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On ternary cubic Diophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3$

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Abstract
The non-homogeneous cubic equation with three unknowns represented by the Diophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3$ is analyzed for its patterns of non – zero integral solutions. A few interesting properties among the solutions are presented.

Keywords: Cubic Equation with Three Unknowns, Integral solutions
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Introduction

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-20] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3$ representing non- homogeneous cubic equation with three unknowns for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

Method of Analysis

Consider the equation

$$3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3 \quad (1)$$

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To start with, it is seen that (1) is satisfied by

$$(\pm 18w^3 - 1, \pm 18w^3 - 1, 3w^2)$$

However, we have other sets of integer solutions to (1) which are illustrated below:
The substitution of the linear transformation,

$$x = u + v; y = u - v \quad (u \neq v \neq 0) \quad (2)$$

In (1) leads to

$$p^2 + 11v^2 = 12z^3 \quad (3)$$

where $u + 1 = p$

The above equation is solved through different methods and using (2), different sets of non zero distinct integer solutions to (1) are obtained.

Set 1
Write 12 as

$$12 = (1+i\sqrt{11})(1-i\sqrt{11}) \quad (4)$$

Assume

$$z = a^2 + 11b^2 \quad (5)$$

where a and b are non zero integers
Using (4) and (5) in (3) and employing the method of factorization, define

$$p + i\sqrt{11}v = (1+i\sqrt{11})(a+i\sqrt{11}b)^3 \quad (6)$$

Equating real and imaginary parts, we have

$$p = u+1 = a^3 - 33ab^2 - 33a^2b + 121b^3$$
$$v = a^3 + 3a^2b - 33ab^2 - 11b^3$$

Substituting the values of u and v in (2), the values of x and y are given by

$$x = x(a,b) = 2a^3 - 66ab^2 - 30a^2b + 110b^3 - 1$$
$$y = y(a,b) = -36a^2b + 132b^3 - 1 \quad (7)$$

Thus (5) and (7) represent non zero distinct integral solutions of (1) in two parameters.

Properties

- $36[6x(a,a) - 5y(a,a) + 1]$ is a cubical integer
- $3z(a,a)$ is a perfect square
- $2z[a,a]$ is a Nasty Number
- $4[x(a,a)+1]$ is a cubical integer
- $6x(1,a) - 5y(1,a) + 396t_{4,a} = 1$
- $y(a,1) + 36t_{4,a} = 131$
- $x(1,b) + y(1,b) - 66so_a - 220p_a^5 + 176t_{4,a} = 0$

Set 2
Consider (3) as

$$p^2 + 11v^2 = 12z^3 * 1 \quad (8)$$

write 12 as

$$12 = (1+i\sqrt{11})(1-i\sqrt{11}) \quad (9)$$

Also, $$1 = \frac{(1+i3\sqrt{11})(1-i3\sqrt{11})}{100} \quad (10)$$

where a and b are non zero integers.
Using (5), (9), (10) in (8) and employing the method of factorization, define

$$(p + i\sqrt{11}v) = (1+i\sqrt{11})(a+i\sqrt{11}b)^3\left(\frac{1+i3\sqrt{11}}{10}\right)$$

Equating real and imaginary parts, we have

$$p = \frac{1}{10}\left[-32a^3 + 1056ab^2 + 484b^3 - 132a^2b\right]$$
$$v = \frac{1}{10}\left[-96a^2b + 352b^3 + 4a^3 - 132ab^2\right]$$

As our interest is on finding integer solutions, choose a and b so that p and v are integers. Replacing a by 10a, b by 10b we have

$$p = u+1 = -3200a^3 + 105600ab^2 + 48400b^3 - 13200a^2b$$
$$v = 400a^3 - 13200ab^2 - 9600a^2b + 35200b^3$$

Also, $$z = 100a^2 + 1100b^2 \quad (11)$$

Substituting the values of u and v in (2) we have

$$x = x(a,b) = -2800a^3 + 92400ab^2 + 83600b^3 - 22800a^2b - 1$$
$$y = y(a,b) = -3600a^3 + 118800ab^2 + 13200b^3 - 3600a^2b - 1 \quad (12)$$

Thus (11) and (12) represent non zero distinct integral solutions of (1) in two parameters.

Properties

- $x(a,1) + y(a,1) + 12800p_a^5 + 20000t_{4,a} \equiv 96798 \pmod{211200}$
- $x(a,1) + 28az(a,1) + 22800t_{4,a} \equiv 83599 \pmod{123200}$
- $2z(a,a) isaNastyNumber$
- $x(1,b) - y(1,b) - 140800p_a^5 + 96800t_{4,a} \equiv 800 \pmod{19200}$
- $44180[x(a,a)+1]$ is a cubical integer
- $y(a,1) + 7200p_a^5 \equiv 13199 \mod(118800)$

Set 3
Consider 1 as

$$1 = \frac{(-1+i3\sqrt{11})(-1-i3\sqrt{11})}{100} \quad (13)$$

where a and b are non zero integers.
Using (9), (13), (5) in (8) and employing the method of factorization, define

$$p+i\sqrt{11}v=(1+i\sqrt{11})(a+i\sqrt{11}b)^3\left(\frac{-1+i3\sqrt{11}}{10}\right)$$

Equating real and imaginary parts, we've

$$p=\frac{1}{10}[-34a^3+1122ab^2-66a^2b+242b^3]$$
$$v=\frac{1}{10}[2a^3-102a^2b-66ab^2+374b^3]$$

As our interest is on finding integer solutions, choose a and b so that p and v are integer.
Replacing a by 10a, b by 10b, we've

$$p=u+1=-3400a^3+112200ab^2-6600a^2b+24200b^3$$
$$v=200a^3-10200a^2b-6600ab^2+37400b^3$$

Also, $z=100(a^2+11b^2)$ (14)

Substituting the values of u and v in (2) we've

$$x=x(a,b)=-3200a^3+105600ab^2-16800a^2b+61600b^3-1$$
$$y=y(a,b)=-3600a^3+118800ab^2+3600a^2b-13200b^3-1$$ (15)

Thus (14) and (15) represent the nonzero distinct integral solutions of (1) in two parameters.

Properties

- $9x(a,1)-8y(a,1)+180000t_{4,a}=659999$
- $5290[x(a,a)+1]$ is a cubical integer
- $21780[y(a,a)+1]$ is a cubical integer
- $3z(a,a)$ is a perfect square
- $y(a,1)+7200p_a^5-7200t_{4,a}\equiv-13201(\mathrm{mod}\,118800)$
- $y(a,1)-z(a,1)+7200p_a^5-7100t_{4,a}\equiv(\mathrm{mod}\,118800)$
- $y(a,1)+z(a,1)+7200p_a^5-7300t_{4,a}\equiv-12101(\mathrm{mod}\,118800)$

Set 4

Also, 1 is represented as

$$1=\frac{(-5+i\sqrt{11})(-5-i\sqrt{11})}{36}$$ (16)

where a and b are nonzero integers.
Using (9), (16), (5) in (8) and employing the method of factorization, define

$$(p+i\sqrt{11}v)=(1+i\sqrt{11})(a+i\sqrt{11}b)^3\left(\frac{-5+i\sqrt{11}}{6}\right)$$

Equating real and imaginary parts we've

$$p=\frac{1}{6}[-16a^3+528ab^2+132a^2b-484b^3]$$
$$v=\frac{1}{6}[-4a^3-48a^2b+132ab^2+176b^3]$$

As our interest is on finding integer solutions, choose a and b so that p and v are integers. Replacing a by 3a, b by 3b, we have

$$p=u+1=-72a^3+2376ab^2+594a^2b-2178b^3$$
$$v=-18a^3-216a^2b+594ab^2+792b^3$$

Also, $z=9(a^2+11b^2)$ (17)

Substituting the values of u and v in (2) we have

$$x=x(a,b)=-90a^3+2970ab^2+378a^2b-1386b^3-1$$
$$y=y(a,b)=-54a^3+1782ab^2+810a^2b-2970b^3-1$$ (18)

Thus (17) and (18) represent the integral solutions of (1) in two parameters.

Properties

- $x(a,1)+10az(a,1)-378t_{4,a}\equiv-1387(\mathrm{mod}\,3960)$
- $y(a,1)+6az(a,1)-810pr_a\equiv-2971(\mathrm{mod}\,1566)$
- $2028[x(a,a)+1]$ is a cubical integer
- $4[y(a,a)+1]$ is a cubical integer
- $3z[a,a]$ is a perfect square
- $x(a,1)+y(a,1)+288p_a^5-1332t_{4,a}\equiv-4358(\mathrm{mod}\,4752)$
- $x(a,1)+z(a,1)+180p_a^5-477t_{4,a}\equiv-1288(\mathrm{mod}\,2970)$

Conclusion

To conclude, one may search for other patterns of solutions and their corresponding properties.

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