ISSN Print: 2394-7500
ISSN Online: 2394-5869
Impact Factor: 5.2
IJAR 2015; 1(8): 318-323
www.allresearchjournal.com
Received: 01-05-2015
Accepted: 02-06-2015

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A Non-Newtonian mathematical model on the two phase Hepatic Systolic blood flow in Hepatic arterioles with special reference to Hepatitis B.

Om Prakash, V. Upadhyay, A.K. Agrawal and P.N. Pandey

Abstract
In this paper, we have presented a model of two phased Systolic blood flow in Hepatic arterioles remote from the heart and proximate to the Liver, Keeping in view the nature of Hepatic blood circulation in human body. The viscosity increases in the arterioles due to formation of rouleaux along axis of red blood cells. P.N. Pandey and V. Upadhyay have considered that the blood flow has two phased, one of which that of red blood cells and other is plasma. They have also applied the Herschel Bulkley Non-Newtonian model in Bio- fluid mechanical setup. We have collected a clinical data in case of Hepatitis B for Hematocrit versus blood pressure. The graphical presentation for a particular parametric value is very close to the clinical observation. The overall presentation is in tensorial form and the solution techanic adopted is analytical as well as numerical. The role of hematocrit is explicit in the determination of blood pressure in case of Hepatic disease Hepatitis B the graphical presentation for particular parametric value is much close to the clinical observation.

Keywords: Structure of the Liver, Hematocrit, rouleaux-structure, Hepatic Blood Flow, Herschel Bulkley Non-Newtonian model.

1. Introduction (Description of Bio- Physical Problem)

The Liver is the largest gland in body. It is a reddish brown in the body with four lobes of unequal size and shape. These are right lobe, left lobe, quadrates and caudate lobe. A human Liver is normally weight 1.44-1.66 kg. Liver lobes are surrounded by a thick capsule, mostly overlaid with reflected periforinum [1].

The Liver has the most complicated circulation of any organ. According to the anatomical peculiarity of the double afferent blood supply of the liver, 75% - 80% of the blood entering the Liver is partially deoxygenated venous blood supplied by the portal vein, which collects all the blood that leaves the spleen, stomach, small and large intestine, gallbladder and pancreas [2-3].

The hepatic artery accounts for the remaining 25% with well-oxygenated blood. Total hepatic blood flow ranges between 800 and 1200 mL/min, which is equivalent to approximately 100 mL/min per 100 g liver wet weight [4]. Although the Liver mass constitutes only 2.5% of the total body weight, the Liver receives nearly 25% of the cardiac output. Liver volume and portal blood flow decreases after the age of 50 [5].

Liver is also prone to many diseases [6]. One of these is Hepatitis B. Hepatitis is irritation and swelling (Inflammation) of Liver due to infection with the Hepatitis B virus (HBV). In 1947 Mac Callum classified viral Hepatitis A & viral hepatitis B. Hepatitis B virus was discovered in 1965 by Dr. Baruch Blumberg. In 1981 The FDA approved sophisticated Plasma, Derived Hepatitis vaccine are human use. In 2001, FDA approved a combination Hepatitis A and Hepatitis B vaccine (Twinrix, Glaxo Smithkline). During 1990-2004, incidence of acute Hepatitis B in the United States declined 75%. The greatest decline (94%) occurred among children and adolescent. *12 March 2015 ¦ GENEVA* - WHO today issued its first-ever guidance for the treatment of chronic Hepatitis B, a viral infection which is spread through blood and body fluids, attacking the Liver and resulting in an estimated 650 000 deaths each year – most of them in low- and middle-income countries. Worldwide, some 240 million people have chronic Hepatitis B virus with the highest rates of infection in Africa and Asia. People with chronic Hepatitis B infection are at increased risk of dying from cirrhosis and Liver cancer. Many Cancer Patients at Risk for Hepatitis B Virus Reactivation [13]

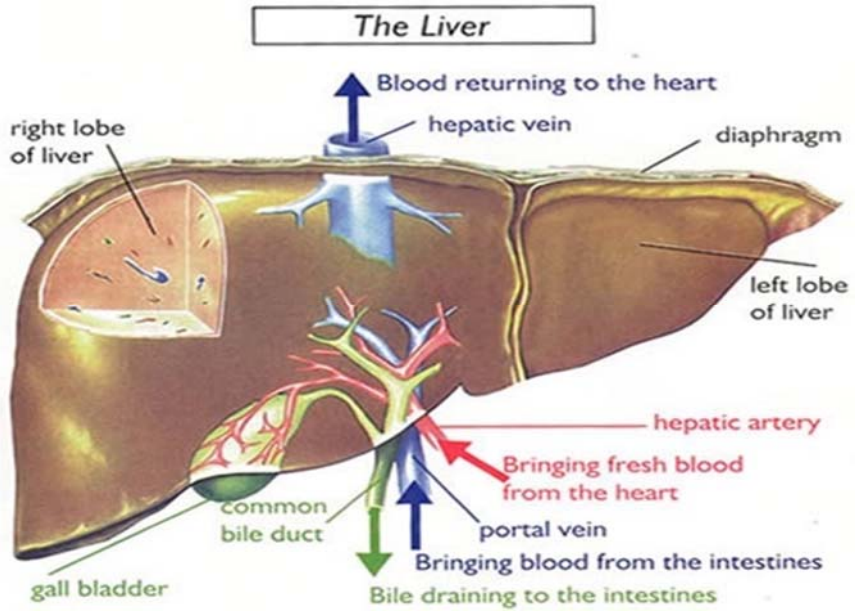


Fig: 1

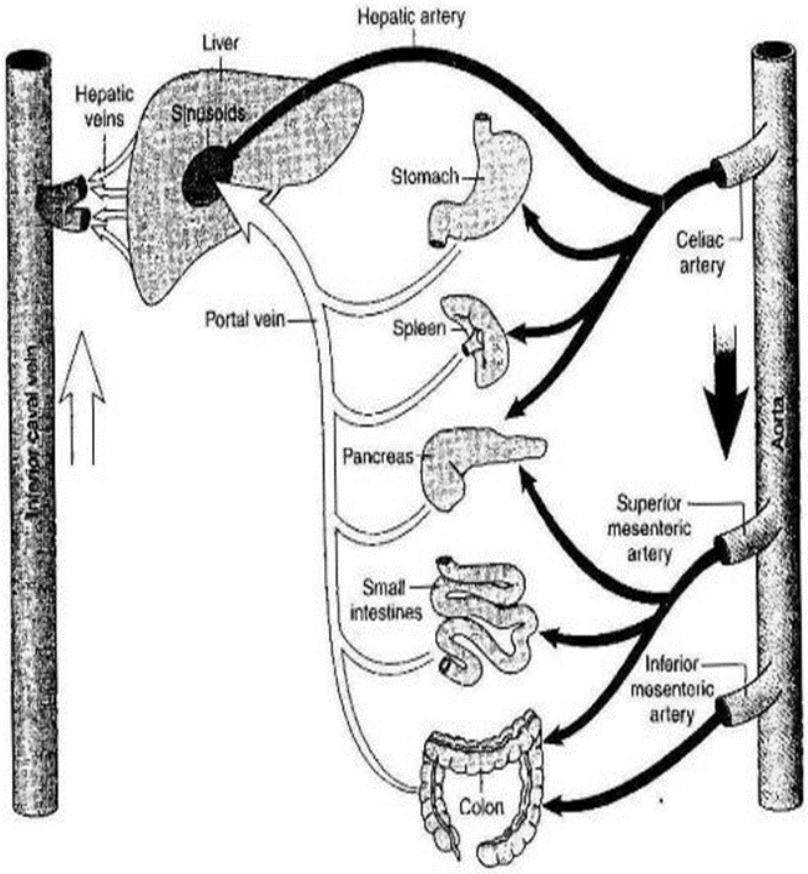


Fig 2: Hepatic Blood Flow

Blood is a complex fluid consisting of particular corpulse suspended in a non-Newtonian fluid. The particular solids are red blood cells (RBCs), white blood cells (WBCs) and platelets. The fluid is plasma, which itself is a complex mixture of proteins and other intergradient in an aqueous base. In blood 98% RBC and remaining 2% WBC and platelets cells i.e. there are a few part of the other cells. Which are ignorable, so one phase of the blood is plasma and 2nd phase of the blood is RBCs.

Two phase Hepatic blood flow is a study of measuring the blood pressure if hemoglobin known. The percentage of volume covered by blood cells in the whole blood is called hematocrit. Hematocrit is three times of hemoglobin concentration (reported as grams per deciliter) [7]. Liver

blood flow is controlled by mechanisms that are independent of extrinsic innervation or vasoactive agents that regulate (1) hepatic arterial inflow; (2) portal venous inflow; and (3) the interrelationship between hepatic arterial and portal venous inflow circuits [8].

This work will focus on two phase hepatic blood flow in arterioles with special reference to Hepatitis B. There are many work is available in that field, but P.N. Pandey and V. Upadhyay (2001) discussed a some phenomena in two phase blood flow gave an idea on the two phase hepatic blood flow in arterioles with a Liver disease Hepatitis B. The work of P.N. Pandey and V. Upadhyay in whole circulatory system but this work will focus on Hepatitic circulatory system, and Hepatitic circulatory system is a sub system of whole circulatory system. In this work, applied the Herschel Bulkley non-Newtonian model.

2. Basic Bio-fluid equation for two phase blood flow

Let us the problem of blood flow in hepatic circulatory system is different from the problems in cylindrical tube and select generalized three dimensional orthogonal curvilinear coordinate system. Briefly described as E^3 called as Euclidean space. According to mishra the biophysical laws thus expressed fully hold good in any co-ordinate system which is a compulsion for the truthfulness of the laws (1990) [9].

According to Sherman I.W. and Sherman V., G Blood is mixed fluid. Mainly there are two phases in blood. The first phase is plasma, while the other phase is that of blood cells are enclosed with a semi-permeable membrance whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma. Thus, blood can be considered as a homogeneous mixture of two phases (1989) [10].

(2.1) Equation of Continuity for two phase blood flow-

According to Singh P. and Upadhyay K.S. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells [11]. Let the volume portion covered by blood cells in unit volume be X, this X is replaced by H/100, where H is the Hematocrit the volume percentage of blood cells. Then the volume portion covered by the plasma will be 1-X. If the mass ratio of blood cells to plasma is r then clearly

$$\mathrm{r} = \frac{X\rho_c}{(1-X)\rho_p} \tag{2.1}$$

where ρ_c and $\rho_p$are densities of blood cells and blood blood plasma respectively. Usually this mass ratio is not a constant, even then this may be supposed to constant in present context (1986)

The both phase of blood, i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher has presented a model for this situation. According to this model, we consider the two phase of blood separately (1958). Hence equation of continuity for two phase according to the principle of conservation of mass defined by J.N and Gupta R.C. as follow

$$\frac{\partial(X\rho_c)}{\partial t} + (\mathrm{X}\rho_c v^i)_{,i} = 0 \tag{2.2}$$

and $$\frac{\partial(1-X)\rho_p}{\partial t} + ((1-\mathrm{X})\rho_p v^i)_{,i} = 0 \tag{2.3}$$

Where v is the common velocity of two phase blood cells and plasma.

Again $(\mathrm{X}\rho_{c\,V^i})_{,\mathrm{i}}$ is co-variant derivative of $(\mathrm{X}\rho_{c\,V^i})$ with respect to X^i . In the same way $(1-X)\rho_c V^i)_{,i}$ is co-variant derivative of $((1-\mathrm{X})\rho_c\, V^i)$ w.r.to X^i.

If we define the uniform density of the blood ρ_m as follow

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad [12] \tag{2.4}$$

Then equation (2.2) and (2.3) can be combined together as follow

$$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,i} = 0 \tag{2.5}$$

(2.2) Equation of Motion for two phase blood flow

According to Ruch, T.C. and H.D. The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973)[14]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for two phase of blood cells as follows:

$$X\rho_c \frac{\partial V^i}{\partial t} + (X\rho_c V^j)V^i_{,j} = -Xp_{,j}g^{ij} + X\eta_{\mathbf{c}}\left(g^{jk}V^i_{,k}\right)_{,j} \tag{2.6}$$

Similarly, taking the viscosity coefficient of plasma to be. The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p V^i\}V^i_{,j} = -(1-X)p_{,j}g^{ij} + (1-X)\eta_{\mathrm{c}}\left(g^{jk}v^i_{,k}\right)_{,j} \tag{2.7}$$

Now adding equation (2.6) and (2.7) and using relation (2.4), the equation of motion for blood flow with the both phases

$$\rho_m \frac{\partial V^i}{\partial t} + \left(\rho_m v^j\right)v^i_{,j} = -p_{,j}g^{ij} + \eta_{\mathrm{m}}\left(g^{jk}v^i_{,k}\right)_{,j} \tag{2.8}$$

Where $\eta_m = \mathrm{X}\,\eta_c + (1-X)\,\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

3. Mathematical Modeling

As the velocity of Blood flow decreases, the viscosity of blood increases. The velocity of blood deceases successively, because of the fact that arterioles, veinules and veins these vessels are relatively a far enough from the heart. Hence the pumping of the heart on these vessels is relatively low [15]. Secondly these vessels relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced.

The Herschel Bulkley law holds good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows:

$\mathrm{T'} = \eta_m e^n + T_p$ $(\mathrm{T'} \geq T_p)$ and $\mathrm{e} = 0$ $(\mathrm{T'} < T_p)$

Where, T_p is the yield stress.

When strain rate e = 0 (T'< T_p) a core region is formed which flows just like a plug. Let the radius of the plug be r_p. The stress acting on the surface of plug will be T_p. Equating the forces acting on the plug, we get

$$P\pi r_p^2 = T_p\, 2\pi r_p \quad \Rightarrow \quad r_p = 2\,\frac{T_p}{P} \qquad (3.1)$$

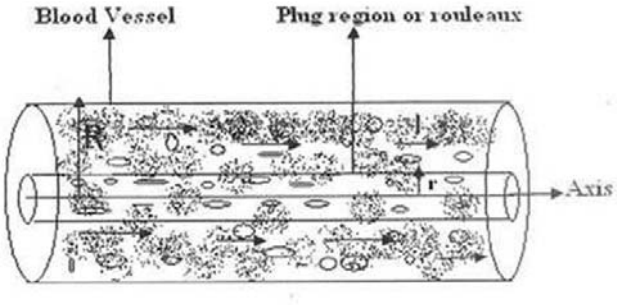


Fig (3): Herschel Bulkley blood flow

The Constitutive equation for test part of the blood vessel is

$$T' = \eta_m e^n + T_p \quad \text{or} \quad T' - T_p = \eta_m e^n = T_e$$

Where, T_e = effective Stress, whose generalized form will be as follows

$$T^{ij} = -Pg^{ij} + T_e^{ij} \quad \text{where,} \quad T_e^{ij} = \eta_m (e^{ij})^n \quad \text{while} \quad e^{ij} = g^{jk} V_k^i$$

Where, the symbols have their usual meanings.
Now we describe the basic equations for Herschel Bulkley blood flow as follows:

(3.1) Equation of Continuity-

$$\frac{1}{\sqrt{g}(\sqrt{g}V^i),i} = 0$$

(3.2) Equation of Motion-

$$\rho_m \frac{\partial vi}{\partial t} + \rho_m\, v^j v^i_{,j} = -T_{e,}{}^{ij}{}_{j} \qquad (3.2)$$

Where all the symbols have their usual meanings.

4. Solution & Discussion

Since, the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:

$$X^1 = r, \quad X^2 = \theta, \quad X^3 = z,$$

Matrix of metric tensor in cylindrical co-ordinates is $[g_{ij}]$ and matrix of conjugate metric tensor is $[g^{ij}]$ whereas the Christoffel's symbols of 2nd kind are as follows:

$\left\{ {1 \atop 2\ 2} \right\} = -r$, $\left\{ {1 \atop 2\ 2} \right\} = \left\{ {1 \atop 2\ 2} \right\} = \frac{1}{r}$ remaining others are zero.

The governing tensorial equations can be transformed into cylindrical forms which are as follows: the equation of Continuity-:

$$\frac{\partial v}{\partial z} = 0$$

The equation of Motion

r - Component: $-\frac{\partial p}{\partial z} = 0$, θ- component: 0 = 0

z - Component: $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial v_z}{\partial r}\right)^n\right]$

Here, this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r, v_z and p do not depend upon θ.

We get $v_z = v(r)$ dp = p(z) and

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r}\left[r\left(\frac{dv}{dr}\right)^n\right] \qquad (4.1)$$

Since, pressure gradient $-\frac{dp}{dz} = P$

$r\left(\frac{dv}{dz}\right)^n = -\frac{p\, r^2}{2\eta_m} +$ A, we apply boundary condition:
At r = 0, v = v_0. then A = 0.

Then $-\frac{dv}{dr} = \left(\frac{Pr}{2\,\eta_m}\right)^{\frac{1}{n}}$ Replace r from r– r_p, for non-plug region

$$\frac{dv}{dr} = -\left(\frac{\frac{1}{2}Pr - \frac{1}{2}Pr_p}{\eta_m}\right)^{\frac{1}{n}} \text{ i.e. } \frac{dv}{dr} = -\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(r - r_p\right)^{\frac{1}{n}} \qquad (4.2)$$

Integrating above equation (4.2) under the no slip boundary condition: v = 0 at r = R, so we get

$$v = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right] \qquad (4.3)$$

This is the formula for velocity of blood flow in arterioles, veinules and veins.
Putting r = r_p to get the velocity v_p of plug flow as follows

$$v_p = \frac{n}{n+1}\left(\frac{P}{2\,\eta_m}\right)^{\frac{1}{n}}(R - r_p)^{\frac{1}{n}+1} \qquad (4.4)$$

Where the value of r_p is taken from (2.7).

5. Result (Bio-physical interpretation)

Observations: Hematocrit Vs. Blood pressure is taken from Lala Lajpat Rai and Associated Hospital, Kanpur by Dr. Richa Giri.

Patient Name: - Mr. Sanjay Sharma, Annual No. 16527
Diagnosis: - Hepatitis B

Date	HB (in gm/dl)	B.P. (Systolic blood pressure in mmhg)	Hematocrit	B.P. (in pascal)
16/06/2013	4.0	80	12.0	10665.60
17/06/2013	4.5	108	13.5	14398.56
18/06/2013	5.8	90	17.4	11998.80
19/06/2013	6.0	100	18.0	13332.00
20/06/2013	6.8	94	20.4	12532.08
26/06/2013	6.0	100	18.0	13332.00

According to Berkow, Robert, The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as per deciliter) [16].

The flow flux of two phased blood flow in arterioles, veinules and veins is

$$Q = \int_0^{r_p} 2\pi r\, v_p\, dr + \int_{r_p}^{R} 2\pi r v dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} dr + \int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}[(R-r_p)^{\frac{1}{n}+1} - (r-r_p)^{\frac{1}{n}+1}]dr$$

Using (4.2) and (4.4), we get

$$Q = \frac{2\pi n}{(n+1)}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1}\left[\frac{r^2}{2}\right]_0^{r_p} + \frac{2\pi n}{(n+1)}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left[\frac{r^2}{2}(R-r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+2} + \frac{(r-r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]_{r_p}^{R}$$

$$= \frac{\pi n}{(n+1)}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3}\left[\frac{r_p^2}{R^2}\left(1-\frac{r_p^2}{R}\right)^{\frac{1}{n}+1} + \left(1+\frac{r_p}{R}\right)\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)} + \frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right] \quad (5.1)$$

Q = 1000 ml. /min = 0.1666 l. /sec. R = 1, $r_p = \frac{1}{3}$ [17]

According to Gustafson, Daniel R. (1980)

η_p = 0.0015 (Pascal-sec.) [18] According to Glenn Elert (2010)

η_m = 0.035 (Pascal-sec.)

H = 16.55, P = 12709.84

By using relation $\eta_m = \eta_c X + \eta_p (1-X)$

Where, $X = \frac{H}{100}$, we get

η_c = 0.2461, and again using same above relation, we get

η_m = 0.0015 + 0.002024 H

Substituting the values of r_p and R in equation (4.1), we get

$$Q = \frac{2}{27}\pi\left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

or, $$\frac{27\times Q}{2\pi} = \left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

or, $$0.071627 = (121046.095)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

Solved by Numerical method, we get n = −2.68851

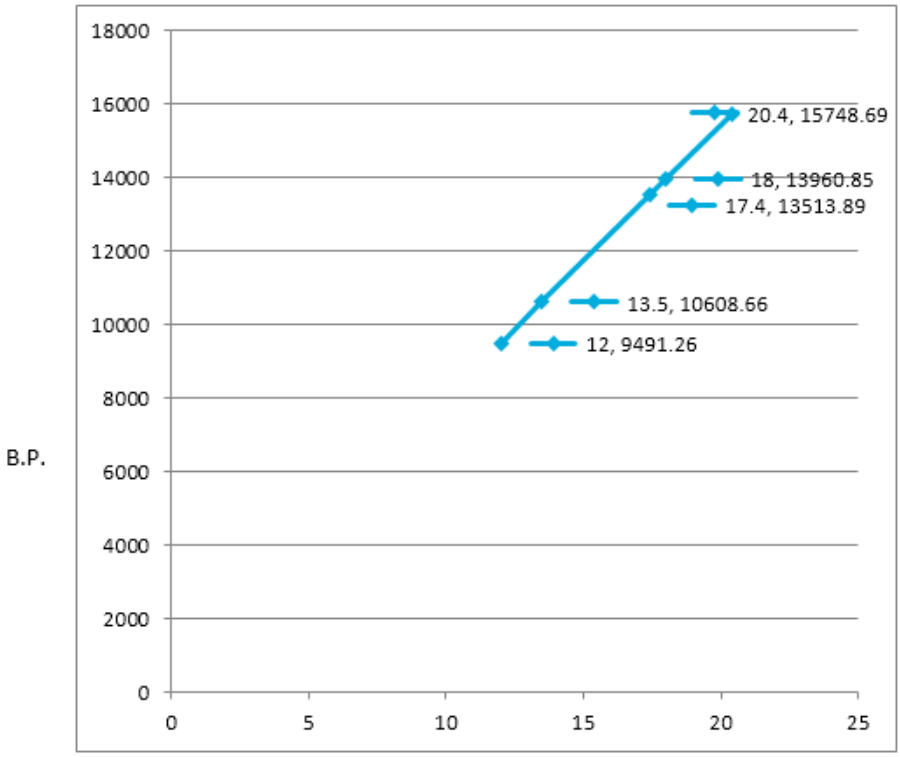
Now using equation (5.1) in equation (5.2), we get

$$P = (0.006072\, H + 0.0045)\left(\frac{27Q}{2\pi}\right)^n \times \left(\frac{6n^3+11n^2+6n+1}{26n^3+33n^2+9n}\right)^n$$

Substituting the values of Q and n, we get

$$P = 744.932\, H + 552.0741$$

H (Hematocrit)	12.0	13.5	17.4	18.0	20.4
BP(Blood Pressure)	9491.26	10608.66	13513.89	13960.85	15748.69



HEMATOCRIT

Graph: 1

6. Conclusion

A simple survey of the graph between blood pressure and hematocrit in Hepatitis B patient shows that when hematocrit increased then Blood pressure is also increased, That is the hematocrit is proportional to Systolic blood pressure.

7. Acknowledgement

I owe my sincere thanks Dr. Richa Giri, physician of Lala Lajpat Rai and Associated Hospital, Kanpur (U.P.).

Remark: If this would have been possible to get blood Pressure on the particular tissue (Liver) during operation of Hepatitis B patient then the relation between blood pressure and hemoglobin has been measured more accurately.

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