note

It is worth to mention here that instead of (8), 1 may be written in general as

$$1=\frac{(m^2-n^2+i2mn)(m^2-n^2-i2mn)}{(m^2+n^2)^2}$$

Or

$$1=\frac{(2mn+i(m^2-n^2))(2mn-i(m^2-n^2))}{(m^2+n^2)^2}$$

The repetition of the above process leads to other choices of integer solutions to (1).

Pattern IV

Instead of (8), write 1 as $1 = \frac{(1+i)^{2n}(1-i)^{2n}}{2^{2n}}$

For this choice, the corresponding integer solutions are found to be

$$x = x(a,b,n) = (a^3 - 3ab^2 + 3a^2b - b^3)\left(6cos\frac{n\pi}{2} - sin\frac{n\pi}{2}\right) + (a^3 - 3ab^2 - 3a^2b + b^3)\left(6sin\frac{n\pi}{2} + cos\frac{n\pi}{2}\right)$$

$$y = y(a,b,n) = (a^3 - 3ab^2 - 3a^2b + b^3)\left(6cos\frac{n\pi}{2} - sin\frac{n\pi}{2}\right) - (a^3 - 3ab^2 + 3a^2b - b^3)\left(6sin\frac{n\pi}{2} + cos\frac{n\pi}{2}\right)$$

$$z = z(a,b,n) = (2a^3 - 6ab^2 + 3a^2b - b^3)\left(6cos\frac{n\pi}{2} - sin\frac{n\pi}{2}\right) + (a^3 - 3ab^2 - 6a^2b + 2b^3)\left(6sin\frac{n\pi}{2} + cos\frac{n\pi}{2}\right)$$

$$w = w(a,b,n) = (2a^3 - 6ab^2 - 3a^2b + b^3)\left(6cos\frac{n\pi}{2} - sin\frac{n\pi}{2}\right) - (a^3 - 3ab^2 + 6a^2b - 2b^3)\left(6sin\frac{n\pi}{2} + cos\frac{n\pi}{2}\right)$$

$$p = p(a,b) = a^2 + b^2$$

Properties

1. $w(b-1,b,3) + 2z(b-1,b,3) - 37CP_b^{12} - 74t_{8,n} = 0$
2. $x(a,2a,3) + y(a,2a,3) + 12CP_a^{23} \equiv 0(mod34)$
3. $x(a,3a,2) - 2y(a,3a,2) + p(a,3a) - 99CP_a^{16} - 2t_{12,a} \equiv 0(mod173)$
4. $2y(2b,b,2) - z(2b,b,2) + 9CP_b^{8} + 3b$ is a cubic integer

Conclusion

In this paper, we have obtained different patterns of non-zero distinct integer solutions to the quintic equation with five unknowns given by $x^4 - y^4 = 37(z^2 - w^2)p^3$. As quintic equations are rich in variety, one may search for integer solutions to quintic equations with variables ≥ 5 and determine their corresponding properties involving polygonal, pyramidal and other special number patterns.

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