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On the non-homogeneous biquadratic equation with four unknowns (x+y+z)xyz= w^2+2z^2

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Abstract

The binary quadratic equation $(x+y+z)xyz = w^2 + 2z^2$ is studied for its non-trivial integral solutions. The recurrence relations satisfied by the solutions x and y are given. A few interesting properties among the solutions are presented.

Keywords: Non-Homogeneous biquadratic equation, Integral solutions. MSC subject classification: 11D25.

Introduction

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular biquadratic Diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous Mathematicians since antiquity [1-5]. In this context one may refer [6-20] for various problems on the biquadratic Diophantine equations with four variables. However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous equation with four unknowns given by $(x+y+z)xyz = w^2 + 2z^2$. A few relations among the solutions are presented.

2. Method of Analysis

The Diophantine equation under consideration is

$$(x+y+z)xyz = w^2 + 2z^2 \quad (1)$$

Introducing the linear transformations

$$x = u+v, y = u-v, z = 2u, w = 4uv, u \neq v \quad (2)$$

in (1) it simplifies to the Pellian equation

$$u^2 = 1 + 3v^2 \quad (3)$$

whose general solution (u_n, v_n) is given by

$$\left.\begin{aligned} u_n &= \frac{1}{2}\left[\left(2+\sqrt{3}\right)^{n+1} + \left(2-\sqrt{3}\right)^{n+1}\right] \\ v_n &= \frac{1}{2\sqrt{3}}\left[\left(2+\sqrt{3}\right)^{n+1} - \left(2-\sqrt{3}\right)^{n+1}\right] \end{aligned}\right\} \quad (4)$$

Taking advantage of (2) and (4) the of integral solutions of (1) can be written as

$$\left.\begin{aligned} x_n &= \frac{f}{2}+\frac{g}{2\sqrt{3}} \\ y_n &= \frac{f}{2}-\frac{g}{2\sqrt{3}} \\ z_n &= f \\ w_n &= \frac{fg}{\sqrt{3}} \end{aligned}\right\} \quad (5)$$

where

$$f=\left[\left(2+\sqrt{3}\right)^{n+1}+\left(2-\sqrt{3}\right)^{n+1}\right]$$

$$g=\left[\left(2+\sqrt{3}\right)^{n+1}-\left(2-\sqrt{3}\right)^{n+1}\right],$$

$$n=0,1,2,3.........$$

The above values of x_n, y_n, z_n and w_n satisfy respectively the following recurrence relations.

$$\left.\begin{aligned} x_{n+2}-4x_{n+2}+x_n &= 0 \\ y_{n+2}-4y_{n+2}+y_n &= 0 \\ z_{n+2}-4z_{n+2}+z_n &= 0 \\ w_{n+2}-14w_{n+2}+w_n &= 0 \end{aligned}\right\} \quad (6)$$

$$n=0,1,2....$$

A few numerical examples are given below

n	x	y	z	w
0	3	1	4	8
1	11	3	14	112
2	41	11	52	1560
3	153	41	194	21728
4	571	153	724	302632
5	2131	571	2702	4215120

Some relations satisfied by the solutions are as follows:

- ❖ The values of x, y are odd and that of z, w are even.
- ❖ $y_{n+1}=x_n$
- ❖ $w_n=x_n^2-y_n^2$
- ❖ $w_n-x_{2n+1}+y_{2n+1}=0$
- ❖ $w_n=(x_n-y_n)z_n$
- ❖ $x_n+y_n=z_n$
- ❖ $x_{2n+1}-y_{2n+1}=x_n^2-y_n^2$
- ❖ The following expressions are nasty numbers:
  - $6(x_{2n+1}+y_{2n+1}+2)$
  - $3(x_{2n+1}+y_{2n+1}+z_{2n+1}+4)$
- ❖ The following expressions are cubic integers
  - $x_{3n+2}+y_{3n+2}+3(x_n+y_n)$
  - $(x_{2n+1}+y_{2n+1}+2)(x_n+y_n)$
  - $2(x_{2n+1}+y_{2n+1}+z_{2n+1}+4)(x_n+y_n+z_n)$
  - $4(x_{3n+2}+y_{3n+2}+z_{3n+2}+3(x_n+y_n+z_n))$
- ❖ Each of the following expressions is a biquadratic integer
  - $(x_{4n+3}+y_{4n+3})+4(x_{2n+1}+y_{2n+1})+6$
  - $8[(x_{4n+3}+y_{4n+3}+z_{4n+3})+4(x_{2n+1}+y_{2n+1}+z_{2n+1})+12]$
  - $8[(x_{4n+3}+y_{4n+3}+z_{4n+3})+8(x_{2n+1}+y_{2n+1})+12]$
- ❖ $x_{3n+2}-y_{3n+2}=(x_n-y_n)(x_{2n+1}+y_{2n+1}+1)$
- ❖ $x_{3n+2}+y_{3n+2}+z_{3n+2}+6x_n+6y_n\equiv 0 \pmod 2$

Remarkable observations:

I: By considering suitable linear transformations between the solutions of (1), one may get integer solutions for the Parabola.

(a) Illustration1: It is to be noted that the Parabola

$$Y^2=X$$

is satisfied by the following two sets of values of X and Y

Set1:

$$Y=x_n+y_n$$
$$X=x_{2n+1}+y_{2n+1}+2$$

Set2:

$$Y=x_{2n+1}+y_{2n+1}+2$$
$$X=x_{4n+3}+y_{4n+3}+4x_{2n+1}+4y_{2n+1}+6$$

(b) Illustration2: It is to be noted that the Parabola

$$Y^2=2X$$

is satisfied by the following two sets of values of X and Y

Set1:

$$Y=x_n+y_n+z_n$$
$$X=x_{2n+1}+y_{2n+1}+z_{2n+1}+4$$

Set2:

$$Y=x_{2n+1}+y_{2n+1}+z_{2n+1}+4$$
$$X=(x_{4n+3}+y_{4n+3}+z_{4n+3})+8(x_{2n+1}+y_{2n+1})+12$$

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4. Conclusion

In this paper, infinitely many integer solutions are obtained for the non-homogeneous quartic equation $(x+y+z)xyz=w^2+2z^2$. As biquadratic equation are rich in variety, one may consider other forms of quadratic equations and search for the patterns of solutions with their corresponding properties.

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