ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 3.4
IJAR 2015; 1(5): 153-156
www.allresearchjournal.com
Received: 03-04-2015
Accepted: 04-04-2015

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Transport equations for turbulent kinetic energy in porous media

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Abstract
The macroscopic transport analysis for the incompressible fluid flow in the porous media based on the volume-average method for the heat transfer was given in the various researches. In the present paper there are the analysis and derivations of equations based on the concept of time-average. This gives a new concepts and method for the analysis of turbulent flow in porous media. The time-averaged transport equations play an important role on analyzing the transportation over the highly permeable media where the turbulent flow occurs in the fluid phase.

Keywords: Heat, Porous media, Turbulent flow, Transportation.

1. Introduction
The concept of macroscopic transportation for the incompressible fluid flow in the porous media was used by Vafai & Tien [10] in 1981 and Whitaker [12] in 1999, based on the volume-average method for the heat transfer by Hsu & Cheng [4] in 1990. The concept of space average in porous media is based on the assumption that although fluid velocities and pressure may be irregular at the pore scale, locally space-averaged measurements of these quantities vary smoothly [12]. Macroscopic equations are commonly derived by spatially averaging the microscopic ones over a Representative Elementary Volume (REV) of the porous media. A REV should be the smallest differential volume, which results in meaningful local average properties. It implies that the length scale of this volume must be sufficiently larger than the pore scale. Also, the dimensions of the system must be considerably larger than the REV's length scale for avoiding the non-homogeneities i.e.

$$p \ll D \ll L$$

where p is the pore scale or microscopic length scale, D is the macroscopic length scale and L is the megascale or scale of the system as represented by figure-1.

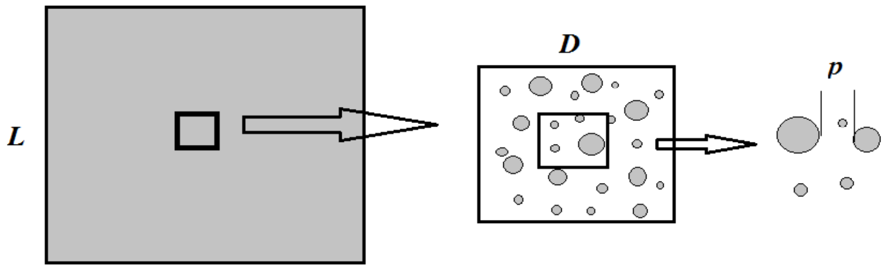

Fig 1: Identification of different length scales.

A schematic representation of a spherical REV consisting of a fixed solid phase saturated with a continuous fluid phase and is shown by the figure-2, here the solid phase is fixed, i.e., the solid phase does not change randomly if different ensembles are considered. The volume of the REV is constant i.e., independent of the space and its value is equal to the sum of the fluid and solid volumes inside the REV, i.e.

$$V=V_s+V_f$$

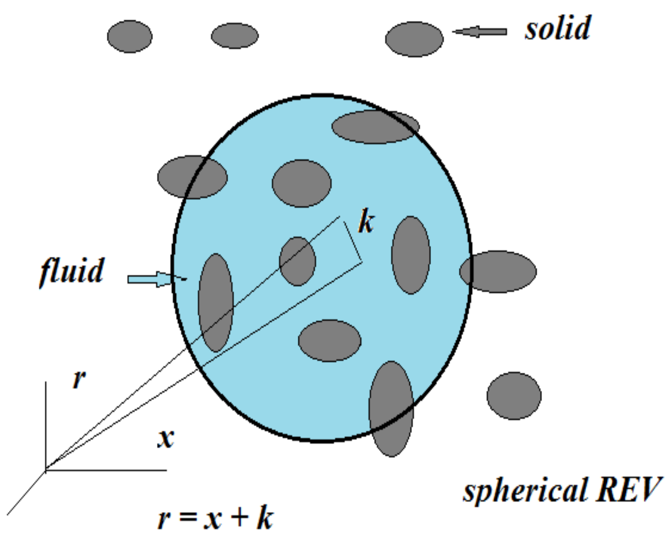

Fig 2: Spherical representative elementary volume (REV).

The spherical representative elementary volume is shown by figure-2. on taking the time fluctuations of the flow properties with spatial deviations, there are generally two methods for deriving and studying the macroscopic equations. The first method based on the time-average operator followed by the volume-averaging initially used by Kuwahara *et al.* [5] in 1998. The second method based on the concept of volume-averaging before time averaging that was used by Lee & Howell [7] in 1987, and the macroscopic transport equations established by these two methods are equivalent. This initial method for the flow variables has been extended to the nonbuoyant heat transfer for the porous media by considering the phenomenon of time variations and spatial deviations was taken by Rocamora & Lemos [8] in 2000. Later, the researches on the natural convection flow on the porous layer, double-diffusive convection for the turbulent flow and heat transfer in the porous media was given by de Lemos *et al.* [2] in 2004. The numerical based analysis for applications of double-decomposition theory to buoyant flow was also reviewed by de Lemos [1] in 2003.

2. Governing Equations

The macroscopic instantaneous transfer equations for the incompressible fluid flow having the constant properties are given as:

$$\nabla.\bar{v} = 0 \tag{1}$$

$$\rho\,\nabla.(\bar{v}.\bar{v}) = -\nabla P + \mu \nabla^2 \bar{v} + \rho\,\bar{g} \tag{2}$$

$$(\,\rho\, C_P)\,\nabla.(\bar{v}\,T) = \nabla.(\lambda\,\nabla\,T) \tag{3}$$

Where $\bar{v}$ is the velocity vector, P is the pressure, μ is the viscosity of the fluid, ρ is the density of the fluid, $\bar{g}$ is the acceleration vector due to gravity, C_P is the specific heat, T is the temperature and λ is the thermal conductivity of the fluid. The mass fraction distribution related to chemical species e is governed by the transport equation given as: $\bar{v}$

$$\nabla.(\rho\,\bar{v}\,m_e + \bar{J}_e) = \rho\,R_e \tag{4}$$

Where m_e is the mass fraction of component e, $\bar{v}$ is the mass-averaged velocity of the fluid mixture, so we have

$$\bar{v} = \sum_e m_e\,\bar{v}_e \tag{5}$$

Where $\bar{v}_e$ is the velocity of species e. The mass diffusion flux $\bar{J}_e$ is due to velocity slip of the species e and is given as:

$$\bar{J}_e = \rho_e\,(\bar{v}_e - \bar{v}) = -\rho\,D_e\,\nabla\,m_e \tag{6}$$

where D_e is the diffusion coefficient of species e for the mixture. The equation (6) is also known as the Fick's law. The R_e represents the generation rate of species per unit mass.
If the density ρ varies with the temperature T for the natural convection flow, the remaining density based on the Boussinesq concept will be given as:

$$\rho_T \cong \rho\,[1 - \beta(T - T_r)] \tag{7}$$

where T_r is the temperature at reference value and β is the thermal expansion coefficient and is defined as:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P \tag{8}$$

By using the equation (2) and (7), we get

$$\rho\,\nabla.(\bar{v}\,\bar{v}) = -(\nabla\,P)^* + \mu\,\nabla^2\,\bar{v} - \rho\,\bar{g}\beta\,(T - T_r\,) \tag{9}$$

Where $(\nabla\,P)^* = \nabla\,P - \rho\,\bar{g}$, represents the modified pressure gradient. From equation (3), we have the equation for fluid as:

$$(\rho\,C_p)_F\,\nabla.(\bar{v}\,T_F) = \nabla.(\lambda_F\,\nabla\,T_F) + S_F \tag{10}$$

Also from equation (3), we have the equation for solid or porous matrix as:

$$\nabla.(\lambda_p\,\nabla\,T_p) + S_p = 0 \tag{11}$$

where the suffix F and p are used for fluid and porous matrix respectively. The factor S_F or S_p vanishes in the absence of heat generation. The volume-averaging in the porous medium was given by Slattery in 1967 [9], Whitaker [11], [12], in 1969 and 1999 and Gray *et al.* [3] in 1977. It makes the concept of *REV* (representative elementary volume) and by using the concept, the equations are integrated.

2.1 Volume and Time Average Operators

The volume average of the general property term φ over REV for the porous medium was given by Gray *et al.* [3] in 1977 and is written as:

$$[\varphi]_V = \frac{1}{\delta V}\int \varphi\;dV \tag{12}$$

Where $[\varphi]_V$ is taken for any point surrounded by REV of size δV . The average is given as:

$$[\varphi_F]_V = \phi\,[\varphi_F]_i \tag{13}$$

where the suffix '*i'* is used for the intrinsic average and ϕ is the porosity of the medium and is defined as:

$$\phi = \frac{\delta V_F}{\delta V}$$

$$\varphi = [\varphi]_i + \varphi_i \tag{14}$$

in addition to the condition that

$$[\varphi_i]_i = 0 \tag{15}$$

where φ_i is the spatial deviation of φ for the intrinsic average φ_i. To derive the flow equations, we have to know the relation between the volume average of derivatives and derivatives of volume average. The relation between these two was presented by Slattery [9] in 1967 & Gray *et al.* [3] in 1977. So we have

$$[\nabla\varphi]_V = \nabla\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\ \varphi\ ds\right]_{\alpha_i} \tag{16}$$

$$[\nabla.\varphi]_V = \nabla.\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\,.\varphi\ ds\right]_{\alpha_i} \tag{17}$$

$$\left[\frac{\partial\varphi}{\partial t}\right]_V = \frac{\partial}{\partial t}\{\phi(\varphi)_i\} - \frac{1}{\delta V}\left[\iint \hat{n}\,.(\bar{v_i}\,\varphi)\ ds\right]_{\alpha_i} \tag{18}$$

where α_i, $\bar{v_i}$ and $\hat{n}$ are interfacial area, velocity and unit vector normal to α_i respectively. If the porous substrate is fixed then $\bar{v_i} = 0$ But if the medium is rigid and heterogeneous then δV_F depends on the space and doesn't depend on time as taken by Gray *et al.* [3]. The time average of φ is given as:

$$\bar{\varphi} = \frac{1}{\delta t}\int_t^{t+\delta t} \varphi\ dt \tag{19}$$

where δt is very small time interval as compared to $\bar{\varphi}$ but sufficient to calculate the turbulent fluctuations of φ Now the time decomposition will be taken as:

$$\varphi = \bar{\varphi} + \varphi' \tag{20}$$

with the condition that

$$\bar{\varphi}' = 0 \tag{21}$$

where φ' is the time fluctuation of φ with respect to $\bar{\varphi}$

3. Time-Averaged Transport Equation

Let us consider the following:

$$v = \bar{v} + v_1, T = \bar{T} + T_1, P = \bar{P} + P_1 \tag{22}$$

The equations (1), (2) and (9) will be

$$\nabla.\bar{v} = 0 \tag{23}$$

$$\rho\nabla.(\bar{v}\,\bar{v}) = -(\nabla\bar{P})^* + \mu\,\nabla^2\bar{v} + \nabla.(-\rho\overline{v_1 v_1}) - \rho\,\bar{g}\,\beta\,(\bar{T} - T_r) \tag{24}$$

$$(\rho\, C_p)\,\nabla.(\bar{v}\,\bar{T}) = \nabla.(K_e\,\nabla\,\bar{T}) + \nabla.(-\rho\, C_p\,\overline{(v_1 T_1)}) \tag{25}$$

Taking,

$$\frac{\{\nabla\,\bar{v} + (\nabla\bar{v})_T\}}{2} = \overline{D_m} = \text{mean deformation tensor} \tag{26}$$

$$\frac{(\bar{v}_1.\bar{v}_1)}{2} = K_e = \text{turbulent kinetic energy per unit mass} \tag{27}$$

By using the eddy-diffusivity concept, we have from equation (24),

$$-\rho\,\overline{(v_1 v_1)} = \mu_t\, 2\overline{D_m} - \frac{2}{3}\,\rho K_e\hat{A} \tag{28}$$

where $\mu_t, \hat{A}$ are the turbulent viscosity and unity tensor respectively.

Again by using the eddy-diffusivity concept for the turbulent heat flux for equation (25), we have

$$-\rho\, C_p\,\overline{(v_1 T_1)} = C_p\,\frac{\mu_t}{\sigma_t}\,\nabla\,\bar{T} \tag{29}$$

where σ_t is the turbulent Prandtl number. The transport equation for turbulent kinetic energy will be founded by taking the multiplication of the difference between the instantaneous and the time-averaged momentum equations v_1 Again, using the time-average operator, the equation takes the form:

$$\rho\,\nabla.(\bar{v}\,K_e) = -\rho\,\nabla.\left\{v_1\frac{P_1}{\rho} + u\right\} + \mu\nabla^2\,K_e + P_K + Q_K - \rho\,e_1 \tag{30}$$

where

$$P_K = -\rho\,\overline{(v_1 v_1)}$$

$\nabla\,\bar{v}$ = generation rate of K_e due to the mean velocity gradient

$$Q_K = -\rho\,\beta\,\bar{g}.\overline{(v_1 T_1)} \tag{31}$$

e_1=dissipation rate of K_e

The term Q_K is the buoyancy generation rate of K_e.

$$u = \frac{v_1.v_1}{2} \tag{32}$$

4. Conclusions

The paper gives a new method for the analysis of turbulent flow in the porous media by using the time-averaged transport equation. This might be better when studying transport over highly permeable media where the turbulent flow occurs in the fluid phase. The analysis gives opportunities for environmental and engineering flows from these derivations.

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