International Journal of Applied Research 2015; 1(3): 86-87

ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 3.4
IJAR 2015; 1(3): 86-87
www.allresearchjournal.com
Received: 22-01-2015
Accepted: 17-02-2015

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Observations on the negative Pellian $y^2 = 86x^2 - 5$

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Abstract
The binary quadratic equation $y^2 = 86x^2 - 5$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special Pythagorean triangle is formed.

Keywords: Binary quadratic equation, Integral solutions.
MSC subject classification: 11D09.

1. Introduction
The binary quadratic Diophantine equations (both homogeneous and non homogeneous) are rich in variety..In [1-9] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions of an another interesting binary quadratic equation given by $y^2 = 86x^2 - 5$. The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

Method of Analysis
Consider the binary quadratic equation

$$y^2 = 86x^2 - 5 \tag{1}$$

with the least positive integer solutions $x_0=1, y_0=9$

To obtain the other solutions of equation (1), consider the Pellian equation

$$y^2 = 86x^2 + 1$$

Whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is given by $\tilde{x}_n = \frac{g}{2\sqrt{86}}$, $\tilde{y}_n = \frac{f}{2}$ in which

$$f = (10405 + 1122\sqrt{86})^{n+1} + (10405 - 1122\sqrt{86})^{n+1}$$
$$g = (10405 + 1122\sqrt{86})^{n+1} - (10405 - 1122\sqrt{86})^{n+1}$$

Where n=-1, 0, 1, 2…………………………

Applying Brahmagupta lemma between the solutions of (x_0,y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the general solutions of equation (1) are found to be

$$x_{n+1} = \frac{f}{2} + \frac{9\sqrt{86}g}{172} \tag{2}$$

$$y_{n+1} = \frac{9f}{2} + \frac{\sqrt{86}g}{2} \quad (3)$$

Thus (2) and (3) represent non-zero distinct integer solutions of (1) which represents a hyperbola.
The recurrence relations satisfied by the values of x and y are respectively

$$x_{n+3} - 20810x_{n+2} + x_{n+1} = 0$$
$$y_{n+3} - 20810y_{n+2} + y_{n+1} = 0$$

A few numerical examples are presented in the table below:

n	x_{n+1}	y_{n+1}
-1	1	9
0	20503	190137
1	426667429	3956750961
2	8878949176987	82339987308273

A few interesting properties are given below:

(i) $y_{n+1} \equiv 0 \pmod 3$

(ii) x_{n+1} and y_{n+1} are always odd

(iii) $x_{n+2} = 10405x_{n+1} + 1122y_{n+1}$

(iv) $860x_{n+3} = x_{n+1}(172c_1 - 1548c_2) + y_{n+1}(172c_2 - 18c_1)$

Where c_1= 1763258*10405 +190137*86*1122
c_2= 1763258*1122 +190137*10405

(v) $y_{n+2} = 96492x_{n+1} + 10405y_{n+1}$

(vi) $10y_{n+3} = x_{n+1}(172D_1 - 1548D_2) + y_{n+1}(172D_2 - 18D_1)$

Where D_1= 190137*10405 +20503*86*1122
D_2=190137*1122 +20503*10405

(vii) $30(172x_{2n+2} - 18y_{2n+2} + 10)$ is a nasty number

(viii) $25(172x_{3n+3} - 18y_{3n+3} + 15(172x_{n+1} - 18y_{n+1}))$ is a cubic integer

(ix) Define $X = 172x_{n+1} - 18y_{n+1}$ and $Y = 172y_{n+1} - 1548x_{n+1}$ then the pair (X,Y) satisfies the hyperbola $Y^2 = 86X^2 - 8600$

Remarkable Observations

Let m(=(x+y)), n(=x) be any two non-zero distinct positive integers. Note that m>n. Treat m,n as the generators of the Pythagorean triangle T (α,β,γ) where $\alpha=2mn$, $\beta=m^2-n^2$, $\gamma=m^2+n^2$ Let A,P represent the area and perimeter of the Pythagorean triangle T. Then the following relations are observed:

a) α+42γ-43β-5=0

b) $44\beta-43\gamma-\frac{4A}{P}+5=0$

c) $44\alpha-\gamma-\frac{172A}{P}-5=0$

Conclusion

In this paper, we have made an attempt to obtain a complete set of non-trivial distinct solutions for the non-homogeneous binary quadratic equation .To conclude, one may search for other choices of solutions to the considered binary equation and further, quadratic equations with multi-variables

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