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Effect of reinforcement gradient on creep response in FG composite cylinders

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Abstract

The paper investigates the effect of varying reinforcement gradation index (m) on steady state creep behaviour of thick walled cylinder made of functionally graded composite under internal and external pressure. The SiC_p reinforcement in the cylinder is assumed to vary radially according to the power law function with reinforcement gradation index (m). The distribution of SiC_p reinforcement is uniform for $m=0$, increasing for $m>0$ and decreasing $m<0$ along the radial direction. The creep behavior of the composite is described by Norton's Power law. The results obtained for FG cylinder are compared with those estimated for a uniform composite cylinder. The study reveals that for the radial stress in cylinder decreases throughout when the SiC_p reinforcement content is decreasing ($m<0$) along the radius in the FGM cylinder. Upon imposing SiC_p gradient, for non-linearly decreasing reinforcement content along the radius, in the composite cylinder, the tangential stress increases near the inner radius but decreases toward the outer radius. Unlike stresses, the tangential and radial strain rates in the composite cylinder reduce over the entire radius when the SiC_p reinforcement content is decreasing ($m<0$) along the radius in the FGM cylinder.

Keywords: Cylinder, Creep, Norton law, Pressure, Functionally Graded Material

1. Introduction

Functionally Graded materials (FGMs) are made of two different materials, in which the mechanical properties vary smoothly and continuously from one surface to the other (Gupta *et al.*, 2003; Guven *et al.*, 2001) ^[1, 2]. Bhatnagar *et al.* (1986) ^[3] have presented the analysis for an orthotropic thick-walled cylinder undergoing creep due to the combined action of internal and external pressure, and rotary inertia. It is observed that the material stronger in radial direction may be beneficial for the design of the cylinder due to lower effective stress. Shukla (1997) ^[5] obtained the expressions for elastic-plastic transitional stresses and strain in a compressible cylinder subjected to internal pressure. The study indicates that the presence of compressibility having lesser value at the internal surface of the cylinder reduces the axial contraction. Gupta *et al.* (2000) ^[6] analyzed creep stresses and strain rates in a rotating non-homogeneous thick-walled cylinder by using Seth's transition theory. The study indicates that for a rotating non-homogeneous cylinder, with compressibility increasing radially, the circumferential stress is maximum at the external surface at lesser angular speed but at some higher angular speed it becomes maximum at the internal surface.

Filho *et al.* (2004) ^[7] derived a lower bound to the creep rupture time of internally pressurized cylinders. They described material behaviour by a phenomenological creep rupture theory, which accounts for creep of all the phases and full coupling between the deformation and damage process. Singla *et al.* (2012) ^[8] investigates the effect of varying anisotropy on the creep performance of a functionally graded composite cylinder by using finite element based Abaqus software. The creep has been described by Norton's Power law. The creep stresses and strains have been estimated in the FGM cylinder for varying degree of anisotropy. The study reveals that by changing the extent of anisotropy the stresses and strains in the FGM cylinder are significantly modified.

The Literature consulted so far as reveals that various studies have been devoted to studying the creep behavior cylinders made of FGM material. The studies regarding creep behavior of non-linear distribution of reinforcement FGM cylinder are rather limited. It is decided to investigate the creep response of anisotropic FGM cylinder having non-linear distribution of reinforcement.

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2. Distribution of reinforcement

The distribution of SiC_p in the FGM cylinder decreases from the inner to outer radius. The content (vol %) of SiC_p, V(r), at any radius r, is given by,

$$V(r) = V_o \left(\frac{r}{b} \right)^m \tag{1}$$

Where m is the gradation index. The values of m are chosen as 0.5 and -0.5 for linear and non-linear FGM cylinders respectively.

The average SiC_p content in the cylinder can be expressed as,

$$V_{avg} = \frac{\int_a^b 2\pi r l V(r) dr}{\pi(b^2 - a^2)l} = \frac{2 \int_a^b r V(r) dr}{(b^2 - a^2)} \tag{2}$$

Where l is the length of cylinder.

Substituting V(r) from Eq. (1) into Eq. (2) and integrating, we get,

$$V_o = \frac{V_{avg} (b^2 - a^2)(m + 2)}{2(b^2 - a^{m+2} - b^{m+2})} \tag{3}$$

3. Creep law and parameters

The creep behavior of the FGM cylinder is described by Norton's power law as,

$$\dot{\epsilon}_e = B \sigma_e^n \tag{4}$$

Where $\dot{\epsilon}_e$ is the effective strain rate, σ_e is the effective stress, B and n are material parameters describing the creep performance in the cylinder.

It is evident from the study of that the values of creep parameters B and n appearing in the Norton's law depend on the content of reinforcement, which vary with the radial distance.

$$B(r) = B_o \left[\frac{V(r)}{V_{avg}} \right]^\phi \tag{5} \text{ and}$$

$$n(r) = n_o \left[\frac{V(r)}{V_{avg}} \right]^{-\phi} \tag{6}$$

where B_o and n_o are respectively the values of creep parameters B and n respectively and ϕ is the grading index.

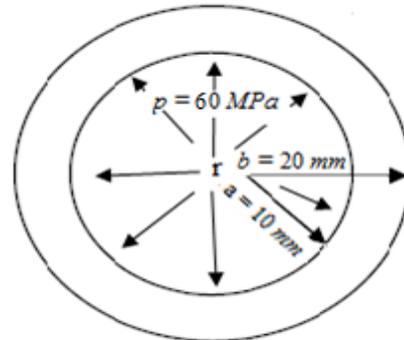
The values of B_o, n_o and ϕ are the taken from the study of Chen *et al.*, 2007.

Table 1: Values of Creep parameters Chen *et al.*, 2007

Creep parameters: $B_o = 2.77 \times 10^{-16} (MP^n/h)$ and $n = 3.75$ Grading index $\phi = 0.7$

3. Mathematical formulation

Consider a FGM thick-walled hollow cylinder with an inner radius a (10 mm) and outer radius b (20 mm) subjected to an internal and external pressures p (70 MPa) and q (10 MPa) respectively. The coordinates axes r, θ and z are taken respectively along the radial, tangential and axial directions of the cylinder.



For the purpose of analysis the following assumptions are made:

- (i) The material of the cylinder is orthotropic and incompressible i.e. $\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = 0$
- (ii) The cylinder is subjected to internal pressure that is applied gradually and held constant during the loading history.
- (iii) Elastic deformations in the cylinder are neglected as compared to creep deformations.
- (iv) The cylinder is sufficiently long and hence is assumed under plain strain condition (i.e. axial strain rate, $\dot{\epsilon}_z = 0$)

The radial ($\dot{\epsilon}_r$) and tangential ($\dot{\epsilon}_\theta$) strain rates in the cylinder are given by:

$$\dot{\epsilon}_r = \frac{d\dot{u}_r}{dr} \tag{7} \text{ and}$$

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} \tag{8}$$

Where $\dot{u}_r = (du/dt)$ the radial displacement is rate and u is the radial displacement.

Eqs (7) and (8) may be solved to get the following compatibility equation,

$$r \frac{d\dot{\epsilon}_\theta}{dr} = \dot{\epsilon}_r - \dot{\epsilon}_\theta \tag{9}$$

The cylinder is subjected to the following boundary conditions,

$$\sigma_r = -p \text{ at } r = a \tag{10}$$

$$\sigma_r = -q \text{ at } r = b \tag{11}$$

Where the negative sign of σ_r implies the compressive nature of radial stress.

By considering the equilibrium of forces acting on an element of the cylinder in the radial direction, we get,

$$r \frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r \tag{12}$$

Since the material of the cylinder is incompressible, therefore,

$$\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = 0 \tag{13}$$

The constitutive equations under multi axial creep in an orthotropic cylinder, when the principal axes are the axes of reference, Bhatnagar and Gupta (1967)^[10] are given by,

$$\dot{\epsilon}_r = \frac{\dot{\epsilon}_e}{(G+H)\sigma_e} [G(\sigma_r - \sigma_z) + H(\sigma_r - \sigma_\theta)] \tag{14}$$

$$\dot{\epsilon}_\theta = \frac{\dot{\epsilon}_e}{(G+H)\sigma_e} [F(\sigma_\theta - \sigma_z) + H(\sigma_\theta - \sigma_r)] \tag{15}$$

$$\dot{\epsilon}_z = \frac{\dot{\epsilon}_e}{(G+H)\sigma_e} [F(\sigma_z - \sigma_\theta) + G(\sigma_z - \sigma_r)] \tag{16}$$

where F , G and H are the anisotropic constants, $\dot{\epsilon}_e$ and σ_e are respectively the effective strain rate and effective stress in the FGM cylinder.

The Hill's yield criterion, when the Principal axes of anisotropy are the axes of reference, Dieter [11], is given by,

$$\sigma_e = \left[\frac{1}{(G+H)} \{ F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2 \} \right]^{1/2} \tag{17}$$

Under plain strain condition ($\dot{\epsilon}_z = 0$), one may get from Eqs. (7), (8) and (13),

$$\dot{u}_r = \frac{C}{r} \tag{18}$$

Where C is a constant of integration. Using Eq. (18) in Eqs. (7) and (8), we get,

$$\dot{\epsilon}_r = -\frac{C}{r^2} \tag{19} \text{ and}$$

$$\dot{\epsilon}_\theta = \frac{C}{r^2} \tag{20}$$

Under plane strain condition, Eq. (16) becomes,

$$\sigma_z = \frac{(G\sigma_r + F\sigma_\theta)}{(F+G)} \tag{21}$$

Substituting σ_z from Eq. (21) in to Eq. (17), we get,

$$\sigma_e = \frac{(\sigma_\theta - \sigma_r)}{\sqrt{(H+G)}} \left[\frac{(FG + GH + HF)}{(F+G)} \right]^{1/2} \tag{22}$$

Substituting $\dot{\epsilon}_r$ and σ_z respectively from Eqs. (19) and (21) into Eq. (14), we obtain,

$$\sigma_\theta - \sigma_r = \frac{(F+G)(H+G)}{(FG + GH + HF)} \frac{\sigma_e C}{\dot{\epsilon}_e r^2} \tag{23}$$

Using Eqs. (4) and (22) in Eq. (23) and simplifying, one gets,

$$\sigma_\theta - \sigma_r = \frac{I_1}{r^{2/n}} \tag{24}$$

Where,

$$I_1 = \left[\frac{(F+G)(H+G)}{(FG + GH + HF)} \right]^{2n} \frac{C^{1/n}}{B^{1/n}} \tag{25}$$

Substituting Eq. (24) into Eq. (12) and integrating, we get,

$$\sigma_r = X_1 - p \tag{26}$$

$$X_1 = \int_a^r \frac{I_1}{r^{n+2}} dr$$

Where,

$$\tag{27}$$

Substituting Eq. (26) into Eq. (24), we obtain,

$$\sigma_\theta = X_1 + \frac{I_1}{r^{2/n}} - p \tag{28}$$

To estimate the value of constant C , needed for estimating I_1 , the boundary conditions given in Eqs. (10) and (11) are used in Eq. (26) with X_1 (Eq. 27) integrated between limits a to b . to get,

$$\int_a^b \frac{I_1}{r^{n+2}} dr - p = -q \tag{29}$$

Substituting the value of I_1 from Eq. (25) in to Eq. (29) and simplifying, we obtain,

$$C = \left[\frac{p - q}{X_2} \right]^n \tag{30}$$

$$X_2 = \int_a^b \frac{(T)^{n+1}}{r^{n+2} B^{1/n}} dr \quad \text{and} \quad T = \sqrt{\frac{(F+G)(H+G)}{(FG + GH + HF)}}$$

Where,

$$\tag{31}$$

Using Eqs. (21) and (22) into Eqs. (14) and (15), one obtains,

$$\dot{\epsilon}_\theta = -\dot{\epsilon}_r = \frac{\dot{\epsilon}_e}{\sqrt{\frac{(F+G)(H+G)}{(FG + GH + HF)}}} \tag{32}$$

The analysis presented above yields the results for isotropic FGM cylinder. When the anisotropic constants are set equal i.e. $F=G=H$.

6. Results and Discussion

The radial stress shown in Fig. 1 remains compressive throughout the cylinder in all the cylinder (Table 1), with a maximum value at the inner radius and zero at the outer radius, under the imposed boundary conditions. The compressive value of the radial stress decreases over the entire radius. The radial stress is minimum for the anisotropic non-linear FGM cylinder C3 and maximum for the anisotropic linear FGM cylinder (C2) when compared with the Non-FGM cylinder C1.

Table 1: Details of different composite cylinders

S. No.	Cylinder	m	V_{avg}	V_o
1.	Non-FGM (C1)	0	20	20
2.	FGM (C2)	0.5	20	22.77
3.	FGM (C3)	-0.5	20	17.40

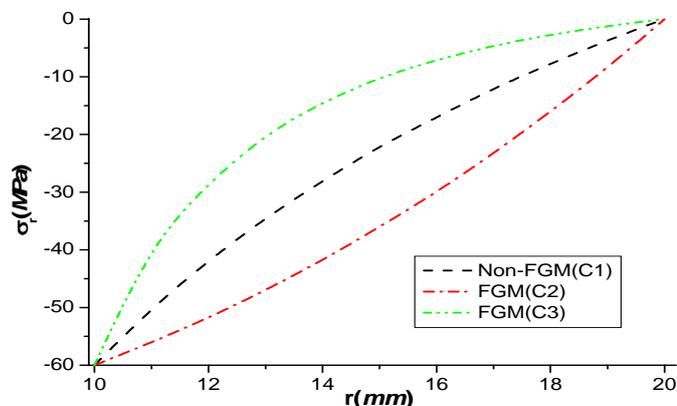


Fig 1: Variation of radial Stress in cylinders

The tangential stress are shown in Fig. 2 remains tensile throughout the cylinders. The tangential stress in non-linear FGM cylinder C3 is increases near the inner radius and decreases towards the outer radius and vice versa for linear FGM cylinder C2 when compared with the Non-FGM cylinder C1. In the tangential stress, Non-FGM cylinder C1 increases at a radius of around 15mm when compared with the linear FGM cylinder C2 and non-linear FGM cylinder C3. In the effective stress, Non-FGM cylinder C1 increases at a radius of around 13mm-14mm when compared with the linear FGM cylinder C2 and non-linear FGM cylinder C3.

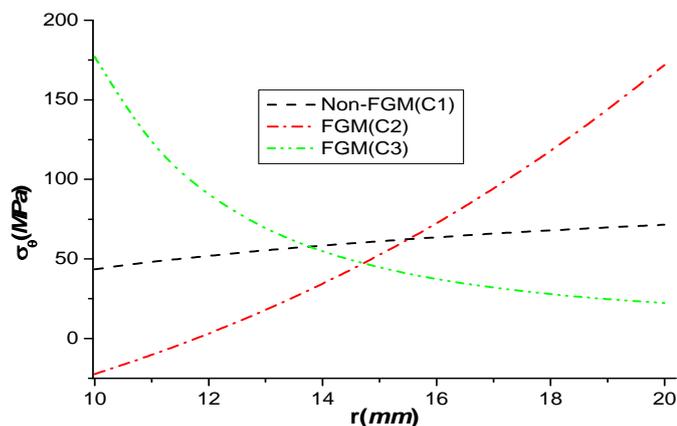


Fig 2: Variation of Tangential Stress in cylinders

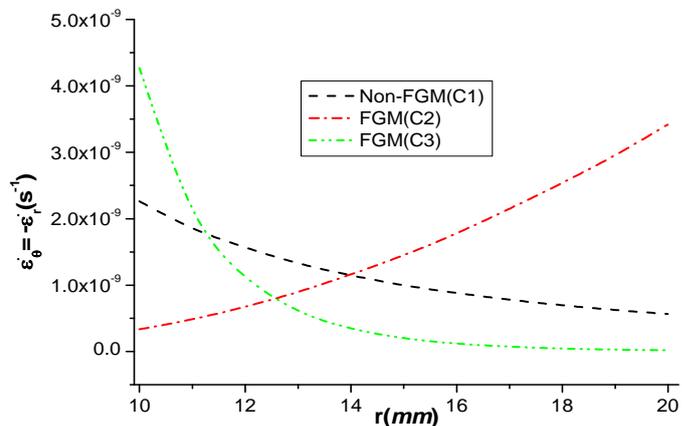


Fig 3: variation of radial and tangential strain rate in cylinders

The radial as well as tangential strains in the composite cylinder decrease over the entire radius in the Non-FGM cylinder and non-linear FGM cylinder and increase over the entire radius in the linear FGM cylinder as shown in Figure 3. The magnitude of strain observed in non-linear FGM cylinders C3 is the lowest over the entire radius when compared to linear FGM cylinder C2 and Non-FGM cylinder C1.

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