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OBSERVATIONS ON THE HYPERPOLA $Y^2 = 35X^2+1$

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Abstract
The binary quadratic equation $y^2 = 35x^2 + 1$ is analysed for its non-zero distinct integer solutions. A few interesting relations among the solutions are presented. Some interesting relations connecting the solutions and polygonal numbers are presented. Employing the integral solutions of the equation under consideration, a special Pythagorean triangle is obtained.

Keywords: Binary quadratic, Hyperbola, Integral solutions, Pell equation. 2010 Mathematics subject classification: 11D09game

1. Introduction
The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer, has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-5]. In [6] infinitely many Pythagorean triangle in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2= 3x^2 + 1$. In [7], a special pythogorean triangle is obtained by employing the integral solutions of $y^2=10x^2+1$. In [8], different patterns of infinitely many pythagorean triangle are obtained by employing the non-trivial solutions of $y^2 = 5x^2 + 1$. In this context one may also refer [9-17]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 35x^2 + 1$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under the consideration, a special Pythagorean triangle is obtained.

Notations
$t_{3,n}$ - Triangular number of rank n.
$Ct_{8,n}$ - Centered octagonal number of rank n

2. Method of Analysis
The binary quadratic equation representing hyperbola under consideration is

$$y^2= 35x^2 + 1 \qquad (1)$$

whose general solution (x_n, y_n) is given by
$x_n = g/2\sqrt{35}$ and $y_n = f/2$

where
$f = (6+\sqrt{35})^{n+1} + (6-\sqrt{35})^{n+1}$
$g = (6+\sqrt{35})^{n+1} - (6-\sqrt{35})^{n+1}$,
n =0,1,2,3,……

The recurrence relations satisfied by x and y are given by
$y_{n+2} - 12y_{n+1} + y_n = 0,\ y_0 = 6,\ y_1 = 71$
$x_{n+2} - 12x_{n+1} + x_n = 0,\ x_0 = 1,\ x_1 = 12$

Some numerical examples of x and y satisfying (1) are given in the following table

n	x_n	y_n
0	1	6
1	12	71
2	143	846
3	1704	10081
4	20305	120126
5	241956	1431431

From the above table, we observe some interesting properties that are presented below:

1. x_n is alternatively odd and even.
2. y_n is alternatively even and odd.
3. $y_{2n} \equiv 0\ (mod\ 6)$.
4. $x_{2n+1} \equiv 0$ (mod 12).

A few interesting properties between the solutions are given below:

1. $2y_{2n+1} + 2$ is a perfect square.
2. $6(2y_{2n+1} + 2)$ is a nasty number.
3. $2y_{3n+2} + 6y_n$ is a cubical integer.
4. $y_{n+1} = 6y_n + 35x_n$
5. $y_{n+2} = 71y_n + 420x_n$
6. $x_{n+1} = 6x_n + y_n$
7. $x_{n+2} = 71x_n + 12y_n$
8. $6y_{n+2} - 71y_{n+1} = y_n$
9. $12y_{n+1} - y_{n+2} = y_n$
10. $70x_ny_{n+1} - y_ny_{n+2} = 2450\ x_n^2 - 71y_n^2$
11. $71y_ny_{n+1} - 35x_ny_{n+2} = 426y_n^2 - 14700x_n^2$
12. $6x_{n+1} - y_{n+1} = x_n$
13. $6y_{n+1} - 35\ x_{n+1} = y_n$
14. $x_ny_{n+1} - y_nx_{n+1} = 35x_n^2 - y_n^2$
15. $y_ny_{n+1} - 35x_nx_{n+1} = 6y_n^2 - 210x_n^2$
16. $71y_{n+1} - 35x_{n+2} = 6y_n$
17. $x_{n+2} - 2y_{n+1} = x_n$
18. $71x_ny_{n+1} - 6y_nx_{n+2} = 2485x_n^2 - 72y_n^2$
19. $12y_ny_{n+1} - 35x_nx_{n+2} = 72y_n^2 - 2485x_n^2$
20. $71x_{n+1} - y_{n+2} = 6x_n$
21. $y_{n+2} - 70x_{n+1} = y_n$
22. $6x_ny_{n+2} - 71\ y_nx_{n+1} = 2520x_n^2 - 71y_n^2$
23. $y_ny_{n+2} - 420x_nx_{n+1} = 71y_n^2 - 2520x_n^2$
24. $71x_{n+2} - 12y_{n+2} = x_n$
25. $71y_{n+2} - 420x_{n+2} = y_n$
26. $x_ny_{n+2} - y_nx_{n+2} = 420x_n^2 - 12y_n^2$
27. $y_ny_{n+2} - 35\ x_nx_{n+2} = 71y_n^2 - 2485x_n^2$
28. $6x_{n+2} - 71x_{n+1} = y_n$
29. $12x_{n+1} - x_{n+2} = x_n$
30. $2y_nx_{n+1} - x_nx_{n+2} = 2y_n^2 - 71x_n^2$
31. $71x_nx_{n+1} - y_nx_{n+2} = 426x_n^2 - 12y_n^2$

Remarkable Observations

1. Let N be any non-zero positive integer such that

$N = (y_{2n+1} - 1)/2$

Then it is observed that

(a) $35x_{2n+1}^2 = 8t_{3,N} = ct_{8,N} - 1$

(b) $y_{2n+1}^2 = 8t_{3,N} + 1 = ct_{8,N}$

2. Let M be any non-zero positive integer such that

$M = x_{2n-1}/2$

It is noted that

$x_{2n}^2 = 8t_{3,M} + 1 = ct_{8,M}$

3. Let p and q be two non-zero distinct positive integers such that

$p = x_n + 2y_n$ and $q = x_n$

Note that $p>q>0$.

Treat p,q as the generators of the pythogorean triangle $T(\alpha,\beta,\gamma)$.
where

$\alpha = 2pq,\ \beta = p^2 - q^2$ and $\gamma = p^2 + q^2$

Let A, P represent the area and perimeter of $T(\alpha,\beta,\gamma)$.
Then the following interesting relations are observed:

(a) $\alpha - 70\beta + 69\gamma + 4 = 0$

(b) $71\alpha - \gamma + 4 = 280(A/P)$

(c) $71\beta - 70\gamma - 4 = 4(A/P)$

3. Conclusion

To conclude, one may search for other choices of hyperbola for patterns of solutions and their corresponding properties.

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