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# Applications of mathematics T of fluid

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### Abstract
The study of these flows has been attached with a wide range of mathematical techniques and, to day, this is a stimulating part of both pure an applied mathematics. Fluid is a material that is infinitely deformable or malleable. A fluid may resist moving from one shape to another but resists the same amount in all directions and in all shapes. The basic characteristic of the fluid is that it can flow. Fluids are divided in two categories, incompressible fluids (fluids that move at far subsonic speeds and do not change their density) and compressible fluids The aim of this Paper is to furnish some results in very different areas that are linked by the common scope of giving new insight in the field of fluid dynamics. Thus, already by the time of the Roman Empire enough practical information had been accumulated to permit quite sophisticated applications of fluid dynamics.

**Keywords:** Fluid dynamics, applications, mathematical, techniques, practical, etc.

### Introduction
We begin by introducing the "intuitive notion" of what constitutes a fluid. As already indicated we are accustomed to being surrounded by fluids both gases and liquids are fluids and we deal with these in numerous forms on a daily basis [1-4]. As a consequence, we have a fairly good intuition regarding what is, and is not, a fluid; in short we would probably simply say that a fluid is "anything that flows." This is actually a good practical view to take, most of the time. But we will later see that it leaves out some things that are fluids, and includes things that are not. So if we are to accurately analyze the behavior of fluids it will be necessary to have a more precise definition [5-8]. It is interesting to note that the formal study of fluids began at least 500 hundred years ago with the work of Leonardo da Vinci, but obviously a basic practical understanding of the behavior of fluids was available much earlier, at least by the time of the ancient Egyptians; in fact, the homes of well-to-do Romans had flushing toilets not very different from those in modern 21-Century houses, and the Roman aqueducts are still considered a tremendous engineering feat [9]. Thus, already by the time of the Roman Empire enough practical information had been accumulated to permit quite sophisticated applications of fluid dynamics.

### Review of Literature
The following example will provide a simple illustration of how to apply the above-stated principle. It will be evident that little is required beyond making direct use of its contents. We consider a cubical object with sides of length h that is floating in water in such a way that $\frac{1}{4}h$ of its vertical side is above the surface of the water, as indicated in Fig. 1. It is required to find the density of this cube.

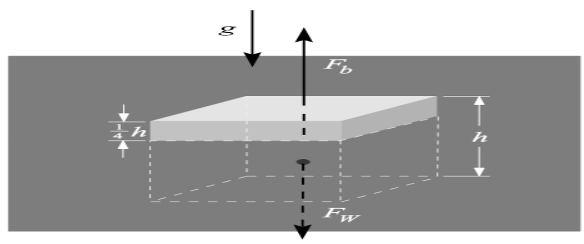


**Fig 1:** Application of Archimedes' principle to the case of a floating object.

From Archimedes' principle it follows directly that the buoyancy force must be

$$F_b = \rho_{water}\frac{3}{4}h^3 g\,,$$

And it must be pointing upward as indicated in the figure. The force due to the mass of the cube, i.e., the weight, is given by

$$F_W = -\rho_{cube}h^3 g\,,$$

With the minus sign indicating the downward direction of the force Now in static equilibrium (the cube floating as indicated), the forces acting on the cube must sum to zero. Hence,

$$F_b + F_W = 0 = \rho_{water}\frac{3}{4}h^3 g - \rho_{cube}h^3 g\,,$$

Or

$$\rho_{cube} = \frac{3}{4}\rho_{water}\,,$$

**1- Bernoulli's Equation**

Bernoulli's equation is one of the best-known and widely-used equations of elementary fluid mechanics [14]. In the present section we will derive this equation from the N.–S. equations again emphasizing the ease with which numerous seemingly scattered results can be obtained once these equations are available [10-12]. Following this derivation we will consider two main examples to highlight applications.

**2. Continuous problem**

❖ **Stokes problem**

Let $\varphi \in \boldsymbol{H}^{1/2}(\partial\Omega)$, $\int_{\partial\Omega} \varphi\cdot\boldsymbol{n}\,\mathrm{d}s = 0$, $\boldsymbol{g} \in \boldsymbol{H}^1(\Omega)$, $\operatorname{div}\boldsymbol{g} = 0$ in $\Omega$, $\boldsymbol{g}|_{\partial\Omega} = \varphi$, $\boldsymbol{f} \in \boldsymbol{L}^2(\Omega), \boldsymbol{u}^* \in \boldsymbol{H}^1(\Omega)$, $\boldsymbol{u}^*|_{\partial\Omega} = \varphi$ and consider the Stokes problem to find $\boldsymbol{u}$ such that

$$\boldsymbol{u} - \boldsymbol{g} \in \boldsymbol{V},$$

$$\nu((\boldsymbol{u},\boldsymbol{v})) = (\boldsymbol{f},\boldsymbol{v}) \quad \forall \boldsymbol{v} \in \boldsymbol{V}$$

This can be reformulated with the aid of the pressure: Find u, p such that

$$\boldsymbol{u} - \boldsymbol{u}^* \in \boldsymbol{H}_0^1(\Omega), \quad p \in L_0^2 = \left\{q \in L^2(\Omega);\ \int_\Omega q\,\mathrm{d}x = 0\right\},$$

$$\nu((\boldsymbol{u},\boldsymbol{v})) - (p, \operatorname{div}\boldsymbol{v}) = (\boldsymbol{f},\boldsymbol{v}) \quad \forall \boldsymbol{v} \in \boldsymbol{H}_0^1(\Omega),$$

$$-(q, \operatorname{div}\boldsymbol{u}) = 0 \qquad \forall q \in L_0^2(\Omega).$$

**Exercise-1**

Let $\boldsymbol{u} \in \boldsymbol{H}^1(\Omega)$, $\boldsymbol{u}|_{\partial\Omega} = \varphi$, $\int_{\partial\Omega} \varphi\cdot\boldsymbol{n}\,\mathrm{d}S = 0$. Prove that $\operatorname{div}\boldsymbol{u} = 0$

$$— \ (q, \operatorname{div}\boldsymbol{u}) = 0 \quad \forall q \in L^2(\Omega) \quad (+) \quad \Leftrightarrow$$

$$— \ (q, \operatorname{div}\boldsymbol{u}) = 0 \quad \forall q \in L_0^2(\Omega). \quad (*)$$

**Proof** The implication ⇒ is obvious.

The implication $\Leftarrow$: Let (*) hold. Then we can write

$$q \in L^2(\Omega) \to \tilde{q} = q - \frac{1}{|\Omega|}\int_\Omega q\,\mathrm{d}x \in L_0^2(\Omega) \Rightarrow$$

$$\Rightarrow 0 = (\tilde{q}, \operatorname{div}\boldsymbol{u}) = (q, \operatorname{div}\boldsymbol{u}) - \frac{1}{|\Omega|}\int_\Omega q\,\mathrm{d}x(1, \operatorname{div}\boldsymbol{u})$$

$$(1, \operatorname{div}\boldsymbol{u}) = \int_\Omega \operatorname{div}\boldsymbol{u}\,\mathrm{d}x = \int_{\partial\Omega} \boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S = \int_{\partial\Omega} \varphi\cdot\boldsymbol{n}\,\mathrm{d}S = 0$$

$(+) \ \Rightarrow \ \operatorname{div}\boldsymbol{u} = 0$ — clear.

❖ **Discrete problem**

For simplicity we assume that $N = 2$, $\Omega$ is a polygonal domain, $\mathcal{T}_h$ is a tri- angulation of Ω with standard properties. This means that $K \in \mathcal{T}_h$ are closed triangles,

$$\overline{\Omega} = \cup K \in \mathcal{T}_h K$$

Over $T_h$ we construct finite dimensional spaces and consider approximations

$$X_h \approx H^1(\Omega), \quad X_{h0} \approx H_0^1(\Omega), \quad \boldsymbol{V}_h \approx \boldsymbol{V},$$
$$\boldsymbol{X}_h \approx \boldsymbol{H}^1(\Omega), \quad \boldsymbol{X}_{h0} \approx \boldsymbol{H}_0^1(\Omega)$$
$$M_h \approx L^2(\Omega), \quad M_{h0} \approx L_0^2(\Omega),$$
$$\boldsymbol{X}_h, \boldsymbol{V}_h, \ldots \subset \boldsymbol{L}^2(\Omega), \ M_h \subset L^2(\Omega), M_{h0} \subset L_0^2(\Omega),$$
$$((\cdot,\cdot))_h \approx ((\cdot,\cdot)), \quad |||\cdot|||_h \approx |||\cdot|||, \quad \boldsymbol{g}_h \approx \boldsymbol{g}, \quad \boldsymbol{u}_h^* \approx \boldsymbol{u}^*,$$
$$\operatorname{div}_h \approx \operatorname{div}, b_h(\cdot,\cdot,\cdot) \approx b(\cdot,\cdot,\cdot)$$

**3. Choice of the finite element spaces**

We define $\boldsymbol{X}_h, M_h$ as spaces of piecewise polynomial functions. However, cannot be chosen in an arbitrary way.

**Example**

$$X_h = \{v_h \in C(\overline{\Omega}); v_h|K \in P^1(K) \ \forall K \in \mathcal{T}_h\},$$
$$X_{h0} = \{v_h \in X_h; v_h|_{\partial\Omega} = 0\}$$
$$\boldsymbol{X}_h = X_h \times X_h$$
$$\boldsymbol{X}_{h0} = X_{h0} \times X_{h0}$$
$$\boldsymbol{V}_h = \{v_h \in \boldsymbol{X}_{h0}; \operatorname{div}(v_h|_K) = 0 \ \forall K \in \mathcal{T}_h\}$$

This is in agreement with the above definition of $V_h$, provided

$$M_h = \{q \in L^2(\Omega); q \ \text{ is constant on each } K \in \mathcal{T}_h\},$$
$$M_{h0} = \left\{q \in M_h; \int_\Omega q \, dx = 0\right\}.$$

In this case we have

$$((\cdot,\cdot))_h = ((\cdot,\cdot)), \quad |||\cdot|||_h = |||\cdot|||$$
$$b_h = b, \quad \operatorname{div}_h = \operatorname{div}$$

Let $\Omega = (0,1)^2$, $\boldsymbol{u}_h \in \boldsymbol{V}_h$. Then

$$\boldsymbol{u}_h|_{\partial\Omega} = 0, \quad \operatorname{div}(\boldsymbol{v}_h|_K) = 0 \quad \forall K \in \mathcal{T}_h.$$

**4. Existence of an approximate solution:**

Now we shall be concerned with the existence of a solution to the discrete Stokes problem [13]. For simplicity we consider the case with zero boundary condition.

**Theorem**

*1.* $((\boldsymbol{v}_h, \boldsymbol{v}_h))_h^{1/2} = |||\boldsymbol{v}_h|||_h$ *is a norm in* $\boldsymbol{X}_{h0}$,

*2. BB condition holds:*

$$\sup_{\boldsymbol{v}_h \in \boldsymbol{X}_{h0}} \frac{(q_h, \operatorname{div}_h \boldsymbol{v}_h)}{|||\boldsymbol{v}_h|||_h} \geq \gamma \|q_h\| \quad \forall q_h \in M_{h0}$$

*Then the discrete Stokes problem has a unique solution* $\boldsymbol{u}_h, p_h$.

**Proof** We want to find $\boldsymbol{u}_h \in \boldsymbol{X}_{h0}$, $p_h \in M_{h0}$ satisfying

$$(+) \qquad \nu((\boldsymbol{u}_h, \boldsymbol{v}_h))_h - (p_h, \operatorname{div}_h \boldsymbol{v}_h) = (\boldsymbol{f}, \boldsymbol{v}_h) \quad \forall \boldsymbol{v}_h \in \boldsymbol{X}_{h0}$$
$$(*) \qquad -q(h, \operatorname{div}_h \boldsymbol{u}_h) = 0 \quad \forall q_h \in M_{h0}.$$

We define

$$\boldsymbol{V}_h = \{\boldsymbol{v}_h \in \boldsymbol{X}_{h0}; (q_h, \operatorname{div}_h \boldsymbol{v}_h) = 0 \ \forall q_h \in M_{h0}\}$$

Is a continuous linear functional on $\boldsymbol{X}_{h0}$ and there exists $\tilde{B}q_h \in \boldsymbol{X}_{h0}$ such that

$$((\tilde{B}q_h, \boldsymbol{v}_h))_h = (q_h, \operatorname{div}_h \boldsymbol{v}_h) \quad \forall \boldsymbol{v}_h \in \boldsymbol{X}_{h0}.$$

By the BB condition,

$$\sup_{0 \neq \boldsymbol{v}_h \in \boldsymbol{X}_{h0}} \frac{((\tilde{B} q_h, \boldsymbol{v}_h))_h}{|||\boldsymbol{v}_h|||_h} \geq \gamma \|q_h\| \quad \forall\, q_h \in M_{h0}.$$

We see that

$$\tilde{B}: \; M_{h0} \to \boldsymbol{X}_{h0} \quad \text{is a linear operator and}$$

$$|||\tilde{B} q_h|||_h \geq \gamma \|q_h\| \quad \forall\, q_h \in M_{h0}.$$

for $\tilde{F}$ there exists a unique $p_h \in M_{h0}$ such that $\tilde{B} p_h = \tilde{F}$. This is equivalent to the relation

$$((\tilde{B} p_h, \boldsymbol{v}_h))_h = ((\tilde{F}, \boldsymbol{v}_h))_h \quad \forall \boldsymbol{v}_h \in \boldsymbol{X}_{h0},$$

which means that

$$(p_h, \operatorname{div}_h \boldsymbol{v}_h) = -F(\boldsymbol{v}_h) = +\nu((\boldsymbol{u}_h, \boldsymbol{v}_h))_h - (\boldsymbol{f}, \boldsymbol{v}_h) \quad \forall\, \boldsymbol{v}_h \in \boldsymbol{X}_{h0},$$

and, thus,

$$\nu((\boldsymbol{u}_h, \boldsymbol{v}_h))_h - (p_h, \operatorname{div}_h \boldsymbol{v}_h) = (\boldsymbol{f}, \boldsymbol{v}_h) \quad \forall\, \boldsymbol{v}_h \in \boldsymbol{X}_{h0},$$

Which we wanted to prove

**Conclusion**

The aim of this paper is to furnish some results in very different areas that are linked by the common scope of giving new insight in the field of fluid dynamics. The study of these flows has been attached with a wide range of mathematical techniques and, today, this is a stimulating part of both pure an applied mathematics. Fluid-mechanics is an “ancient science” that is incredibly alive today. The modern technologies require a deeper understanding of the behavior of real fluids; on the other hand new discoveries often pose new challenging mathematical problems. The basic characteristic of the fluid is that it can flow. Fluids are divided in two categories, incompressible fluids (fluids that move at far subsonic speeds and do not change their density) and compressible fluids The aim of this paper is to furnish some results in very different areas that are linked by the common scope of giving new insight in the field of fluid dynamics.

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