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S. Vidhyalakshmi
Professor, Department of mathematics, sigc, trichy-620002, India

T. Geetha
Lecturer, Department of mathematics, sigc, trichy-620 002, India

R. Sridevi
PG Scholar, Department of mathematics, sigc, trichy-620002, India

Correspondence:
R. Sridevi
PG Scholar, Department of mathematics, sigc, trichy-620002, India

On ternary quadratic diophantine equation $2x^2-7y^2=25z^2$

S. Vidhyalakshmi, T. Geetha, R. Sridevi

Abstract

The ternary quadratic Diophantine equation representing cone given by $2x^2 - 7y^2 = 25z^2$ is analyzed for its non-zero distinct integer points. A few interesting relations between the solutions and special figurate numbers are obtained.

Notations

Polygonal number of rank 'n' with size m:

$$t_{m,n} = n[1 + \frac{(n-1)(n-2)}{2}]$$

Pronic number of rank 'n':

$$pr_n = n(n+1)$$

Octahedral number of rank n:

$$OH_n = \frac{1}{3}n(2n^2+1)$$

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1. Introduction

The quadratic Diophantine equations with three unknowns offer an unlimited field for research because of their variety [1-3]. For an extensive review of various problems on ternary quadratic Diophantine equations representing specific 3 dimensional surfaces, one may refer [4-14].

In this communication, we present general formulas for obtaining sequences of non-zero integer solutions to the ternary quadratic Diophantine equation $2x^2 - 7y^2 = 25z^2$. Also, a few interesting relations among the solutions are presented.

2. Method of Analysis

The ternary quadratic equation to be solved is $2x^2 - 7y^2 = 25z^2$ (1)

To start with, it is seen that (1) is satisfied by the following triples of integers (x,y,z):(774,369,99),(-360,45,99),(198,63,45),(180,45,45),(230,85,47),(176,-31,47),(428, 193, 65), (-230,-5, 65).

However, we have other patterns of solutions which are illustrated as follows:

Method 1

Introducing the linear transformations

$$x = X + 7T, \; y = X + 2T \tag{2}$$

in (1), it is written as

$$14T^2 = 5z^2 + X^2 \quad (3)$$

Assume

$$T = a^2 + 5b^2 \quad (4)$$

write 14 as

$$14 = (3 + i\sqrt{5})(3 - i\sqrt{5}) \quad (5)$$

Substituting (4), (5) in (3) and write it in the factorizable form as

$$(3 + i\sqrt{5})(3 - i\sqrt{5})(a + i\sqrt{5}b)^2 (a - i\sqrt{5}b)^2$$

$$= (X + i\sqrt{5}z)(X - i\sqrt{5}z)$$

Equating real and imaginary parts, the values of z and X are

$$X = 3a^2 - 15b^2 - 10ab$$

$$z = a^2 - 5b^2 + 6ab \quad (6)$$

Then the corresponding integer solution of (1) are given by

$$x(a,b) = 10a^2 + 20b^2 - 10ab$$

$$y(a,b) = 5a^2 - 5b^2 - 10ab$$

$$z(a,b) = a^2 - 5b^2 + 6ab$$

Properties

1) $15[x(a,a) - 2y(a,a)]$ is a nasty number

2) $x(a,a) - 2y(a,a) - 40t_{4,a} = 0$

3) $3\{z(a,a) - y(a,a)\}$ is a perfect square

4) $4\{z(a,a^2) - y(a,a^2)\}$ + perfect square is a cubical integer

Also, instead of (2) one may consider the linear transformation as

$$x = X - 7T, \; y = X - 2T \quad (7)$$

For this choice, the corresponding integer solutions to (1) are given by

$$x(a,b) = -4a^2 - 50b^2 - 10ab$$

$$y(a,b) = a^2 - 25b^2 - 10ab$$

$$z(a,b) = a^2 - 5b^2 + 6ab$$

Method 2

Write 14 as

$$14 = \frac{(11 + i\sqrt{5})(11 - i\sqrt{5})}{9} \quad (8)$$

Use (8),(4) in (3) and write it in the factorization form as,

$$\frac{(11 + i\sqrt{5})(11 - i\sqrt{5})}{9}(a + i\sqrt{5}b)^2 (a - i\sqrt{5}b)^2$$

$$= (X + i\sqrt{5}z)(X - i\sqrt{5}z)$$

Equating real and imaginary parts, the values of X, z are

$$X = \frac{1}{3}[11a^2 - 55b^2 - 10ab]$$

$$z = \frac{1}{3}[a^2 - 5b^2 + 22ab] \quad (9)$$

For X and z to be integers, the values of a and b should be multiples of 3. Therefore, replacing a by 3A, b by 3B in (4), (9), we have

$$X = 33A^2 - 165B^2 - 30AB$$

$$z = 3A^2 - 15B^2 + 66AB$$

$$T = 9A^2 + 45B^2$$

In view of (2), it is seen that

$$x(A,B) = 96A^2 + 150B^2 - 30AB$$

$$y(A,B) = 51A^2 - 75B^2 - 30AB$$

$$z(A,B) = 3A^2 - 15B^2 + 66AB$$

Properties

1) $5[x(a,a) - y(a,a)]$ is a nasty number

2) $x(a,a) + 2y(a,a) - 108t_{4,a} = 0$

3) $5z(a,a) - y(a,a)$ is a perfect square

4) $x(a,a+1) + 10z(a,a+1) - 126t_{4,a} - 630pr_a = 0$

Also, using (7), the corresponding integer solutions to (1) are given by

$$x(A,B) = -30A^2 - 480B^2 - 30AB$$

$$y(A,B) = 15A^2 - 225B^2 - 30AB$$

$$z(A,B) = 3A^2 - 15B^2 + 66AB$$

Method 3

Write 14 as

$$14 = \frac{(9 + i11\sqrt{5})(9 - i11\sqrt{5})}{49} \quad (10)$$

Use (10), (4) in (3) and write it in the factorization form as

$$\frac{(9 + i11\sqrt{5})(9 - i11\sqrt{5})}{49}(a + i\sqrt{5}b)^2 (a - i\sqrt{5}b)^2$$

$$= (X + i\sqrt{5}z)(X - i\sqrt{5}z)$$

Equating real and imaginary parts, the values of X and z are

$$X = \frac{1}{7}[9a^2 - 45b^2 - 110ab]$$

$$z = \frac{1}{7}[11a^2 - 55b^2 + 18ab] \quad (11)$$

For X and z to be integers, the values of a and b should be multiples of 3. Therefore, replacing a by 7A, b by 7B in (4), (11), we have

$$X = 63A^2 - 315B^2 - 770AB$$

$$z = 77A^2 - 385B^2 + 126AB$$

$$T = 49A^2 + 245B^2$$

In view of (2), it is seen that

$$x(A,B) = 406A^2 + 1400B^2 - 770AB$$

$$y(A,B) = 161A^2 + 175B^2 - 770AB$$

$$z(A,B) = 77A^2 - 385B^2 + 126AB$$

Properties

1) $x(a,a)-y(a,a)-1470t_{4,a}=0$

2) $23[x(a,a)-8y(a,a)]$ is a perfect square

3) $x(a,2a^2+1)-8y(a,2a^2+1)+882t_{4,a}-5390OH_a=0$

4) $x(a,a+1)-8y(a,a+1)+882t_{4,a}-5390pr_a=0$

Also using (*), the corresponding integer solutions to (1) are given by

$$x(A,B)=280A^2-2030B^2-770AB$$
$$y(A,B)=-35A^2-805B^2-770AB$$
$$z(A,B)=77A^2-385B^2+126AB$$

Method 4

Write (3) as

$$X^2=14T^2-5z^2 \quad (12)$$

Introducing the linear transformations

$$T=\alpha+5\beta,\ z=\alpha+14\beta \quad (13)$$

In (12), it becomes

$$X^2=9\alpha^2-70*9\beta^2$$

Replacing X by 3U in the above equation, it is written as

$$\alpha^2=70\beta^2+U^2 \quad (14)$$

Assume

$$\alpha=a^2+70b^2 \quad (15)$$

Using (15) in (14) and expressing in the factorizable form, we have

$$(a+i\sqrt{70}b)^2(a-i\sqrt{70}b)^2$$
$$=(U+i\sqrt{70}\beta)(U-i\sqrt{70}\beta)$$

Equating real and imaginary parts, we have

$$U=a^2-70b^2,\beta=2ab \quad (16)$$

Note that

$$X=3U=3a^2-210b^2 \quad (17)$$

Using (15) and (16) in (13), we get

$$T=a^2+70b^2+10ab$$
$$z=a^2+70b^2+28ab \quad (18)$$

Put (17) & (18) in (2), we get the integer solutions of (1) to be

$$x(a,b)=10a^2+280b^2+70ab$$
$$y(a,b)=5a^2-70b^2+20ab$$
$$z(a,b)=a^2+70b^2+28ab \quad (19)$$

Also using (7), the corresponding integer solutions to (1) are given by

$$x(a,b)=-4a^2-700b^2-490ab$$
$$y(a,b)=a^2-350b^2-20ab$$
$$z(a,b)=a^2+70b^2+28ab$$

Note: It is worth mentioning have that, instead of (15), one may also consider the transformations

$$T=\alpha-5\beta\ ,\ z=\alpha-14\beta$$

Following the analysis presented above, a different choice of integer solutions to (1) is obtained.

3. Conclusion

In this paper, we have obtained infinitely many non-zero distinct integer solutions to the ternary quadratic Diophantine equation represented by

$$2x^2-7y^2=25z^2$$

As quadratic equations are rich in variety, one may search for their choices of quadratic equation with variables greater than or equal to 3 and determine their properties through special numbers.

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