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A qualitative analysis of technological innovation diffusion

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Abstract
Mathematical modeling is well known in the art. Presently, mathematical models are in widespread use in nearly all forms of technologies such as in computer hardware and software and as an aide in the optimizing and improving of practically every development and manufacturing effort. As a result, mathematical models play in integral role in most technologies in use today. These mathematical models have been developed and applied to a wide variety of technologies depending upon the intended need at the implementation site. The mathematical modeling is an interdisciplinary approach to problem solving characterized by the technological change, testing, and use of models. It uses mathematical methods to analyze the process of technological innovation diffusion also a qualitative analysis of innovation diffusion deepening and development. This studies a good tool for the analysis technological innovation diffusion far fast growing industry and to obtain a mathematical model for the dissemination of an innovative product. With the use of mathematical methods it is proposed to control the function of external influence and to obtain the optimal price for the sale of goods at the maximum balance profit.

Keywords: technological innovation diffusion model, bass model, fisher- pry model, sharif- kabir model, flyod model

1. Introduction

Diffusion is the communication process through which an innovation travels or spreads through certain channels from a person, an organization, or any unit of adoption to another within a social system over time. An innovation can be an object, technology, behavior, practice, program, idea, and/or meme perceived as new to potential adopters. Adoption is the decision and the subsequent implementation, discontinuance, and/or modification by an individual or an organization. Therefore, adoption is an individual or organizational process that leads to diffusion as a systemic process. The term diffusion has been used differently in two groups of literatures. Within economics and most non marketing disciplines, diffusion is defined as the spread of an innovation across social groups over time Brown [1], Stoneman [15]. As such, the phenomenon is separate from the drivers, which can be consumer income, the product's price, word-of-mouth communication, and so on. In marketing and communication, diffusion typically has come to mean the communication of an innovation through the population Golder and Tellis [6]. Mahajan, Muller, and Wind, [12]; Mahajan, Muller, and Bass, [11], Rogers [14]. In this sense, the phenomenon (spread of a product) is synonymous with its underlying driver (communication).

In today's fast-pacing world we are always surrounded by newly emerged products: cars, color television mobile phones, personal computers, tablets, etc .An industry manufactures is a new technological product and want to know at what rate the new process or product can penetrate the market so that it can regulate the production strategy accordingly or so that government can license to produce it in order to satisfy the future demands. The diffusion of innovation is a process by which an innovation is communicated through certain channels over time among the members of a social system .It is noted that the diffusion processes in general follow an S curve.

Rogers [14] defines diffusion as a spread of an innovation over time, in which communication between members of one social system proceeds through certain communication channels. Rogers' innovation diffusion refers to both the planned and the spontaneous ideas spread. The novelty of these ideas is assumed. Rogers' diffusion theory describes the spread of innovation in a social system.

According to Rogers, the concept of innovation diffusion includes four basic elements: innovation, communication channels, time and the social system. Individuals play a major role in the diffusion process of innovation, as innovation spreads through the diffusion networks established by individuals. This innovation can be already known, but it has not yet been accepted or rejected by the person.
The theory of innovation diffusion is concerned with the demand for an innovation by relevant groups of potential users, Rogers and Shoemaker [3] define the innovation in the following words: in the context of the developing countries where most of the technological innovation have been transferred from the developed countries, they can be perceived as new and be treated as an innovation for the purpose of diffusion in the as and innovation for the purpose of diffusion recipient society. The extent to a new idea or product spread is a measure of diffusion. The diffusion itself is referred, more conventionally to the process by which an innovation is communicated to possible jobs. This technological substitution may be categorized in three groups depending upon the characteristics of substitutes. In the first category we notice technological substitution where the capabilities of the technology are enhanced i.e. memories of the computer or speed of aero-planes. In the second categories the attributes of the technology are changed or replaced such as speed to fuel ratio. In third case product is substituted. This case is widely known as technology substitution (graph 5) and it is manifestation of technological development. Each type of technology is followed by the apparition of new products, Specifically, products that rapidly progress through their life cycle Viardot [4]. Due to rapidly recorded life cycle somehow obsolete. This is the result of shortened life cycles,consumers change in lifestyle,and a rapid process of continuous change that characterizes the industy,such as the entire economy Demetrescu [10]. Beginning with the 90,diffusion modeling research has reexamined this interpretation to identify and discuss about other social types of interaction Ivanov,Avasilcai and Procedia [2], Cioca and Cioca [9]. Clarck and Goldsmith [13] examine the relationship between innovation and social influence. The varibale factors were analyzed separtely,although in the marketing research they are considerd part of a global concept named consumer susceptibility to interpersonal influence. The aim of the study is to obtain a mathematical models for the dissemination of an innovative product. With the use of mathematical methods it is proposed to control the function of external influence and to obtain the optimal price for the sale of goods at the maximum balance profit. .In this studies attempt has been made to explain the process of technological change and diffusion with the help of mathematical models.

2. Diffusion Models

It is assumed that adoption of an innovation is essentially the outcome of learning or communication process and that the diffusion regime or social system is the one in which all the individuals have equal opportunity to adopt. The inherent assumption in these models is that the old technology is completely replaced by the new one. Where as in many cases particularly in developing countries more than one completing technologies coexist. Cases of multiple substitutions may be seen in many others. Multiple substitutions are the result of frequent innovation in a particular area i.e. economic viability of old technology, energy resources wood-coal, oil, natural gas and non conversional energy sources like biogas, solar energy.

Modelling technology diffusion processes was initially derived from theory of growth of a colony of biologicali cell in a medium.since the growth of acell would be limited due to limited space.similarly, technology diffusion models assume that the growth of a technology or an innovation is dependent on the total potential adopters and rate of increase is represented by the following diferential equation referred to as the internal influence diffusion model

$$\frac{dn}{dt} = c\, n(t)[N - n(t)] \qquad \ldots\ldots(1)$$

Where n(t) is the cumulative adoption at time t, N is the total number of potential adopter and c is the coefficient of diffusion .The equation is called logistic growth curve and is directly used in technological diffusion which assumes that diffusion process is influenced by previous adopters

Let $\frac{n(t)}{N} = f(t)$

$$\frac{df}{dt} = c' f(`1 - f) \qquad \ldots\ldots.(2)$$

$f(t)$ or $n(t)$ increases at increasing rate when $n(t) < \frac{N}{2}$ and it increases but at decreasing rate when $n(t) > \frac{N}{2}$ and there is a point of inflexion when $f(t) = \frac{1}{2}$ or $n(t) = \frac{N}{2}$.
This statement can be represented by graph (1).
In graph (1) AB is the convex part, in which number of adopters $n(t)$ rises fast at an increasing rates. B us point of inflexion at which almost half of the potential adopter have adopted the innovation but the number $n(t)$continues to increases with decreasing rate and this continues till almost all the potential adopters adopt the innovation. For different technology the curve are similar but are not identical.
The model (2) is called Fisher-Pry model and is very successful model in the study of innovation diffusion. In this model point of inflexion is $n = \frac{N}{2}$ while in real life situation it can occur before or after $\frac{N}{2}$.

If the influence on diffusion is external,the equation for the external model is given by

$$\frac{dn}{dt} = b\,[N - n(t)] \qquad \ldots\ldots\ldots(3)$$

Where n(t) is the cumulative adoption at time t, N is the total number of potential adopter and b is the coefficient of diffusion.
A mixed influence model which combines the equations (1) and (3) was first presented by Bass to represent the first purchase growth of a new product durable in marketing [5].
The Bass model is a mixed influence model with three parameters p, q, N represents the coefficient of innovation, coefficient of imitation of imitation and is the total number of adopters . The Bass diffusion model is given by

$$\frac{dn}{dt} = p\big(N - n(t)\big) + q\,\frac{n(t)}{N}\big(N - n(t)\big) \ldots\ldots\ldots\ldots(4)$$

where p is external influence and q is internal influence .The function n(t) and $\frac{dn}{dt}$ strongly depend on the coefficients p, q and N.

n(t)= n(t, p, q, N) and $\frac{dn}{dt}$ = n(t,p, q, N)

n(t, p, q, N)= N $\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}$, t≥0

lim (t → ∞)n(t, p, q, N)= N

$$\frac{df}{dt} = p\big(1 - f(t)\big) + qf(t)\big(1 - f(t)\big)$$

Where $\frac{n(t)}{N} = f(t)$

$$= (p + qf)(1 - f)$$

$$f(t) = \frac{(p+q)^2}{p}\frac{e^{-(p+q)t}}{(1+\frac{q}{p}e^{-(p+q)t})^2}$$

For the inflection points

$$\frac{df(t)}{dt} = \frac{(p+q)^3}{p}\frac{e^{-(p+q)t}}{(1+\frac{q}{p}e^{-(p+q)t})^3}\left(1-\frac{q}{p}e^{-(p+q)t}\right)$$

$$\frac{df(t)}{dt} = 0$$

$$t = t^* = \left(\frac{1}{(p+q)}\right)\ln\left(\frac{q}{p}\right)$$

$$f(t^*) = \frac{1}{2} - \frac{1}{2}\frac{p}{q}$$

$$\frac{d^2f}{dt^2} = \frac{(p+q)^3}{p}\frac{d}{dt}\left[\frac{e^{-(p+q)t}}{(1+\frac{q}{p}e^{-(p+q)t})^3}\left(1-\frac{q}{p}e^{-(p+q)t}\right)\right]$$

Put $\gamma(t) = e^{-(p+q)t}$

$$\frac{d^2f}{dt^2} = \frac{(p+q)^3}{p}\frac{d}{dt}\left[\frac{\gamma(t)}{(1+\frac{q}{p}\gamma(t))^3}\left(1-\frac{q}{p}\gamma(t)\right)\right]$$

$$= \frac{(p+q)^3}{p}\frac{d}{dt}\left[\frac{1}{(1+\frac{q}{p}\gamma(t))^3}\left(\gamma(t)-\frac{q}{p}\gamma(t)^2\right)\right]$$

$$= \frac{(p+q)^3}{p}\left[\frac{1}{(1+\frac{q}{p}\gamma(t))^6}\right]\Big[(1+\frac{q}{p}\gamma(t))^3\left(\frac{2q}{p}\gamma(t)-1\right)\gamma(t) - 3\left(1+\frac{q}{p}\gamma(t)\right)^2\frac{q}{p}\frac{d\gamma(t)}{dt}\left(\frac{q}{p}\gamma(t)^2-\gamma(t)\right)\Big]$$

$\frac{d^2f}{dt^2}$ =0, gives

$$\left(\frac{q}{p}\right)^2 \cdot \gamma(t)^2 - 4\left(\frac{q}{p}\right)\gamma(t) + 1 = 0$$

$$\gamma(t) = \frac{p}{q}(2\pm\sqrt{3})$$

$$e^{-(p+q)t} = \frac{p}{q}(2\pm\sqrt{3})$$

$$t = t^{**} = \frac{\ln(\frac{p}{q})-\ln(2\pm\sqrt{3})}{(p+q)} = \frac{\ln(\frac{p}{q})-\ln(2\pm\sqrt{3})}{(p+q)}$$

$$t^{**} = t^* \pm \ln\frac{(2+\sqrt{3})}{(p+q)}$$

In this case the point of inflexion occurs before half the final population size is reached. The model is not sufficiently flexible since it cannot represent those situations in which the point of inflexion occurs after half the population size is reached. The model has worked quite well in predicting the growth curves, and one major advantage is that f(t) can be easily solved . In Bass model, there are three parameters required to be estimated, the potential market N, the innovation coefficient p , and imitation coefficient q. In this study, we use the assumption that the market potential N is changes over time with the decision variables.

If $\frac{q}{p} > 1,$ then the adopting curve will reach its peak points and the diffusion of the products is successful (the product is successful and sales experience growth and then decline due to saturation)

If $\frac{q}{p} < 1,$ then the adopting curve will not reach its peak points (the product is unsuccessful and sales will start at a certain level and keep declining)

Bass also presents the influence of (*p* + *q*) values, between 0.3 and 0.9, on the growth rate. Sultan [16] performed a meta-analysis of applications of diffusion models and they compare several parameters estimation methods and also how the number of sampling points influences accuracy. They found that the average *p* value is 0.03, while the average *q* value is 0.38. Van den Bulte [18] explored how *p* and *q* vary across products and countries, based on a database containing 1586 sets of *p* and *q* parameters, from 113 papers published between 1969 and 2000. He explains that the parameters *p* and *q* provide information about the speed of diffusion. A high value for *p* indicates that the diffusion has a quick start, but also tapers off quickly. A high value of *q* indicates that the diffusion is slow at first, but accelerates after a while. He also notes that, when *q* is larger than *p*, the cumulative number of adopters follows the type of S curve often observed for radically innovative product categories. When *q* is smaller than *p*, the cumulative number of adopters follows an inverse J-curve often observed for less risky innovations such as .The time between the inflection points and between them to peak depends only on (*p* +*q*) and not on (*p/q*).

3. Analysis of Parameters

The external and internal influences represented by p and q are taken as constant but they may not be constant as the mode of publishing becoming stronger. We take p and q as function of "f" and the earlier model is in the general form as

p = p(f) and q =q(f)

$$\frac{df}{dt} = [p(f) + fq(f)](1 - f) \quad \ldots\ldots\ldots\ldots(5)$$

$p(f)$ and $q(f)$ cannot be any arbitrary functions since we want (3) to give an S-shaped curve with a point of inflexion lying between f_0 and 1.

3.1 The basic expression of the Bass model is used to enrich the impact of external environmental factors on the diffusion of leading technology of new industry 'because the diffusion of the leading technology of new an industry will be greatly affected by the external environment. Considering that researchers use markets and policies as environmental factors when they use the Bass model to study technolgy diffusion within the industry,the research in this paper mainly considers the environmental impact of market and policy environment on the dominant technology diffusion of new industries. Now considers the innovation coefficient p and the imitation coefficient q in Bass model, and introduces external influence factors to make it a function equation with environmental characteristics;

$p(f) = (a_t + s_t)i_p + \varepsilon_p$ $q(f) = (a_t + s_t)i_q + \varepsilon_q$

Where a_i represents the market acceptance of technology in a space,s_t represents the policy follow-up speed after technology entering the market in same space and i_p and i_q represent the influence coefficients of the policy comprehensive interaction coefficient on the imitation coefficient q and the innovation coefficient p, ε_p and ε_q are the random error term,

$$\frac{df}{dt} = \{[(a_t + s_t)i_p + \varepsilon_p] + [(a_t + s_t)i_q + \varepsilon_q] f(t)]\} \quad (1 - f(t))$$

$$\frac{df}{dt} = a_t\,(1 - f(t))[i_p + i_q f(t)] + s_t(1 - f(t))[i_p + i_q f(t)] + (1-f)[\varepsilon_p + f(t)\ \varepsilon_q]$$

When $a_t \neq 0, s_t$ =,0, then in a certain area the domestic policy has a small or no comprehensive role, them the diffusion is government –oriented .

When $a_t = 0, s_t \neq 0$, then the markets comprehensive role in a certain area is small or not,then diffusion is market- oriented .When $a_t \neq 0$, $s_t \neq 0$, then diffusion is mixed –oriented.

3.2 $p(f) = 0$, $q(f) = cf^{\delta}$

$$\frac{df}{dt} = c\,f^{\delta}(1 - f)$$

$$\frac{d^2f}{dt^2} = c\,f^{\delta-1}[\delta - (1+\delta)f]\,\frac{df}{dt}$$

Point of inflexion is

$$f = f^* = \frac{\delta}{1+\delta}$$

f^* varies from 0 to 1 as δ goes from 0 to ∞.

$\delta = 0$ signifies that external influence is a constant function and depending on f and point of inflexion is zero.

$\delta = \infty$ signifies that external influence is maximum and point of inflexion occurs when almost all the potential adopters have accepted then new innovation.

This model is perfectly flexible.

3.3,

$$p(f) = 0,\ q(f) = \frac{cf(1-f)}{1-(1-\sigma)f}$$

$$\frac{df}{dt} = \frac{cf(1-f)^2}{1-(1-\sigma)f}$$

This is Sharif-Kabir Model. Sharif –Kabir can accommodate symmetric as well as nonsymmetric diffusion patterns,it produces a point of inflection that must be in the range 0.33.

If $\sigma = 1$

$$\frac{df}{dt} = cf(1-f)^2$$

This model is called Flyod model. This model is a non symmetric and possesses a fixed possesses a fixed point of inflection at f = $\frac{1}{3}$.

If $\sigma = 0$

$$\frac{df}{dt} = cf(1-f)$$

This is Fisher-Pry model.

4. Variation in total number of potential adopters

In the previous models it was supposed that total number of potential adopter of new innovation is constant over a time period. But when we think about the increase in the population size and increase in the economic prosperity of the classes producing potential adopters, then N will not be constant any more will change with time. The model (1) now can be written as

$$\frac{dn}{dt} = c\,n(t)[N(t) - n(t)]$$

$$\frac{1}{n^2}\frac{dn}{dt} - \frac{c}{n(t)}.N(t) = -C$$

Let $u(t) = \frac{1}{n(t)}$

This gives

$$\frac{du\ (t)}{dt} + cu\ (t)N(t) = C$$

On integration we get u.

$$\exp\int CN\,dt = C\exp\int CN\,d + K$$

from this expression if $N(t)$ is known we can always calculate $u(t)$ or the total number of adopters $n(t)$ till time 't'.

5. Delay effect in communication

The models described above is based on the instantaneous effect of communication on the non-adopters. This mean that as soon as $n(t)$ adopters adopted the innovation at time 't' and communicated it to $N - n(t)$ non adopters they immediately accepted the innovation. This seems not reasonable. Because logical human brain takes some time to accept or reject any situation under consideration.

Let at time 't' number of adopters is n (t) communicated the innovation to $N - n(t)$ non-adopters after delay time T innovation is adopted. In other words we can say that $N - n(t)$ non-adopters are influenced by $n(t-T)$ of adopter at time $t - T$, so that model (2) becomes

$$\frac{dn}{dt} = c\, n(t-T)\big(N - n(t)\big)$$

This model is delay differential equation model.

6. Diffusion of innovation over space and time

The scientific discoveries have reduced the span of space and time in such a way that the globe an which we live is reduced to a real table globe. A new product may be developed in one country and after a lapse of time it spread by publicity to the potential adopters of another countries. Thus the number of adopters depends not only on time but on space variable also. There is technological innovation diffusion in time and space. The total number of potential adopters N can be function of space variable only that is country to country N will very hence our model (1) is reduced to

$$\frac{\partial n(x,t)}{\partial t} = C\, n(x,t)[N(x) - n(x,t)]$$

with initial condition

$$n(x,0) = No$$

No is number of adopters at the time of launch of the product in the market and this is taken as zero time. On integration we get

$$\frac{n(x,t)}{N(x) - n(x,t)} = \frac{N}{N(x) - No} e^{ct}$$

at any time t if we know the total potential adopters of different countries, we can calculate number of adopters of the innovation in the respective countries.

7. Application of models

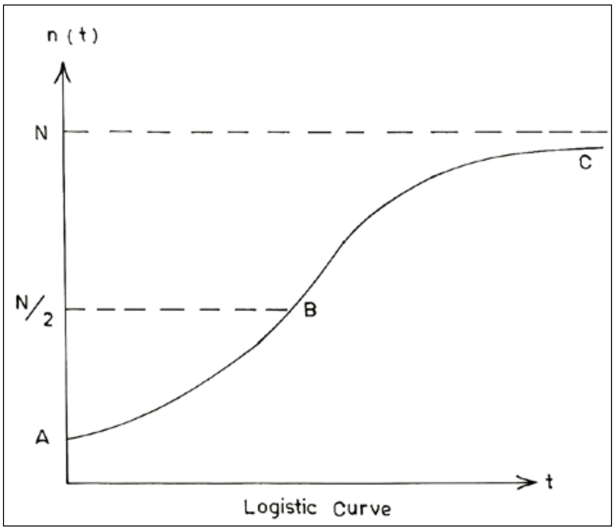
The application of Bass model has been made to analyse the diffusion of a new products from electronic industry. We obtained the data from "Trends in Industrial production" CMIE Oct. 1994 for programme logic controllers (PLC) Electronics Training Boards (ETB) and large local exchanges operating under three systems strowger, crass Bar and Electronic. The trend of these electronic industries are represented by graphs (2 & 3). If we analyse the production graph for local exchange we conclude that for newer technology production increases fast with time compared to older one. Stronger and crass Bar are nearly in the same production range where as the electronic are increasing very fast after 1990. If we see trend of five years 1995-2000 electronic industry is declining may be also in near future. This may be the result of short life cycle of the products in electronic industry and also because of the enhanced capacity of individual items like electronic exchanges.

It is assumed that electronics exchanges are replacing stronger and crass Bar, its share in the market f has been calculated in fisher and pry model and log $\frac{f}{1-f}$ plated against time't'. It gives a linear trend. The results show that there is declining trend in the production of electronic exchange but the share is increasing from declining trends we interpret the following possibilities:

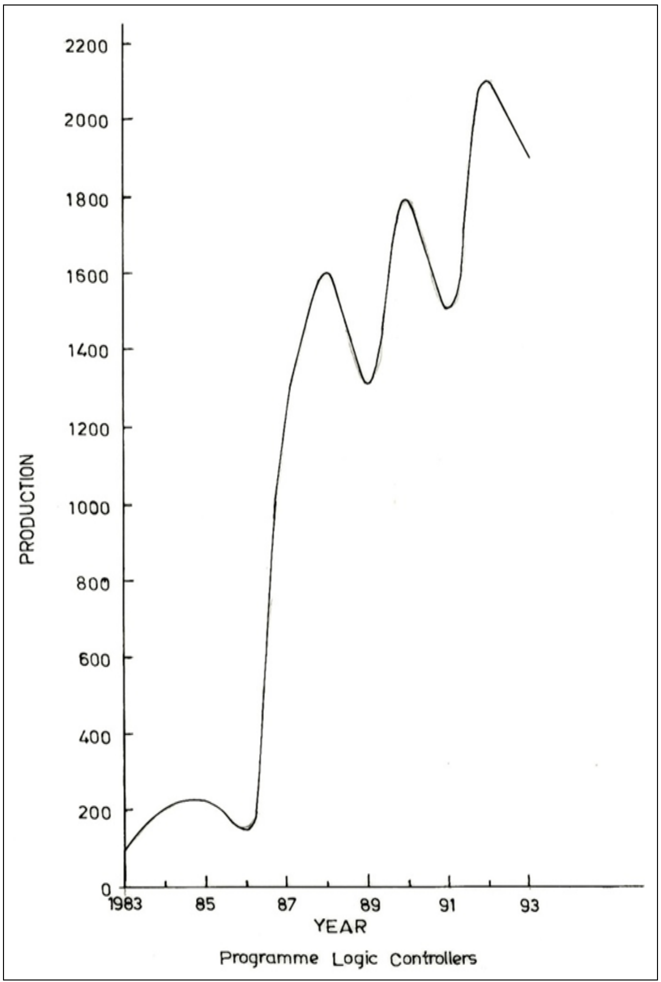
Generally the saturation in the technology can be seen in a decreasing rate of diffusion and at this point technology transfer take place graph (4),a declining trend also suggest that new technology is replacing old one, the short product life cycle is also responsible for declining trend.

8. Results

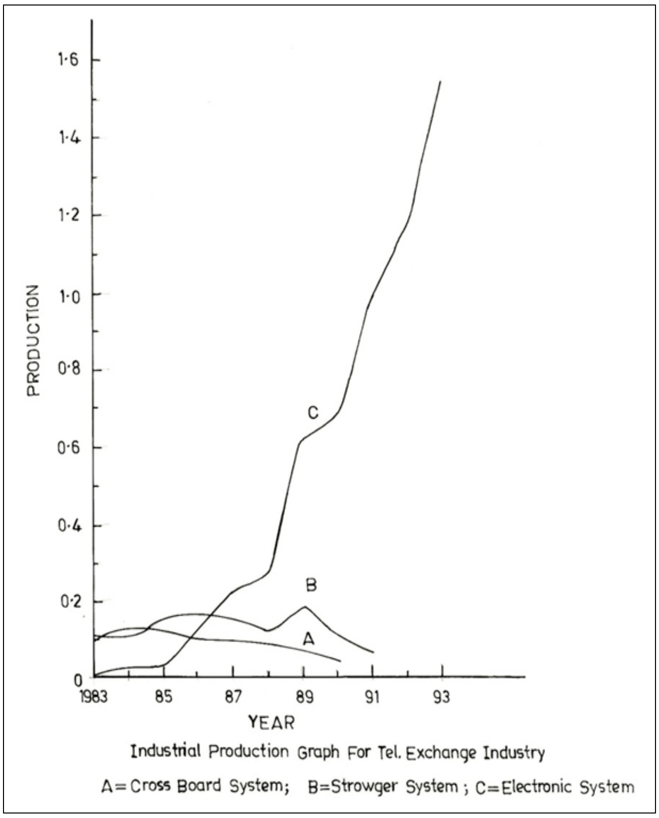
We have studied the models of innovation diffusion starting from simple case and taken into account influences which are coming from real life situation. In mathematical form these models are differential equation, time space, delay differential equations. If we take many parameters at a time the model becomes complex and the object of modeling is not the display of mathematical power but obtaining the insight and understanding. These models and such type of studies help in process of talking decision about the launching of new products in the market making right kind of investment in the product as it influences the number of potential adopters. The aim of this paper is to understand the diffusion of products belonging to different cluster technologies in terms of the available chaotic and stochastic models. After reviewing the relevant mathematical models, we categorize their use according to selected technology clusters. Furthermore, the development and adjustment of mathematical models is compared with relevant changes of the products/services, communication channels, needs, demands.



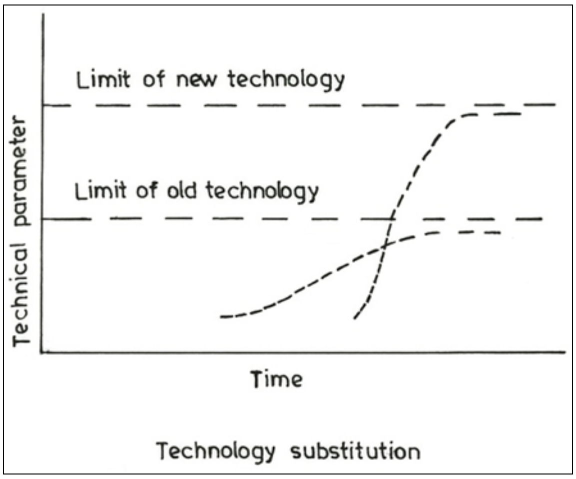
Graph 1: Logistic curve



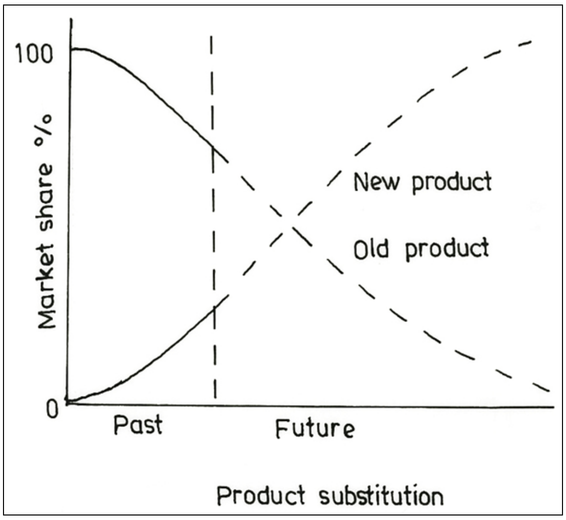
Graph 2: Year



Graph 3: Industrial production graph for tel. exchange industry



Graph 4: Technology substitution



Graph 5: Product substitution

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