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Transformation technique to solve multi-objective linear fractional programming problem

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Abstract
In this paper we studied different techniques on technique to solve multi-objective linear fractional programming problem. In mathematical optimization, linear-fractional programming (LFP) is a generalization of linear programming (LP). As the objective functions in linear programs are linear functions, the objective function in a linear-fractional program is a ratio of two linear functions. A linear program can be regarded as a special case of a linear-fractional program in which the denominator is the constant function one. Multi-objective linear fractional programming (MOLFP) technique is a very useful decision-making tool for modeling problems with more than one objective such as profit/cost, actual cost/standard cost, debt/equity, inventory/sales, etc. Linear fractional programming problem (LFPP) deals with problems in which objective function is a ratio of two linear functions.

Keywords: Linear fractional, problem, programming, techniques, Transformation, Multi-Objective.

Introduction
Linear fractional problems may be found in different fields such as data development analysis, tax programming, risk and portfolio theory, logistic and location theory [7, 6, 5, 4, 3]. Also, linear fractional programming is used to achieve the highest ratio of outcome to cost, the ratio representing the highest efficiency.
Several methods for solving linear fractional program by transforming it to an equivalent linear program considered updated objective functions method to solve linear fractional program by solving a sequence of linear programs whereas Dinkelbach [9] used parametric approach to solve a linear fractional programming problems. Later on, several authors extended the approach to solve fractional programming problems, *e.g.*, generalized fractional programming problems [10, 11] and the minimum spanning tree with sum of ratios problems [12]. The parametric approach of Dinkelbach [9, 8] to solve sum of ratios problems the approach in [13] does not always lead to appropriate solutions and they extended the parametric approach of Dinkelbach to solve sum of ratios, product of ratios and product of linear functions in [14]. Tammer *et al.,* [15] considered Dinkelbach approach to solve Multi-objective linear fractional programming problems by estimating the parameters. However, their approach does not necessarily guarantee an efficient solution. Gomes *et al.,* [1] focused on Multi-objective linear programming problem having weights established some optimality conditions.

Review of literature
During recent years, complexity of problems arising in different fields prompted researchers to develop efficient algorithms to solve linear fractional programs [2]. suggested an iterative parametric approach for solving Multi-objective linear fractional programming (MOLFP) problems. Cambini *et al.,* [3] reviewed methods for solving objective linear fractional programming. Recently, [11] suggested an iterative method based on conjugate gradient projection method for solving linear fractional programming problem.
A linear fractional programming problem occurs when a function is minimized or maximize and the objective function is ratio (numerator and denominator) and the constraints are linear type function. Consider a linear fractional programming problem:

$$Max/Min(Z) = \frac{c^T x+\alpha}{d^T x+\beta}$$
$$s.t. \qquad Ax \le b$$
$$x \ge 0$$

This problem can be applied to find an optimal productivity solution, minimizing or maximizing the ratio between storage cost and production cost under the storage constraints, Z=the total ratio cost between storage cost and production cost,

c^T= storage cost per unit,

d^T=production cost per unit,

α and β are constant.

Transformation of Objective:

$$Max(Z) = \frac{c^T x+\alpha}{d^T x+\beta}$$
$$\beta$$

Multiplying on both numerator and denominator we have,

$$Max(Z) = \frac{c^T x\beta+\alpha.\beta}{\beta(d^T x+\beta)}$$
$$= \frac{c^T x\beta+d^T x\alpha-d^T x\alpha+\alpha\beta}{\beta(d^T x+\beta)}$$
$$= (c^T - d^T.\frac{\alpha}{\beta}).\frac{x}{d^T x+\beta} + \frac{\alpha}{\beta}$$
$$Max(Z) = P^T y + \frac{\alpha}{\beta}$$
where $P^T = (c^T - d^T.\frac{\alpha}{\beta})$,$y = \frac{x}{d^T x+\beta}$,$g = \frac{\alpha}{\beta}$
$$Max(Z) = P^T y + g$$

Transformation of constraints:

$$(Ax-b) \le 0,$$
$$= \quad \frac{\beta(Ax-b)}{\beta(d^T x+\beta)} \le 0,$$
$$= \quad \frac{A\ x\beta-b\ \beta}{\beta\ (d^T x+\beta)} \le 0,$$
$$= \quad \frac{A\ x\ \beta+b\ d^T x-b\ d^T x-b\ \beta}{\beta(d^T x+\beta)} \le 0,$$
$$= \quad (A+d^T.\frac{b}{\beta}).\frac{x}{d^T x+\beta} \le 0,$$
$$= \quad Gy \le h.$$
where $G = (A+d^T.\frac{b}{\beta})$,$h = \frac{b}{\beta}$

Now consider the linear programming problem from the above transformation of objective and transformation of constraints we have,

$$MaxF(x) = P^T y + g$$
$$s.t \quad Gy \le h$$
$$y \ge 0.$$

where $g = \frac{\alpha}{\beta}.y$is the variable and F(x) is the optimal value.

Example

Consider the following LFP problem:

$$MaxF(x) = \frac{x_1+3}{x_2+1}$$
$$s.t. \qquad -x_1+x_2 \le 1$$
$$2x_1 \le 3$$
$$3x_1+2x_2 \le 7$$
$$x_1, x_2 \ge 0$$

First we solve the LFP problem by using our proposed technique to its corresponding linear programming problem,

$$MaxF(x) = y_1 - 2y_2 + 3$$
$$s.t. \qquad -y_1+2y_2 \le 1$$
$$2y_1 \le 3$$
$$3y_1+9y_2 \le 7$$
$$y_1, y_2 \ge 0$$

Now this problem is our linear programming problem and solved by regular simplex method. We find the variable is $\frac{3}{2}$, 0 and the optimal solution for the relaxed linear programming with optimal value F(x) = 9/2.This result is same as the result of [12]. The method is very useful because of his calculations involved are very simple and take least time as compare as other method for solving linear fractional programming problem. We also solved this problem by LINGO software and find objective solutions is same as our proposed method result.

Conclusion

We hope that the proposed FGP approach can be applied in the field of multi-level applied in the field of multi-level optimization such as de centralized MO-MLFPP, integer MO-MLFPP, MO- MLFPP, and decentralized MO-MLFPP with fuzzy parameters for practical decision making problems. In this study, we present a transformation method for solving linear fractional programming problem when the objective function is ratio function and the set of constraints is in the form of linear inequality. Our proposed method based upon transformation technique. Our new method can be applied to any linear fractional programming problem, since it is a special thing of the mathematical program. In line with this, we discussed FGP models for dealing with MLFPP to seek compromise optimal solution. The pro-posed approach is also extended to solve MO-MLFPP.

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