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A few results on linear space with the notion of semi convex set

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Abstract
In this work we have establish a few results on linear space with the notion of semi convex set.

Keywords: Linear space, convex set

Introduction
In this present paper we have establish a few results extending the notion of semi convex set introduced by Sharan, [1].

Definition: For definition we refer to Sharan, [1]. However we give below some of the definition to serve as ready reference.

Semi Convex Set: A non-empty subset C of linear space E is said to be semi convex set if for x,y Є C, $\alpha,\beta \geq 0$, $\alpha x+\beta y$ is in C for $\alpha+\beta \leq 1$.
Clearly every semi convex set is a convex set. The converse may or may not be.

Zero Space: A linear space may consist solely of the vector 0 (zero) with scalar multiplication defined by $\alpha \cdot 0 = 0$for every α we call this linear space at zero space and we always denoted it by {0}.
Clearly every zero space is semi convex set.

In this section we establish a few results using the definitions given in the section 2.

Theorem (3.1): Any finite linear combination of semi convex set is again a semi convex set.
Proof: Let $A_1, A_2, A_3 \dots A_n$be n semi convex sets.
Also let $a_1, a_2, a_3 \dots a_n$ be a scalars.
Then our problem is to show that
$a_1A_1 + a_2A_2 + a_3A_3 + \dots + a_nA_n$is also a semi convex set.
For, let z_1, z_2Є $a_1A_1 + a_2A_2 + a_3A_3 + \dots + a_nA_n$then
$Z_1 = a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n$
Where x_i Є A_i for I = 1, 2, 3, …., n
Similarly $z_2 = a_1x'_1 + a_2x'_2 + a_3x'_3 + \dots + a_nx'_n$
wherex'_iЄ A_i for i = 1,2,3,…,n
Again let, $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$
Then$\alpha z_1 + \beta z_2 = \alpha (a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n) + \beta(a_1x'_1 + a_2x'_2 + a_3x'_3 + \dots + a_nx'_n)$
$= (\alpha x_1 + \beta x'_1)a_1 + (\alpha x_2 + \beta x'_2)a_2 + \dots + (\alpha x_n + \beta x'_n)a_n$
Since $x_i$andx'_iЄ A_i andit is supposed already that $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$.
Also A_i is semi convex set for each i.
Thus$\alpha x_i + \beta x'_I$Є A_i
Hence$\alpha z_1 + \beta z_2 = (a_1A_1 + a_2A_2 + a_3A_3 + \dots + a_nA_n)$
Therefore $a_1A_1 + a_2A_2 + a_3A_3 + \dots + a_nA_n$is also semi convex set.

Theorem (3.11): Let M be a semi convex set in a linear space E. if $x_1, x_2, x_3, \ldots, x_n \in M$ then all elements of the form $\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \ldots + \alpha_n x_n \in M$
Where all $\alpha_i \geq 0$ and $\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_n \leq 1$.

Proof: We prove the theorem by the method of induction.
If n=2, the theorem holds because M is a semi convex set thus we assume that the theorem holds good for (n=1) then it is sufficient to show that thetheorem holds good for also,
For, we may assume that $\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_{n-1} > 0$
Because then $\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_{n-1} = 0$ each of $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_{n-1}$ is zero and hence $\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_{n-1} x_{n-1} + \alpha_n x_n = \alpha_n x_n = x_n \in M$ and which proves the results
Thus we suppose that

$\lambda = \alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_{n-1} > 0$

Then $\alpha_1/\lambda + \alpha_2/\lambda + \alpha_3/\lambda + \ldots + \lambda_{n-1}/\lambda = 1$

Thus by the method of the assumption of the induction

$y = (\alpha_1/\lambda)x_1 + (\alpha_2/\lambda)x_2 + (\alpha_3/\lambda)x_3 + \ldots + (\alpha_{n-1}/\lambda)x_{n-1} \in M$

Thus $\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n$

$= \lambda((\alpha_1/\lambda)x_1 + (\alpha_2/\lambda)x_2 + (\alpha_3/\lambda)x_3 + \ldots + (\alpha_{n-1}/\lambda)x_{n-1}) + \alpha_n x_n$

$= \lambda y + \alpha_n x_n \in M$

But M is supposed to be a semi convex set. Thus $y, x_n \in M$ and $\lambda + \alpha_n + \leq |$
Hence the theorem is established.

Reference

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